Preface

Differential-algebraic equations (DAEs) have been the object of increasing attention in the last three decades. Nowadays, they provide a valuable tool for system modeling and analysis within different fields, including nonlinear electric and electronic circuit theory, constrained mechanics, and control theory, among others.

The first part of this book addresses analytical properties of such differential-algebraic systems. The few existing monographs on DAEs are mainly focused on numerical aspects and, in most cases, they are restricted to a specific approach, structured around the differentiation, geometric, tractability, perturbation or strangeness indices, respectively. By contrast, the present book attempts to discuss a variety of analytical frameworks for the study of DAEs. The emphasis will be on projector methods based upon the tractability index, and also on reduction techniques supported on the geometric index. The differentiation index will also be briefly examined; note that it has received comparatively more attention in the DAE literature than the projector-based and reduction frameworks, in spite of the many benefits displayed by these approaches.

Projector-based methods, introduced for linear DAEs in Chapter 2, allow for precise input-output functional descriptions and explicit solution characterizations in terms of the original problem variables: this holds for linear time-varying DAEs with arbitrary index, under mild smoothness requirements. These methods have been mainly developed by Roswitha März and, accordingly, the material in Chapter 2 is crucially based on her work. Nevertheless, some recent or new contributions can also be found in this Chapter, concerning e.g. the so-called Π-projectors, a simplification of the decoupling of DAEs with properly stated leading term, or a detailed characterization of standard form linear problems.

DIFFERENTIAL-ALGEBRAIC SYSTEMS - Analytical Aspects and Circuit Applications

Reduction methods, based on the research of Rabier, Rheinboldt and other authors, define a powerful framework for the analysis of nonlinear DAEs. Chapter 3 is mainly focused on these techniques, introducing in particular a local approach for quasilinear DAEs in settings where the global assumptions of Rabier and Rheinboldt do not hold, and paving the way for subsequent analyses of singular problems. The differentiation index is also discussed in this Chapter; cf. Sections 3.1, 3.2 and 3.7.

From a dynamical point of view, the essential differences between DAEs and explicit ordinary differential equations (ODEs) arise in so-called singular problems, which lead to new dynamic phenomena such as those displayed at impasse points or singularity-induced bifurcations. Recent results on the classification and analysis of singularities are extensively discussed in Chapter 4. The topics covered on singularities of linear time-varying problems are the result of recent research, whereas the material on singular points of quasilinear DAEs is completely new.

The second part of the present monograph is focused on the analysis of DAEs arising in electrical circuit theory, emphasizing modeling aspects and considering both linear and nonlinear problems. Chapter 5 discusses nodal analysis methods, widely used in circuit simulation programs, with special attention to index characterizations. Chapter 6 addresses so-called branch-oriented analysis methods. The models arising from these techniques, which are not as well-suited as nodal ones for numerical simulation purposes, provide however several advantages from an analytical point of view. In particular, they make it possible to frame in the DAE context the state formulation problem for electrical circuits, including normal tree methods used to tackle it, and also to analyze different qualitative aspects of circuit dynamics.

I have tried to write this book in a self-contained manner, making it accessible to as many interested readers as possible. Some background material has been added with this aim; this includes regular matrix pencil theory, Schur complements and Cauchy-Binet expansions from matrix analysis, several elementary aspects of differential geometry, and rather detailed introductions to digraphs and to circuit theory fundamentals. See, specifically, Sections 2.1.1, 3.3 and 5.1. With this material, the prerequisites for reading this book virtually amount to undergraduate courses on differential calculus, linear algebra and ordinary differential equations. However, readers with more background on mathematical analysis, differential equations, and circuit and system theory will probably get a deeper insight into the contents of this monograph.

In the same spirit, technicalities have been intended to be kept at a minimum. In particular, the local properties discussed in Chapter 3 are not stated in terms of germs in order to make these results available to readers unfamiliar with this language. Examples have been chosen to be as simple as possible, being just focused on the ideas that they are intended to illustrate. In Part II, controlled sources are excluded from most analyses for the sake of simplicity, although subsection 6.2.6 indicates how to extend the results to circuits including a broad family of these sources. Notational and terminological issues are explained or recalled at the points where they show up; this should make the reading easier, specially regarding the different variable splittings appearing throughout.

This book can be used in a graduate course on DAEs. Additionally, different reader profiles might benefit from its contents. Some hints are given in the section *How to read this monograph* on page 22. In particular, the book may be of interest for applied mathematicians and analysts. It is worth indicating in this regard that, very often, the main goal in DAE analyses is to unveil their behavior in terms of some kind of related ODE; from this perspective, DAEs pose many problems which can be better framed within linear algebra or mathematical analysis contexts, rather than in the theory of differential equations. The book can also be helpful within numerical mathematics, covering the analysis which usually precedes the numerical simulation of differential and differential-algebraic equations.

Regarding applications, the present book may be useful for scientists and engineers interested in singular system theory and modeling issues. Part II may be of help in electrical and electronic circuit analysis and design. On the other hand, the general theory discussed in Part I can be used in other applications of DAEs arising for instance in mechanics, chemical processes or control theory. Other scientists aiming to get a glimpse on DAEs would profit from certain parts of the book.

Unfortunately, other aspects of DAE theory which might be of interest for some readers are beyond the scope of this monograph. These topics include numerical issues, normal form theory, over- and underdetermined problems, partial differential-algebraic equations (PDAEs), stochastic DAEs, singularity-induced bifurcations, or impasse phenomena and singularities in electrical circuits. Hopefully, the references cited on these topics will be of some help for the interested readers.

I wish to thank Ms Anna Tong, Commissioning Editor of World Scientific, for her kind invitation to undertake this project, and Ms Lai Fun Kwong for her support in the book production process. I am greatly indebted to Roswitha März not only for her comments and suggestions on this material, but also for the continued and pleasant research collaboration on these topics. Actually, I have a very nice memory of my research stays with the DAEs group at the Humboldt University of Berlin, where I also benefited from many fruitful discussions with René Lamour and Caren Tischendorf. The help of Diana Estévez-Schwarz is gratefully acknowledged, as well as that of Uri Ascher and Steve Campbell, who guided my first steps on DAEs. I am also thankful to Clara Ministral for many useful English tips. The efficient bibliographic support offered by the personnel of the Library of the ETSI Telecomunicación (UPM) has been very much appreciated. Finally, I want to express my gratitude to my wife, Raquel, for her warm understanding, patience and support during the writing of this book.

Ricardo Riaza
Madrid, December 2007