Preface

The modern theory of dynamical systems originates from the Poincarè's qualitative analysis, which focuses on the complexity and stability of motions in such dynamical systems. In 1892, Poincarè discovered that the motion of nonlinear coupled oscillators is sensitive to the initial condition and qualitatively presented that the inherent characteristics of the motion in vicinity of unstable fixed points of nonlinear oscillation systems may be *stochastic* under regular applied forces. This is because the separatrix exists in nonlinear dynamical systems. Such a separatrix connected with hyperbolic points is generic, which is called the generic separatrix. However, another kind of separatrix is generated by the nonlinear resonance between the system and periodic forcing or between the two oscillation interactions. Such a separatrix is called the resonant separatrix. The natural frequency of the separatrix connected with hyperbolic points is zero and the natural frequencies of motion on both sides of the separatrix are different, which makes such a stochastic motion exists in the neighborhood of separatrix. This book will discuss the global transversality of a flow to the separatrix from a domain to another domain in order to understand the mechanism for the onset, growth and destruction of chaos in vicinity of separatrix. The author would like to present a different point of view in order to look into a fundamental theory on global transversality, resonance and chaotic dynamics in nonlinear dynamic systems. The ideas presented in this book are less formal and rigorous in an informal and lively manner. The author hopes the initial ideas may give some inspirations in the field of nonlinear dynamics.

To measure the complex behaviors in a nonlinear dynamical system, the corresponding well-behaved dynamical system is employed. For doing so, the differential geometrical relations of two flows in the two nonlinear dynamical systems are presented. Based on such differential geometric relations, this book presents a theory of global transversality, resonance and chaos in n-dimensional nonlinear dynamics. The history and recent development of nonlinear dynamics is briefly discussed first, and then the global transversality of a flow to the separatrix is investigated to determine motion complexity in n-dimensional dynamical systems. The resonant mechanism of chaos in n-dimensional dynamical systems is discussed in general. Further, the resonant theory of the stochastic layer in 2-dimensional dynamical systems is presented. In addition, the stochasticity of the resonant separatrix layers for $2n$-dimensional, nonlinear Hamiltonian

systems is also presented, and nonlinear dynamics on a $(2n\text{-}1)$-dimensional equi-energy surface is briefly discussed. For dissipative, nonlinear dynamical systems, the stability and grazing bifurcation are addressed. The global dynamics of 2-dimensional dissipative dynamical systems is presented. Finally, the switchability of a flow from a domain to its adjacent domain in discontinuous dynamical systems is discussed. The objective of this book is to throw out some original ideas on global transversality, resonant dynamics and chaos in nonlinear dynamics. The author believes that some ideas may not be very mature and some typos may exist in the book. The author sincerely hopes that readers can forgive such unavoidable errors here and where in the book. The author really appreciates readers for providing suggestions and comments to improve the theory presented in this book.

This book is dedicated to people who are challenging difficult and unsolved problems in natural science. Finally, I also dedicate this book to my wife (Sherry X. Huang) for support and to my lovely children (Yanyi Luo, Robin Ruo-Bing Luo, and Robert Zong-Yuan Luo) for their happiness to stimulate my inspiration.

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