

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>I</b>	<b>GENERAL THEORY OF OPEN QUANTUM SYSTEMS</b>	<b>5</b>
<b>2</b>	<b>Diverse limited approaches: a brief survey</b>	<b>5</b>
2.1	Langevin equation for a damped classical system . . . . .	5
2.2	New schemes of quantization . . . . .	7
2.3	Traditional system-plus-reservoir methods . . . . .	8
2.3.1	Quantum-mechanical master equations for weak coupling . . . . .	8
2.3.2	Operator Langevin equations for weak coupling . . . . .	12
2.3.3	Quantum and quasiclassical Langevin equation . . . . .	13
2.3.4	Phenomenological methods . . . . .	14
2.4	Stochastic dynamics in Hilbert space . . . . .	15
<b>3</b>	<b>System-plus-reservoir models</b>	<b>18</b>
3.1	Harmonic oscillator bath with linear coupling . . . . .	19
3.1.1	The Hamiltonian of the global system . . . . .	19
3.1.2	<b>The road to the classical generalized Langevin equation</b> . . . . .	21
3.1.3	Phenomenological modeling . . . . .	24
3.1.4	Quasiclassical Langevin equation . . . . .	25
3.1.5	Ohmic and frequency-dependent damping . . . . .	27
3.1.6	Rubin model . . . . .	30
3.2	The Spin-Boson model . . . . .	31
3.2.1	The model Hamiltonian . . . . .	31
3.2.2	Josephson two-state systems: flux and charge qubit . . . . .	35
3.3	Microscopic models . . . . .	38
3.3.1	Acoustic polaron: one-phonon and two-phonon coupling . . . . .	40
3.3.2	Optical polaron . . . . .	41
3.3.3	Interaction with fermions (normal and superconducting) . . . . .	43
3.3.4	Superconducting tunnel junction . . . . .	46
3.4	Charging and environmental effects in tunnel junctions . . . . .	47
3.4.1	The global system for single electron tunneling . . . . .	49
3.4.2	Resistor, inductor and transmission lines . . . . .	53
3.4.3	Charging effects in Josephson junctions . . . . .	54
3.5	Nonlinear quantum environments . . . . .	55

<b>4</b>	<b>Imaginary-time path integrals</b>	<b>57</b>
4.1	The density matrix: general concepts . . . . .	58
4.2	Effective action and equilibrium density matrix . . . . .	62
4.2.1	Open system with bilinear coupling to a harmonic reservoir . . . . .	63
4.2.2	State-dependent memory-friction . . . . .	67
4.2.3	Spin-boson model . . . . .	68
4.2.4	Acoustic polaron and defect tunneling: one-phonon coupling . . . . .	69
4.2.5	Acoustic polaron: two-phonon coupling . . . . .	75
4.2.6	Tunneling between surfaces: one-phonon coupling . . . . .	77
4.2.7	Optical polaron . . . . .	79
4.2.8	Heavy particle in a metal . . . . .	80
4.2.9	Heavy particle in a superconductor . . . . .	86
4.2.10	Effective action for a Josephson junction . . . . .	88
4.2.11	Electromagnetic environment . . . . .	95
4.3	Partition function of the open system . . . . .	96
4.3.1	General path integral expression . . . . .	96
4.3.2	Semiclassical approximation . . . . .	97
4.3.3	Partition function of the damped harmonic oscillator . . . . .	98
4.3.4	Functional measure in Fourier space . . . . .	99
4.3.5	Partition function of the damped harmonic oscillator revisited . . . . .	100
4.4	Quantum statistical expectation values in phase space . . . . .	102
4.4.1	Generalized Weyl correspondence . . . . .	103
4.4.2	Generalized Wigner function and expectation values . . . . .	105
<b>5</b>	<b>Real-time path integrals and dynamics</b>	<b>106</b>
5.1	Feynman-Vernon method for a product initial state . . . . .	108
5.2	Decoherence and friction . . . . .	112
5.3	General initial states and preparation function . . . . .	115
5.4	Complex-time path integral for the propagating function . . . . .	116
5.5	Real-time path integral for the propagating function . . . . .	120
5.5.1	Extremal paths . . . . .	123
5.5.2	Classical limit . . . . .	124
5.5.3	Semiclassical limit: quasiclassical Langevin equation . . . . .	125
5.6	Stochastic unraveling of influence functionals . . . . .	127
5.7	Brief summary and outlook . . . . .	130
<b>II</b>	<b>FEW SIMPLE APPLICATIONS</b>	<b>131</b>
<b>6</b>	<b>Damped harmonic oscillator</b>	<b>131</b>
6.1	Fluctuation-dissipation theorem . . . . .	132
6.2	Stochastic modeling . . . . .	135
6.3	Susceptibility for Ohmic friction and Drude damping . . . . .	138

6.3.1	Strict Ohmic friction . . . . .	138
6.3.2	Drude damping . . . . .	138
6.4	The position autocorrelation function . . . . .	139
6.4.1	Ohmic damping . . . . .	140
6.4.2	Algebraic spectral density . . . . .	142
<b>6.5</b>	<b>Partition function, internal energy and density of states . . . . .</b>	<b>143</b>
6.5.1	<b>Partition function and internal energy . . . . .</b>	<b>143</b>
6.5.2	<b>Spectral density of states . . . . .</b>	<b>146</b>
6.6	Mean square of position and momentum . . . . .	147
6.6.1	General expressions for coloured noise . . . . .	147
6.6.2	Strict Ohmic case . . . . .	149
6.6.3	Ohmic friction with Drude regularization . . . . .	150
6.7	Equilibrium density matrix . . . . .	152
6.7.1	Purity . . . . .	154
<b>7</b>	<b>Quantum Brownian free motion . . . . .</b>	<b>156</b>
7.1	Spectral density, damping function and mass renormalization . . . . .	157
7.2	Displacement correlation and response function . . . . .	159
7.3	Ohmic damping . . . . .	160
7.4	Frequency-dependent damping . . . . .	163
7.4.1	Response function and mobility . . . . .	163
7.4.2	Mean square displacement . . . . .	165
<b>8</b>	<b>The thermodynamic variational approach . . . . .</b>	<b>167</b>
8.1	Centroid and the effective classical potential . . . . .	167
8.1.1	Centroid . . . . .	167
8.1.2	The effective classical potential . . . . .	169
8.2	Variational method . . . . .	170
8.2.1	Variational method for the free energy . . . . .	170
8.2.2	Variational method for the effective classical potential . . . . .	171
8.2.3	Variational perturbation theory . . . . .	174
8.2.4	Expectation values in coordinate and phase space . . . . .	176
<b>9</b>	<b>Suppression of quantum coherence . . . . .</b>	<b>178</b>
9.1	Nondynamical versus dynamical environment . . . . .	179
9.2	Suppression of transversal and longitudinal interferences . . . . .	180
9.3	Localized bath modes and universal decoherence . . . . .	182
9.3.1	A model with localized bath modes . . . . .	182
9.3.2	Statistical average of paths . . . . .	184
9.3.3	Ballistic motion . . . . .	185
9.3.4	Diffusive motion . . . . .	186

<b>III QUANTUM STATISTICAL DECAY</b>	<b>189</b>
<b>10 Introduction</b>	<b>189</b>
<b>11 Classical rate theory: a brief overview</b>	<b>192</b>
11.1 Classical transition state theory . . . . .	192
11.2 Moderate-to-strong-damping regime . . . . .	193
11.3 Strong damping regime . . . . .	195
11.4 Weak-damping regime . . . . .	197
<b>12 Quantum rate theory: basic methods</b>	<b>199</b>
12.1 Formal rate expressions in terms of flux operators . . . . .	200
12.2 Quantum transition state theory . . . . .	202
12.3 Semiclassical limit . . . . .	203
12.4 Quantum tunneling regime . . . . .	205
12.5 Free energy method . . . . .	207
12.6 Centroid method . . . . .	211
<b>13 Multidimensional quantum rate theory</b>	<b>212</b>
<b>14 Crossover from thermal to quantum decay</b>	<b>216</b>
14.1 Normal mode analysis at the barrier top . . . . .	216
14.2 Turnover theory for activated rate processes . . . . .	218
14.3 The crossover temperature . . . . .	222
<b>15 Thermally activated decay</b>	<b>223</b>
15.1 Rate formula above the crossover regime . . . . .	224
15.2 Quantum corrections in the preexponential factor . . . . .	227
15.3 The quantum Smoluchowski equation approach . . . . .	228
15.4 Multidimensional quantum transition state theory . . . . .	230
<b>16 The crossover region</b>	<b>233</b>
16.1 Beyond steepest descent above $T_0$ . . . . .	235
16.2 Beyond steepest descent below $T_0$ . . . . .	236
16.3 The scaling region . . . . .	239
<b>17 Dissipative quantum tunneling</b>	<b>242</b>
17.1 The quantum rate formula . . . . .	242
17.2 Thermal enhancement of macroscopic quantum tunneling . . . . .	245
17.3 Quantum decay in a cubic potential for Ohmic friction . . . . .	246
17.3.1 Bounce action and quantum prefactor . . . . .	247
17.3.2 Analytic results for strong Ohmic dissipation . . . . .	248
17.4 Quantum decay in a tilted cosine washboard potential . . . . .	250
17.5 Concluding remarks . . . . .	257

<b>IV THE DISSIPATIVE TWO-STATE SYSTEM</b>	<b>259</b>
<b>18 Introduction</b>	<b>259</b>
18.1 Truncation of the double-well to the two-state system . . . . .	261
18.1.1 Shifted oscillators and orthogonality catastrophe . . . . .	261
18.1.2 Adiabatic renormalization . . . . .	263
18.1.3 Renormalized tunnel matrix element . . . . .	264
18.1.4 Polaron transformation . . . . .	269
18.2 Pair interaction in the charge picture . . . . .	269
18.2.1 Analytic expression for any $s$ and arbitrary cutoff $\omega_c$ . . . . .	269
18.2.2 Ohmic dissipation and universality limit . . . . .	271
<b>19 Thermodynamics</b>	<b>272</b>
19.1 Partition function and specific heat . . . . .	272
19.1.1 Exact formal expression for the partition function . . . . .	272
19.1.2 Static susceptibility and specific heat . . . . .	274
19.1.3 The self-energy method . . . . .	275
19.1.4 The limit of high temperatures . . . . .	277
19.1.5 Noninteracting-kink-pair approximation . . . . .	277
19.1.6 Weak-damping limit . . . . .	279
19.1.7 The self-energy method revisited: partial resummation . . . . .	280
19.2 Ohmic dissipation . . . . .	281
19.2.1 General results . . . . .	282
19.2.2 The special case $K = \frac{1}{2}$ . . . . .	283
19.3 Non-Ohmic spectral densities . . . . .	288
19.3.1 The sub-Ohmic case . . . . .	288
19.3.2 The super-Ohmic case . . . . .	289
19.4 Relation between the Ohmic TSS and the Kondo model . . . . .	290
19.4.1 Anisotropic Kondo model . . . . .	290
19.4.2 Resonance level model . . . . .	292
19.5 Equivalence of the Ohmic TSS with the $1/r^2$ Ising model . . . . .	293
<b>20 Electron transfer and incoherent tunneling</b>	<b>294</b>
20.1 Electron transfer . . . . .	295
20.1.1 Adiabatic bath . . . . .	296
20.1.2 Marcus theory for electron transfer . . . . .	298
20.2 Incoherent tunneling in the nonadiabatic regime . . . . .	302
20.2.1 General expressions for the nonadiabatic rate . . . . .	302
20.2.2 Probability for energy exchange: general results . . . . .	304
20.2.3 The spectral probability density for absorption at $T = 0$ . . . . .	307
20.2.4 Crossover from quantum-mechanical to classical behaviour . . . . .	308
20.2.5 The Ohmic case . . . . .	312
20.2.6 Exact nonadiabatic rates for $K = 1/2$ and $K = 1$ . . . . .	314

20.2.7	The sub-Ohmic case ( $0 < s < 1$ ) . . . . .	315
20.2.8	The super-Ohmic case ( $s > 1$ ) . . . . .	317
20.2.9	Incoherent defect tunneling in metals . . . . .	319
20.3	Single charge tunneling . . . . .	322
20.3.1	Weak-tunneling regime . . . . .	322
20.3.2	The current-voltage characteristics . . . . .	326
20.3.3	Weak tunneling of 1D interacting electrons . . . . .	328
20.3.4	Tunneling of Cooper pairs . . . . .	330
20.3.5	Tunneling of quasiparticles . . . . .	331
<b>21</b>	<b>Two-state dynamics</b> . . . . .	<b>333</b>
21.1	Initial preparation, expectation values, and correlations . . . . .	333
21.1.1	Product initial state . . . . .	333
21.1.2	Thermal initial state . . . . .	336
21.2	Exact formal expressions for the system dynamics . . . . .	340
21.2.1	Sojourns and blips . . . . .	340
21.2.2	Conditional propagating functions . . . . .	343
21.2.3	The expectation values $\langle \sigma_j \rangle_t$ ( $j = x, y, z$ ) . . . . .	344
21.2.4	Correlation and response function of the populations . . . . .	346
21.2.5	Correlation and response function of the coherences . . . . .	348
21.2.6	Generalized exact master equation and integral relations . . . . .	349
21.3	The noninteracting-blip approximation (NIBA) . . . . .	352
21.3.1	Symmetric Ohmic system in the scaling limit . . . . .	355
21.3.2	Weak Ohmic damping and moderate-to-high temperature . . . . .	359
21.3.3	The super-Ohmic case . . . . .	365
21.4	Weak-coupling theory beyond the NIBA for a biased system . . . . .	368
21.4.1	The one-boson self-energy . . . . .	369
21.4.2	Populations and coherences (super-Ohmic and Ohmic) . . . . .	371
21.5	The interacting-blip chain approximation . . . . .	373
21.6	Ohmic dissipation with $K$ at and near $\frac{1}{2}$ : exact results . . . . .	376
21.6.1	Grand-canonical sums of collapsed blips and sojourns . . . . .	376
21.6.2	The expectation value $\langle \sigma_z \rangle_t$ for $K = \frac{1}{2}$ . . . . .	377
21.6.3	The case $K = \frac{1}{2} - \kappa$ ; coherent-incoherent crossover . . . . .	379
21.6.4	Equilibrium $\sigma_z$ autocorrelation function . . . . .	380
21.6.5	Equilibrium $\sigma_x$ autocorrelation function . . . . .	385
21.6.6	Correlation functions in the Toulouse model . . . . .	387
21.7	Long-time behaviour at $T = 0$ for $K < 1$ : general discussion . . . . .	388
21.7.1	The populations . . . . .	389
21.7.2	The population correlations and generalized Shiba relation . . . . .	389
21.7.3	The coherence correlation function . . . . .	391
21.8	From weak to strong tunneling: relaxation and decoherence . . . . .	392
21.8.1	Incoherent tunneling beyond the nonadiabatic limit . . . . .	392

21.8.2	Decoherence at zero temperature: analytic results . . . . .	395
21.9	Thermodynamics from dynamics . . . . .	396
<b>22</b>	<b>The driven two-state system</b>	<b>399</b>
22.1	Time-dependent external fields . . . . .	399
22.1.1	Diagonal and off-diagonal driving . . . . .	399
22.1.2	Exact formal solution . . . . .	400
22.1.3	Linear response . . . . .	402
22.1.4	The Ohmic case with Kondo parameter $K = \frac{1}{2}$ . . . . .	403
22.2	Markovian regime . . . . .	403
22.3	High-frequency regime . . . . .	404
22.4	Quantum stochastic resonance . . . . .	407
22.5	Driving-induced symmetry breaking . . . . .	409
<b>V</b>	<b>THE DISSIPATIVE MULTI-STATE SYSTEM</b>	<b>411</b>
<b>23</b>	<b>Quantum Brownian particle in a washboard potential</b>	<b>411</b>
23.1	Introduction . . . . .	411
23.2	Weak- and tight-binding representation . . . . .	412
<b>24</b>	<b>Multi-state dynamics</b>	<b>413</b>
24.1	Quantum transport and quantum-statistical fluctuations . . . . .	413
24.1.1	Product initial state . . . . .	414
24.1.2	Characteristic function of moments and cumulants . . . . .	414
24.1.3	Thermal initial state and correlation functions . . . . .	415
24.2	Poissonian quantum transport . . . . .	416
24.2.1	Dynamics by incoherent nearest-neighbour tunneling moves . . . . .	416
24.2.2	The general case . . . . .	418
24.3	Exact formal expressions for the system dynamics . . . . .	419
24.3.1	Product initial state . . . . .	421
24.3.2	Thermal initial state . . . . .	423
24.4	Mobility and Diffusion . . . . .	426
24.4.1	Exact formal series expressions for transport coefficients . . . . .	426
24.4.2	Einstein relation . . . . .	427
24.5	The Ohmic case . . . . .	428
24.5.1	Weak-tunneling regime . . . . .	429
24.5.2	Weak-damping limit . . . . .	429
24.6	Exact solution in the Ohmic scaling limit at $K = \frac{1}{2}$ . . . . .	431
24.6.1	Current and mobility . . . . .	431
24.6.2	Diffusion and skewness . . . . .	434
24.7	The effects of a thermal initial state . . . . .	435
24.7.1	Mean position and variance . . . . .	435

24.7.2	Linear response . . . . .	436
24.7.3	The exactly solvable case $K = \frac{1}{2}$ . . . . .	439
<b>25</b>	<b>Duality symmetry</b>	<b>439</b>
25.1	Duality for general spectral density . . . . .	440
25.1.1	The map between the TB and WB Hamiltonian . . . . .	440
25.1.2	Frequency-dependent linear mobility . . . . .	443
25.1.3	Nonlinear static mobility . . . . .	444
25.2	Self-duality in the exactly solvable cases $K = \frac{1}{2}$ and $K = 2$ . . . . .	446
25.2.1	Full counting statistics at $K = \frac{1}{2}$ . . . . .	446
25.2.2	Full counting statistics at $K = 2$ . . . . .	448
25.3	Duality and supercurrent in Josephson junctions . . . . .	450
25.3.1	Charge-phase duality . . . . .	450
25.3.2	Supercurrent-voltage characteristics for $\rho \ll 1$ . . . . .	453
25.3.3	Supercurrent-voltage characteristics at $\rho = \frac{1}{2}$ . . . . .	454
25.3.4	Supercurrent-voltage characteristics at $\rho = 2$ . . . . .	454
25.4	Self-duality in the Ohmic scaling limit . . . . .	455
25.4.1	Linear mobility at finite $T$ . . . . .	456
25.4.2	Nonlinear mobility at $T = 0$ . . . . .	457
25.5	Exact scaling function at $T = 0$ for arbitrary $K$ . . . . .	459
25.5.1	Construction of the self-dual scaling solution . . . . .	459
25.5.2	Supercurrent-voltage characteristics at $T = 0$ for arbitrary $\rho$ . . . . .	462
25.5.3	Connection with Seiberg-Witten theory . . . . .	462
25.5.4	Special limits . . . . .	463
25.6	Full counting statistics at zero temperature . . . . .	464
25.7	Low temperature behaviour of the characteristic function . . . . .	467
25.8	The sub- and super-Ohmic case . . . . .	468
<b>26</b>	<b>Charge transport in quantum impurity systems</b>	<b>470</b>
26.1	Generic models for transmission of charge through barriers . . . . .	471
26.1.1	The Tomonaga-Luttinger liquid . . . . .	471
26.1.2	Transport through a single weak barrier . . . . .	472
26.1.3	Transport through a single strong barrier . . . . .	474
26.1.4	Coherent conductor in an Ohmic environment . . . . .	476
26.1.5	Equivalence with quantum transport in a washboard potential . . . . .	478
26.2	Self-duality between weak and strong tunneling . . . . .	478
26.3	Full counting statistics . . . . .	479
26.3.1	Charge transport at low $T$ for arbitrary $g$ . . . . .	479
26.3.2	Full counting statistics at $g = \frac{1}{2}$ and general temperature . . . . .	482
	<b>Bibliography</b>	<b>483</b>
	<b>Index</b>	<b>503</b>