## Preface to the Chinese Edition

Elasticity has been one of the most complex fields in various branches of mathematical physics involving partial differential equations. The solution procedure of such problems has been a bottleneck in the development of elasticity. Considering "Theory of Elasticity" of S.P. Timoshenko as an example, the solutions of various elasticity problems using the semi-inverse method constitute a large portion of the text. The application of semi-inverse method is due to the complexity of the system of equations. The conventional method of solution is always confined to the solution of a higher-order partial differential equation with a single variable by eliminating the various unknown functions. From the viewpoint of mathematical systems, the solution of single variable systems belongs to the Lagrangian approach which inevitably results in a higher-order partial differential equation. Hence, the effective methods in mathematical physics such as variable separation and expansion of eigenfunctions become inapplicable. Consequently, the semi-inverse method has been unable to achieve major breakthrough for a long period.

In this book, the symplectic space formed by the original and dual variables is introduced into elasticity in accordance with analogy theory between structural mechanics and control optimization. As a result, it is possible to apply the direct analytical methods of variable separation and expansion of symplectic eigenfunctions. It then forms the symplectic analytical systems in elasticity and it is the breaking and unified approach that is emphasized throughout the book. The solution of symplectic analytical systems is based on a rational, systematic approach with clear step-be-step derivation procedure. It alters the classical practice in elasticity of using the semi-inverse method by presenting a new systematic and rational method of solution. The many previous problems unsolvable or too complicated to be solved using the semi-inverse approach can hence be resolved accordingly. For instance, we have presented solutions for plate bending problems with various boundary conditions, laminated composite plates and anisotropic

problems. In addition, the Saint-Venant problems for plain elasticity and elastic cylinders can be described in a new system of equations and solved. The difficulty of satisfying end boundary conditions in conventional problems which could only be covered using the Saint-Vanent principle can also be solved.

Due to the difference in basic principle for elasticity problems using the classical semi-inverse method and the symplectic analytical systems, the rational approach of the latter can be completely and directly generalized to more complex problems. More problems can thus be solved analytically and larger solution domain can thus be obtained. The solution procedure for various symplectic analytical systems is identical and only the algebraic derivation becomes more complicated for more complex problems. Such algebraic complexity can be overcome by using symbolic mathematical softwares.

The analytical concept for symplectic systems and conventional approach for partial differential equation are just opposite. The conventional concept tries to eliminate unknown variables as many as possible thus increasing the order of partial differential equations. Since higher-order differential equations are not conducive to numerical solution methods such as finite element method, such elimination will cause problems in numerical analysis. On the contrary, although there are more variables in symplectic analytical systems, the order of differential equations is lower. There are numerical advantages in dealing with lower-order differential equations and an increase in the number of variables will not have significant impact on the system. In other words, the association of symplectic analytical systems with numerical methods will not only greatly highlight the excellence of the former but also enhances the usefulness of computers for solving engineering problems.

The book aims at introducing to the readers the methodology of symplectic analytical systems in elasticity in a systematic manner. From basic equations of elasticity and classical variational principle, it first describes in detail the procedure of constructing Hamiltonian mixed energy variational principle and Hamiltonian dual system of equations and subsequently the symplectic analytical systems through discussion of various basic problems in elasticity. The eigenvalue problem in the transverse direction, i.e. the symplectic eigenvalue problem, can then be derived by applying the method of separation of variables. Hence, the solution can be obtained by expanding the eigenvectors. Many solutions with specific interpretation in physics can be obtained via the eigenfunctions of specific eigenvalues and their

corresponding Jordan form eigenfunctions. In general, a similar solution methodology is applied throughout this book so that the readers are able to master the solution procedure of symplectic analytical systems. Furthermore, the differences with respect to the classical semi-inverse can also be clearly observed.

Analytical method is emphasized in this book because it is a major first step in deriving new solutions for a system. The problems discussed in this book are all fundamental problems in elasticity such as Timoshenko beam, plane elasticity, laminated composite plate, plate bending, etc. It is emphasized here that the method of symplectic analytical system is absolutely applicable to three dimensional problems such as cylindrical bodies. Such complicated subjects are, however, not described in this book.

Introducing a new concept without sophisticated mathematics is a unique feature in this book. The related contents of symplectic mathematics as required in substitution of systems are discussed in detail in Chapter 1. The mathematical preliminaries for calculus and matrix algebra are at graduate level and omitting this part will not hinder the understanding and master of symplectic analytical system in elasticity. The contents in this book have been repeatedly introduced in courses for graduates and higher level undergraduates. It not only helps the students familiarize a new solution methodology but also widens very much their research vision. The effect has been remarkable.

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Although every care has been taken to ensure correctness of contents, incompleteness is bound to exist in this book. The authors thank the readers in advance for giving critical comments and pointing out mistakes.

*Weian Yao*
*Wanxie Zhong*
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