Preface

Academician Mitropolsky's commentary on the scientific research of Ukrainian scientist Professor Alexey Stakhov, Doctor of Engineering Sciences

I have followed the scientific career of Professor Stakhov for a long time—seemingly since the publication of his first book, *Introduction into Algorithmic Measurement Theory* (1977), which was presented by Professor Stakhov in 1979 at the scientific seminar of the Mathematics Institute of the Ukrainian Academy of Sciences. I became especially interested in Stakhov's scientific research after listening to his brilliant speech at a session of the Presidium of the Ukrainian Academy of Sciences in 1989. In his speech, Professor Stakhov reported on scientific and engineering developments in the field of "Fibonacci computers" that were conducted under his scientific supervision at Vinnitsa Technical University.

I am very familiar with Stakhov's scientific works as many of his papers were published in various Ukrainian academic journals at my recommendation. In April 1998, I invited Professor Stakhov to report on his scientific research at a meeting of the Ukrainian Mathematical Society. His lecture produced a positive reaction from the members of the society. At the request of Professor Stakhov, I wrote the introduction to his book, *Hyperbolic Fibonacci and Lucas Functions,* which was published in 2003 in small edition. In recent years, I have been actively corresponding with Professor Stakhov, and we have discussed many new scientific ideas. During these discussions I became very impressed with his qualifications and extensive knowledge in regard to his research in various areas of modern science. In particular, I am impressed by his knowledge in the field of mathematics history.

The main feature of Stakhov's scientific creativity consists of his unconventional outlook upon ancient mathematical problems. As an example, I shall begin with my review of his book *Introduction into Algorithmic Measurement Theory* (1977). This publication rewarded Professor Stakhov with recognition in the field of modern theoretical metrology. In this book, Professor Stakhov introduced a new mathematical direction in measurement theory—the Algorithmic Measurement Theory.

In 1993, I recommended a publication of an innovative paper, prepared by Professor Alexey Stakhov and Ivan Tkachenko, entitled "Fibonacci Hyperbolic Trigonometry," for publication in the journal *Reports of the Ukrainian Academy of Sciences*. The paper addressed a new theory of hyperbolic Fibonacci and Lucas functions. This paper demonstrated the uniqueness of Stakhov's scientific thinking. In fact, the classical hyperbolic functions were widely known and were used as a basis of non-Euclidean geometry developed by Nikolay Lobachevsky. It is quite peculiar that at the end of 20th century Ukrainian scientists Stakhov and Tkachenko discovered a new class of the hyperbolic functions based on the Golden Section, Fibonacci and Lucas numbers that has "strategic" importance for the development of modern mathematics and theoretical physics.

In 1999, I also recommended Stakhov's article "A Generalization of the Fibonacci Q-Matrix"—which was presented by the author in English—to be published in the journal *Reports of the Ukrainian Academy of Sciences* (1999, Vol. 9). In this article, Professor Stakhov generalized and developed a new theory of the *Q*-matrix which had been introduced by the American mathematician Verner Hoggatt—a founder of the Fibonacci-Association. Stakhov introduced a concept of the Q_p-matrices (p=0, 1, 2, 3...), which are a new class of square matrices (a number of such matrices is infinite). These matrices are based on so-called Fibonacci *p*-numbers, which had been discovered by Stakhov while investigating "diagonal sums" of the Pascal triangle. Stakhov discovered a number of quite unusual properties of the Q_p-matrices. In particular, he proved that the determinant of the Q_p-matrix or any power of that matrix is equal to +1 or -1. It is my firm belief that a theory of Q_p-matrices could be recognized as a new fundamental result in the classic matrix theory.

In 2004, *The Ukrainian Mathematical Journal* (Vol. 8), published Stakhov's article "The Generalized Golden Sections and a New Approach to Geometrical Definition of Number." In this article, Professor Stakhov obtained mathematical results in number theory. The following are worth mentioning:

1. A Generalization of the Golden Section Problem. The essence of this generalization is extremely simple. Let us set a non-negative integer (p=0, 1, 2, 3, ...) and divide a line segment AB at the point C in the following proportion:

$$\frac{CB}{AC} = \left(\frac{AB}{CB}\right)^p$$

We then get the following algebraic equation:

$$x^{p+1} = x^p + 1.$$

The positive roots of this algebraic equation were named the *Generalized Golden Proportions* or the *Golden p-proportions* t_p. Let's ponder upon this result. Within several millennia, since Pythagoras and Plato, mankind widely used the known classical Golden Proportion as some unique number. And at the end of the 20th century, the Ukrainian scientist Stakhov has generalized this result and proved the existence of the infinite number of the Golden Proportions; as all of them have the same right to express Harmony, as well as the classical Golden Proportion. Moreover, Stakhov proved that the golden p-proportions τ_p $(1 \le \tau_p \le 2)$ represented a new class of irrational numbers, which express some unknown mathematical properties of the Pascal triangle. Undoubtedly, such mathematical result has fundamental importance for the development of modern science and mathematics.

2. Codes of the Golden *p*-proportions. Using a concept of the golden p-proportion, Stakhov introduced a new definition of real number in the form:

$$A = \sum_i a_i \tau_p^i, (a_i \in \{0,1\})$$

He named this sum the "Code of the golden p-proportion." Stakhov proved that this concept, which is an expansion of the well-known Newton's definition of real number, could be used for the creation of a new theory for real numbers. Furthermore, he proved that this result could also be used for the creation of new computer arithmetic and new computers—*Fibonacci computers*. Stakhov not only introduced the idea of Fibonacci computers, but he also organized the engineering projects on the creation of such computer prototypes in the Vinnitsa Polytechnic Institute from 1977-1995. 65 foreign patents for inventions in the field of Fibonacci computers have been issued by the state patent offices of the United States, Japan, England, France, Germany, Canada, and other countries; these patents confirmed the significance of Ukrainian science and of Professor Stakhov's work in this important computer area.

In recent years, the area of Professor Stakhov's scientific interests has moved more and more towards the area of mathematics. For example, his lecture "The Golden Section and Modern Harmony Mathematics" delivered at the Seventh International Conference on Fibonacci Numbers and their Applications in Graz, Austria in 1996, and then repeated in 1998 at the Ukrainian Mathematical Society, established a new trend in Stakhov's scientific research. This lecture was impressive and it created wide discussion on Stakhov's new research.

Currently, Professor Stakhov is an actively working scientist who publishes his scientific papers in many internationally recognized journals. Most recently, he has published many fundamental papers in the international journals: *Computers & Mathematics with Applications*; *The Computer Journal*; *Chaos, Solitons & Fractals*; *Visual Mathematics*; and others. This fact demonstrates, undoubtedly, tremendous success not only for Professor Stakhov, but also for Ukrainian science.

Stakhov's articles are closing a cycle of his long-term research on the creation of a new direction in mathematics: **Mathematics of Harmony**. One may wonder what place in the general theory of mathematics this work may have. It seems to me that in the last few centuries as Nikolay Lobachevsky said, "Mathematicians have turned all their attention to the advanced parts of analytics, and have neglected the origins of Mathematics, and are not willing to dig the field that has already been harvested by them and left behind." As a result, this has created a gap between "Elementary Mathematics"—the basis of modern mathematical education—and "Advanced Mathematics." In my opinion, the Mathematics of Harmony developed by Professor Stakhov fills that gap. **Mathematics of Harmony** is a huge theoretical contribution to the development of "Elementary Mathematics," and as such should be considered of great importance for mathematical education.

It is imperative to mention that Professor Stakhov focuses his organizational work on stimulating research in the field of theory surrounding Fibonacci numbers and the Golden Section; he also assists in spreading knowledge among broad audiences inside the scientific community. In 2003, under Professor Stakhov's initiative and scientific supervision, the international conference on "Problems of Harmony, Symmetry, and the Golden Section in Nature, Science, and Art" was held. At this conference, Professor Stakhov was elected as President of the International Club of the Golden Section, confirming his official status as leader of a new scientific direction that is actively progressing the modern science.

Professor Stakhov proposed the discipline "Mathematics of Harmony and the Golden Section" for the mathematical faculties of pedagogical universities. In essence, this mathematical discipline can be considered the beginning of mathematical education reform—which is based on the principles of Harmony and the Golden Section. It should be noted that such discipline was delivered by Professor Stakhov during 2001-2002 for the students and faculty of physics and mathematics at Vinnitsa State Pedagogical University. I have no doubts about the usefulness of such discipline for future teachers in mathematics and physics. I believe that Professor Stakhov has the potential to write a textbook

on this discipline for pedagogical universities, and also a textbook on *Mathematics of the Golden Section* for secondary schools.

It is clear to me that "Mathematics of Harmony," created by Professor Stakhov, has huge interdisciplinary importance as this mathematical discipline touches the bases of many sciences, including: mathematics, theoretical physics, and computer science. Stakhov suggested mathematical education reform based on the ideas of Harmony and the Golden Section. This reform opens the doors for the development of mathematical and general education curriculum. It would greatly contribute to the development of the new scientific outlook based on the principles of Harmony and the Golden Section.

Yuri Mitropolsky
Doctor of Sciences in Theoretical Mechanics, Professor
Academician of the National Academy of Sciences of Ukraine
Academician of the Russian Academy of Sciences
Honorable Professor: The Mathematics Institute of the National
Academy of Sciences of Ukraine
Editor-in-Chief of the *Ukrainian Mathematical Journal*