Preface

The notion of renormalization is at the core of several spectacular achievements of contemporary physics; originated in the context of Quantum Field Theory, where it appeared in order to solve the problem of the "ultraviolet divergences", it becomes later on (partly in a form known as "renormalization group") central in many other areas, like in the analysis of the critical properties close to phase transitions in classical statistical mechanics or in the theory of quantum liquids in condensed matter.

Renormalization is generally presented in a purely perturbative context (with no control of convergence of the series expansions), but in the last years, new mathematical techniques have been developed, allowing to put it on a firm mathematical basis. The aim of this book is to provide an introduction to rigorous non-perturbative renormalization in Quantum Field Theory, Statistical Physics and Condensed Matter. With respect to previous books on renormalization, the focus is mainly on fermionic (rather than bosonic) functional integrals, whose theory has been developed more recently and for which the structure of renormalization is not obscured by too many technicalities. Another important novelty is the implementation of Ward Identities based on local symmetries in the context of multiscale analysis, which allows the rigorous analysis of models with non trivial fixed points and anomalous behaviour. The book is devoted either to mathematicians and physicists aiming to enter into contact with the modern theory of renormalization; prerequisites are then limited to a minimum.

We start with an introduction to renormalization in physics and to the mathematical techniques for treating fermionic functional integrals, including multiscale decomposition techniques, tree expansions and determinant bounds. Such methods allow a unified treatment of models coming from Quantum Field Theory, Statistical Physics and Condensed matter. In particular, the first part of this book is devoted to constructive Quantum Field Theory, providing a mathematical construction of models at low dimensions and discussing the removal of the ultraviolet and infrared cut-off, the verification of the axioms and the validity of Ward Identities with the relative anomalies. The second part is devoted to lattice 2d Statistical Physics, analyzing in particular the theory of universality in perturbed Ising models and the computation of the non-universal critical indices in Vertex or Ashkin-Teller models. Finally in the third part, the theory of Quantum liquids like Luttinger or Fermi liquids is developed, considering models of interest in Condensed Matter like the Hubbard model in $1d$ or $2d$ or the Heisenberg spin chain.

Most of the material presented in this book grew out from common work with G. Benfatto and G. Gallavotti, and with the researchers composing the Roman school of rigorous renormalization, namely F. Bonetto, P. Falco, G. Gentile, G. Giuliani, A. Procacci and B. Scoppola. I have also benefitted from important discussions, which strongly influenced my point of view on renormalization, with J. Magnen and V. Rivasseau in Paris, with T. Spencer in Princeton and with K. Gawedzki in Lion.

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