

Foreword to the Chinese Edition

Theoretical mechanics has been for a long time the leading discipline in the history of science. The development for centuries has resulted in multiple branches and the achievement has been remarkable. Mechanics as a fundamental subject has helped in the development of many engineering disciplines such as aeronautical and astronautical engineering, mechanical engineering, civil engineering, chemical engineering, natural resources engineering, material engineering, etc. Mean while, many theories and methodologies have been developed in applied mechanics as compelled by the requirement in various engineering applications. From the viewpoint of applied mathematics, a problem is clearly described once the basic differential equations are constructed. The remaining task is to look for a solution. Nevertheless, in many circumstances the solutions are extremely difficult although the basic equations are already constructed.

Elasticity has been one of the most complex fields in various branches of mathematical physics involving partial differential equations. The fundamental systems of equations in elasticity were established as early as the beginning of the nineteenth century. However, for over a century of development, the solutions were far from complete. Solution methodology has been a bottleneck in the development of elasticity. The difficulty for strict solutions in elasticity, in turn, drove the development of some applied branches such as structural mechanics, thin-walled structures, plate and shell theories, as well as structural dynamics, stability, soil mechanics, fluid mechanics, etc. These branches form the various systems in applied mechanics. Although these applied theories simply the governing equations, analytical solutions are still difficult to a large extent. Research collaboration between mathematicians and mechanicians not only enriched mathematical physics but also developed applied mechanics. Some representative works in this period are *Methods of Mathematical Physics* by R. Courant and D. Hilbert, and a set of texts by S.P. Timoshenko including *Theory of Elasticity*, *Theory of Elastic Stability*, *Theory of Plates and Shells*, *Vibration Problems*

in Engineering, Mechanics of Materials, etc. This set of analytical solutions became the classical solution systems in the field. The achievement in the period was marvelous and it influenced the subsequent research development in the field.

With the advent of computing machinery and high level programming languages in the second half of the twentieth century, the surface of finite element method in applied mechanics changed the situation rapidly. Based on the theory of applied mechanics and powerful computational ability, versatile numerical methods in finite element method were developed for solving structural mechanics, solid mechanics, etc., described by linear equations. Large scale finite element systems were geared to solve sets of linear algebraic equations with tens of thousands of unknown variables. It became a powerful analytical tool for engineers and the status of computational mechanics was established. The successful application of finite element method in structural analysis swiftly extended to various aspects in computational mechanics, engineering and science with significant achievement.

The success of finite element method has not weakened the significance of analytical methods because (i) it is a kind of numerical approximation and its theory is based on analytical methods; and (ii) many problems require analytical solutions such as crack tip singularity element in fracture mechanics, element for infinite domain, etc. Besides, the application of finite element method for analyses with local effect such as boundary effects in shell theory, and the free boundaries and boundary singular points in composite materials, etc., result in stiffness problems and therefore analytical methods are still of significant importance.

Considering “Theory of Elasticity” of S.P. Timoshenko as an example, the solutions of various elasticity problems using the semi-inverse method constitute a large portion of the text. This method was introduced by Saint-Venant in 1855–1856 to obtain certain solutions for torsion and bending of elastic columns. Since then it became the classical solution methodology for elasticity and its influence extends to the present moment. It is a trial method which is valid only for a specific problem without generality. It often obtains a certain solution but it cannot ensure complete solutions. What bothers one is the way to obtain a specific trial in order to solve the problem in hand.

The application of semi-inverse method is due to the complexity of the governing system of equations. The conventional analytical method is confined to the domain of single variable, using either stress function (method of force) or displacement method (only the shallow shell theory applies

the hybrid method). The various unknown functions are eliminated thus resulting in a higher-order partial differential equation with a single variable which is then solved. From the viewpoint of mathematical systems, the solution of single variable systems belongs to the Lagrangian approach which inevitably results in a higher-order partial differential equation. Hence, the effective methods in mathematical physics such as variable separation and expansion of eigenfunctions become inapplicable. Consequently, the semi-inverse method has been unable to achieve major breakthrough for a long period.

Hence, a question arises. Is it absolutely necessary to employ this classical approach of eliminating variables? In reality, the classical approach is not the only avenue and dual theory and state symplectic space is the answer.

Recalling the many years we learnt applied mechanics, we may observe some problems. Classical analytical mechanics is the most fundamental system. Lagrange equation, the principle of minimum action, Hamilton's canonical equation, canonical transformation, Hamilton-Jacoby theory, etc., are all very beautiful theoretical systems. Classical analytical mechanics is also the basis of some fundamental branches of science such as statistical mechanics, electrodynamics, quantum mechanics, etc. It is, however, insufficiently appears in courses in applied mechanics. This is because it is less relevant to courses in elasticity, structural mechanics, fluid mechanics, vibration and stability, etc. Although control theory is originated from mechanics, it is seldom introduced in courses in applied mechanics. For instance, many existing texts in elasticity do not have much relevance with analytical mechanics. Systems of theory and methodologies for these fields are independent to a certain extent.

Control theory developed into modern control theory with the impact of computational techniques. The modern control theory is not merely an extension of the classical control theory but it has undergone fundamental changes in the basic theory with major breakthrough. The state space method based on modern control theory can be traced back to the system of Hamilton's canonical equation which is in principle a system with dual variables and dual equations.

Control theory underwent changes in system representation regulated by its own rules during development. Its system of theory was thought to have deviated further from applied mechanics. However, the real situation is not. It has been proven that the mathematical problems of modern control theory and structural mechanics map one-to-one and they are mutually

similar. From the viewpoint of mathematics, the similarity is based on the theory and fundamentals of dual variables and Hamiltonian systems. It indicates that mechanics is able to gain advantage from the successful experience of control theory. As a matter of fact, the teaching and research in applied mathematics develops more and more towards systems of duality. From the observation above, the systems of dual variables should be implemented in the various branches of applied mechanics in a natural and systematic manner.

The development of information techniques of contemporary science and technology has been applied to intelligent materials, intelligent structures, precision weapon, etc., and the influence of control and remote sensing via various channels has been observed. Structural control has received increasing attention from now and then. Such development trend should not be neglected in the teaching of applied mechanics. As the world now moves towards “smartness”, mechanics will not be able to be “smart” if it is not linked to control theory. In the United States, the mismatch in designs by structure engineers and control engineers has not been beneficial for overall rational design and there are voices to call for a “control-structure overall design”. In fact, symplectic analytical systems can be applied to many other subjects such as vibration, wave propagation, etc. The engagement of an identical system of theory will encourage and make easy the assimilation and association of various branches of science such as mechanics and modern control, etc. It is also beneficial to teaching.

The advance from Lagrangian systems to Hamiltonian systems means the advance from the conventional Euclidean geometry to symplectic geometry. It is a breakthrough of the conventional concept which causes the application of dual and mixed variables into the vast fields of mechanics. In addition, symplectic systems can also be applied to mathematical physics and further to other related disciplines. Introducing the application of this approach in elasticity as a professional fundamental course to students will unquestionably help them to achieve greater heights in research in the future.