Preface

Knot theory is a new and rich field of mathematics. Although "real" knots are familiar to everyone and many ideas in knot theory can be formulated in everyday language, it is an area abundant with open questions.

One of the main ideas of this book is to avoid obvious classification of knots and links according to their number of components. For this reason knots and links are referred to as KLs and treated together whenever possible.

KLs are denoted by Conway symbols, a geometrical-combinatorial way to describe and derive KLs. The same notation is used in the *Mathematica* based computer program *LinKnot* that represents an integral part of this book. *LinKnot* is not only a supplementary computer program, but the best and most efficient tool for obtaining almost all of the results presented in the book, that belong to the field of *experimental mathematics*.

Hands-on computations using *Mathematica* or the *webMathematica* package *LinKnot* along with detailed illustrations facilitate better learning and understanding. The program *LinKnot* can be downloaded from the web address http://www.mi.sanu.ac.yu/vismath/linknot/ and used as a powerful educational and research tool for experimental mathematics– implementation of Caudron's ideas and the Conway notation enables working with large families of knots and links. The electronic version of this book and the program *LinKnot* that provides *webMathematica* on-line computations are available at the address http://math.ict.edu.yu/.

Each knot theory problem described in this book is accompanied with the corresponding *LinKnot* function that enables the reader to actively use the program *LinKnot*, not only for illustrating some problems, but for computations and experimentation. *LinKnot* is software open to future development: a reader can change it or add new functions. For the systematic and exhaustive derivation of KLs we have accepted the concept proposed by J.H. Conway and A. Caudron, supported and used in a form adapted for computer implementation. As a prerequisite for the use of the Conway notation, the complete list of basic polyhedra up to 20 crossings is given in the program *LinKnot*.

The key idea is the "vertical" classification of KLs into well-defined categories– worlds, subworlds, classes, and *families*, according to new sets of recursively computed invariants. Patterns obtained from computing KL invariants imply the existence of more general KL family invariants that agree with all proposed conjectures. We strongly believe that the concept of family invariants will be placed on a firm theoretical foundation in the future. New KL tables, organized according to KL families, are given in Appendix A that can be downloaded from the address http://www.mi.sanu.ac.yu/vismath/Appendix.pdf.

After a short graph-theoretical introduction, we consider different notations for KLs: Gauss, Dowker, and Conway notation, along with their advantages and disadvantages. All basic KL invariants such as the minimum crossing number, minimum writhe, linking number, unknotting or unlinking number, cutting number, and KL properties such as chirality, periodicity, unlinking gap, and braid family representatives of KLs are discussed in Chapter 1.

In Chapter 2 we address two important problems: recognition and generation of KLs. As recognition criteria we consider KL colorings, KL groups, and more powerful tools such as polynomial KL invariants. Again, we try to show that polynomial KL invariants can be recovered from the Conway notation and recursively computed for KL families.

Chapter 3 contains a short excursion into the history of knot theory and places an emphasis on the beauty, universality, and diversity of knot theory through various non-standard applications such as mirror curves, fullerenes, self-referential systems, and KL automata.

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