Preface

The concepts of independence for systems of events or for collections of random variables belong to the principal ones in the Probability Theory. There are numerous beautiful results established for families of independent random variables. One can say that such achievements form a core of the Modern Probability Theory. However in XIX and especially XX centuries interesting stochastic models arose involving dependent random variables. Phenomena studied in physics, chemistry, biology, economics and technics were main sources for these models. Thus the theory of stochastic processes and random fields has emerged and evolved intensively. Of course there were also intrinsic factors in mathematics leading to deep generalizations and new constructions.

Now there are important classes of stochastic processes and random fields, for example, Gaussian, Markov, martingales, mixing ones etc. For each class the appropriate methods of investigation were developed, so we have different complementary tools to describe and analyze various stochastic models.

As far back as the 60 s of the last century the new important classes of positively (and later negatively) dependent random variables were introduced in the pioneering papers by Harris, Lehmann, Esary, Proschan, Walkup, Fortuin, Kasteleyn, Ginibre, Alam, Saxena and Joag-Dev. The interest in such models is to a large extent connected with applications in mathematical statistics, reliability theory, percolation and statistical physics. The concept of association is the basic one here. Note that any family of independent real-valued random variables is automatically associated.

Starting from the seminal paper by Newman (1980), during the last 25 years quite a number of classical limit theorems of Probability Theory, such as central limit theorem (CLT), strong law of large numbers (SLLN), law of the iterated logarithm (LIL), functional LIL, weak and strong invariance principles (IP) etc. were established for these new models and their modifications. The main advantage of dealing with sums of positively or negatively associated random variables is the simplicity of the conditions which ensure the limit theorems. Namely, one can assume for summands the existence of absolute moment of order $s \in (2, 3]$ and specify the behavior of the covariance function, e.g., its rate of decrease.

The goal of this book is to introduce the reader to the vast area of recent progress in that research domain. As far as we know, this is the first self-contained exposition of the results obtained during the whole period of development till nowadays. The authors intended to provide detailed proofs rather than reproductions of original journal papers. The bibliography consists of more than 350 items. The word "related" in the title of the book means "related to associated random fields", as there are various modifications of the association concept and a number of such extensions are used below in essential way. In each Chapter we also give some references for further reading as the volume of the book does not permit to include all interesting results.

The book is supplemented with six Appendices. Here one can find an extension of the classical Hoeffding formula given by Khoshnevisan and Lewis, the general information on Markov processes and Poisson spatial process needed to build various examples of dependent random fields, the proofs of auxiliary results from linear algebra and graph theory, the proof of the Móricz inequality, the version of the Berkes–Philipp theorem on normal approximation and some results used to construct the strong approximation of random fields.

Inside any Section the theorems, lemmas, definitions, remarks and examples are numerated in succession. The numeration of formulas takes the form (Section.formula) if the cited formula belongs to current Chapter, and (Chapter.Section.formula) otherwise. Theorems and other statements are numerated analogously. For example, Theorem 1.3.2 belongs to Chapter 1 and Section 3, having the number 2 in this Section. In Appendices instead of the Section number one writes A.1—A.6. The sign □ indicates the end of the proof.

Parts of the book are based on the lectures delivered by the authors at the Moscow State University. The book is addressed to wide audience of probabilists and statisticians who are interested in analysis of various stochastic models. It can be useful for researchers, graduate students and also for academic staff of the Universities.

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