

# Preface

Markov-modulated processes are processes that are modulated (or driven) by an underlying Markov process. The earliest of these processes studied in the literature are Markov-additive processes, namely, Lévy processes with a Markov component. Semiregenerative phenomena are regenerative phenomena that have, in addition, a Markov component. Such phenomena are important in the study of Markov-additive processes and arise in several important applications.

Our presentation emphasizes the interplay between theory and applications of stochastic processes. Accordingly, the first six chapters are devoted to theory along with illustrative examples, while the last three chapters treat applications to queueing, data communication, and storage systems. As a result, the book may be used both as a research monograph and as a textbook for a graduate course in applied stochastic processes focusing on Markov-modulated processes, semiregenerative phenomena, and their applications. Although the book is more easily read in a linear fashion, care was taken to give the reader the possibility of reading a chapter separately as proper referencing of results used from other chapters is done in a systematic manner.

The book starts with a brief introduction to recurrence and regeneration, which contains historical insights and brings out the connection between recurrent phenomena and renewal processes. This introduction, given in Chapter 1, prepares the reader for subsequent results in the book on semiregenerative phenomena, which are linked regenerative phenomena that have, in addition, a Markov property.

Following the introduction to recurrence and regeneration, we present in Chapter 2 an exposition of the basic concepts and main properties of Markov renewal processes (MRPs), along with the properties of some spe-

cial cases of Markov-additive processes (MAPs) which are of importance in applied probability. Our approach makes it clear that an MRP is a family of Markov-dependent renewal processes, whose corresponding family of renewal counting processes gives rise in a natural fashion to the so-called Markov renewal equation. We establish the connection between the existence and uniqueness of the solution of this equation and the finiteness of the total number of renewals. Among the MAPs considered in the chapter is the so called MMPP (Markov-modulated Poisson process), in which the rate of occurrence of Poisson events changes instantaneously with the changes of state of the modulating Markov chain, along with other Markov-compound Poisson processes. We also illustrate the use of infinitesimal generators for MAPs.

The theory of semiregenerative processes is developed in Chapter 3, where we explore the correspondence between semiregenerative sets and the range of a Markov subordinator with a unit drift, in the continuous-time case, or an MRP in the discrete-time case. In the former case, we construct a Markov subordinator with a unit drift whose range turns out to be a semiregenerative set. In the case where the label parameter set is finite we prove the converse, that every semiregenerative set corresponds to the range of a Markov subordinator. In the latter case, we show that the semirecurrent set of a semirecurrent phenomena corresponds to the range of an MRP and, conversely, a semirecurrent set can only arise in this manner.

In Chapter 4 we present some basic theory of the Markov random walk (MRW) viewing it as a family of imbedded standard random walks and consider a time-reversed version of the MRW which plays a key role in developing results analogous to those of the classical random walk. One of the central themes of the chapter is on the fluctuation theory of MRWs, first studied by Newbould in the 1970's for nondegenerate MRWs with a finite state ergodic Markov chain. Here, more general results for MRWs with countable state space are presented and a fundamental definition for degenerate MRW is given so that the probabilistic structure of MRWs can be firmly established. Together with the main result containing Wiener-Hopf factorization of the underlying transition measures based on the associated semirecurrent sets, it provides better insights for models with Markov sources and facilitates potential applications.

In Chapter 5 we establish the central limit theorem and of iterated logarithm for the additive component of an MRW. Our proofs are probabilistic using i.i.d. sequences of random variables denoting normalized increments of the additive component of the MRW between successive visits of the

modulating Markov chain to a given state. We express the parameters involved in terms of the transition measure of the MRW, and indicate an application of the theorems derived in the chapter to finance models.

In Chapter 6 we address MAPs of arrivals, and investigate their partial lack of memory property, interarrival times, moments of the number of counts, and limit theorems for the number of counts. We further consider transformations of MAPs of arrivals that preserve the Markov-additive property, such as linear transformations, patching of independent processes, linear combinations, and some random time transformations. Moreover, we consider secondary recordings that generate new arrival processes from an original MAP of arrivals; these include, in particular, marking, colouring and thinning. For Markov-Bernoulli recording we show that the secondary process in each case turns out to be an MAP of arrivals.

In Chapter 7 we consider single server queueing systems that are modulated by a discrete time Markov chain on a countable state space. The underlying stochastic process is an MRW whose increments can be expressed as differences between service times and interarrival times. We derive the joint distributions of the waiting and idle times in the presence of the modulating Markov chain. Our approach is based on properties of the ladder sets associated with this MRW and its time-reversed counterpart. The special case of a Markov-modulated M/M/1 queueing system is then analyzed and results analogous to the classical case are obtained.

In Chapter 8 we investigate a storage model whose input process  $(X, J)$  is an MAP, where, in addition to a nonnegative drift  $a(j)$  when the modulating Markov chain  $J$  with countable state space is in state  $j$ ,  $X$  has nonnegative jumps whose rate and size depend on the state transitions of  $J$ . This formulation provides for the possibility of two sources of input, one slow source bringing in data in a fluid fashion and the other bringing in packets. As for the demand for transmission of data we assume that it arises at a rate  $d(j)$  when the current state of  $J$  is  $j$ , and the storage policy is to meet the demand if physically possible. We prove the existence and uniqueness of the solution of the basic integral equation associated to the model and investigate various processes of interest, showing in particular that the busy-period process is a Markov-compound Poisson process. Transforms of the various processes of interest are obtained and the steady state behavior of the model is investigated.

Finally, in Chapter 9 we investigate a storage model where the input and the demand are continuous additive functionals on a modulating Markov chain, with the instantaneous input and demand rates being only a func-

tion of the state of that chain. As in the model considered in Chapter 8, the storage policy is to meet the largest possible portion of the demand. Our analysis is based on the net input process imbedded at the epochs of transitions of the modulating Markov chain, which is an MRW. We use a Wiener-Hopf factorization for this MRW, which also gives results for the busy-period of the storage process, aside from direct results for the unsatisfied demand. Our analysis allows for the Markov chain to have infinite state space and we derive the time dependent as well as the steady state behavior of both the storage level and the unsatisfied demand.

To a great extent, the book's contents that have been summarily described constitute an updated review with many new insights of research work started twenty years ago at Cornell University and originally published in [81, 82, 88–90, 93–95, 112]. António Pacheco and L.C. Tang are grateful respectively to Instituto Superior Técnico and the National University of Singapore for their support to study at the School of Operations Research and Industrial Engineering of Cornell University with Professor N.U. Prabhu as their PhD thesis advisor, and are very honored to have been his last two PhD students. We would like to thank our friends and colleagues from whom we have benefited greatly through some of their earlier works and discussions. This includes, in particular, Dr. Yixin Zhu and Arnold Buss, who have worked on similar problems in the 1980's.

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