Preface

There is already a vast selection of text books in engineering mathematics, and one can be excused for emitting a groan at the appearance of yet another. However, the two authors of this text sense a need for an additional text for several reasons.

Most of the current text books in engineering mathematics are exactly that, books targeted at engineers irrespective of their particular engineering discipline. The variety of engineering (chemical, civil, electrical, mechanical and so forth) disciplines today, however, have become sufficiently different from one another that we feel book with a focus on just chemical engineering mathematics is justified. Although the engineering disciplines all share a common set of mathematical tools, the art of writing mathematical models varies from discipline to discipline because they invoke different physical laws. By focusing only on models in chemical engineering, we can provide an adequate number of modeling examples to illustrate the theory without having them taking up most of the book.

Chapter 1 consists primarily of illustrations of the techniques required for writing models of chemical engineering systems, and additional model formulation examples are scattered throughout the remaining chapters. The chapter serves not just to illustrate modeling but also serves as brief review of chemical engineering concepts. We hope that mathematics students may also find the chapter valuable as physical motivation to explore and study these equations.

Like chapter 1, chapter 2 will be a review of material for many readers. It covers solution of various common ordinary differential equations without going deep into the theory of these equations.

The relatively recent emergence of symbolic mathematical software has significantly altered the way engineers do mathematics. Cumbersome evaluations of integrals, solutions of ordinary differential equations, and simplifications of tangled expressions are now performed routinely by computers instead of by hand, which allows the engineer and applied mathematician to attack thornier problems. Nonetheless, a symbolic solution obtained by a computer requires an understanding of the relevant mathematical structures in order to write the code. Engineering students should therefore strive to understand the mathematical structures that are important to solving a given problem rather than spend the time and effort to learn every trick and rule for solving integrals and equations. Chapter 3, which covers finite dimensional vector spaces, is therfore quite rigorous and goes into considerably more mathematical detail about the structure of finite dimensional linear spaces than most engineering students are familiar with. The chapter contains the proofs of almost all the relevant results and the student is urged to read and understand these proofs as part of the learning process. This chapter as well as the remaining chapters should be studied while having access to a PC with symbolic mathematical software and the students should learn the relevant commands and programming steps used to solve problems on the platform of choice.

The primary goal of chapter 4 is to introduce tensors. It also gives a brief introduction to curvilinear coordinate systems.

Chapters 5 and 6, linear difference and differential equations, are again heavily dependent on the material covered in the chapter on finite dimensional vector spaces. Linear difference and linear differential equations share a very similar solutions structure, a fact that the student should strive to appreciate.

Chapter 7 introduces Hilbert spaces, and generalizes the ideas from finite dimensional vector spaces to infinite dimensional spaces. The algebraic structure from finite dimensional theory provides a backdrop, but by itself, is shown to be inadequate to fully appreciate infinite dimensions. One needs additional structure to develop a theory suitable for applications, which is the concept of an inner product. Of central importance is the concept of a self adjoint operator, the infinite dimensional analog of a real symmetric matrix. Self adjointness is a type of symmetry and problems with self adjoint operators are typically far easier to solve that problems that are not.

Chapter 8, the last chapter, describes solution methods for various types of differential equations. Solution of self adjoint problems, usually second order PDEs, are described in great detail and introductions are given to solution of first order PDEs and to solution of PDEs by similarity transformation.

In this text, only a small amount of effort will be spent on nonlinear systems. When nonlinear systems are mentioned, it is primarily to illustrate how complicated and different these systems are from linear systems and to make the reader appreciate the simplicity and beauty of linear systems.

As an illustration of the use of symbolic mathematical software for PCs, many of the examples in the text have been coded in Maple and are available on the books web site, http://www.che.lsu.edu/faculty/hjortso/mathbook/index.htm.

Acknowledgment

During the many years that this book took shape, starting from a set of typed lecture notes, many students have contributed with suggestions and corrections. They have all helped to improve the text and we are grateful for them all. We have also received many suggestions form colleagues and are particularly grateful

to our colleagues and former teachers who have given us permission to use their old homework problems in this book.

Nomenclature

The material covered in these notes is so diverse that a consistent nomenclature that does not clash with established nomenclature in some area is virtually impossible. The first chapter, in particular, makes use of a lot of different physical quantities and these are indicated using common chemical engineering nomenclature, such as C for concentrations, ρ for density etc. This nomenclature is given in a Table 0.1 below. The remaining chapters are primarily abstract mathematics and the nomenclature used in these is given in Tables 0.2, 0.3 and 0.4.

Table 0.1 Nomenclature for physical (dimensioned) quantities.

Symbol	variable	Units
A	Area	length^2
C	Concentration	moles/volume
D	Diffusion coefficient	length^2/time
E	Activation energy	energy/mole
F	Molar flowrate	moles per time
h	Heat transfer coefficient	energy/(length^2 time degree)
ΔH_r	Heat of reaction	energy/mole
k	Reaction rate constant	Depends on the rate expression
k, k_T	Thermal conductivity[a]	energy/(length time degree)
k_m	Overall mass transfer coefficient	length/time
$k_{m,I}$	Mass transfer coefficients for phase I	length/time
k_0	Frequency factor of reaction rate constant	Depends on the rate expression
M_n	Molecular weight of compound n	
P	Pressure	force/area
$\dot{q}$	Specific rate of heat generation	energy/(time volume)
Q	Volumetric flow rate	volume/time
μ	Viscosity	mass/(length time)
$r(C)$	Specific reaction rate	moles/(time volume)
R	Gas Constant	energy/(mole temperature)
t	time	time
T	Temperature	temperature
U	Overall heat transfer coefficient	energy/(time lenght^2 temperature)
V	Volume	length^3
ω	Mole fraction	number between 0 and 1
ρ	Density	mass/volume

[a]The simpler notation k is used whenever possible. However, in problems that include chemical reactions the k_T-notation is used to distinguis thermal conductivity from reaction rate constants.

Table 0.2 Latin letters.

Symbol	Common use
$a_{n,m}$	Element in row n, column m of the matrix A. Matrices themselves are indicated by capital letters and their elements by the same letter in lower case. The size of a matrix, i.e. the number of rows and columns, may be indicated as a subscript, e.g. $A_{N\times M}$.
B_x	Basis, identified by x, of a finite dimensional vector space.
C_n	Arbitrary constants
$\{e_n\}$	Usual set of basis vectors in $\mathbb{R}^N$. Thus e_n is an N-dimensional vector for which all elements equals zero, except for element n which equals 1.
J	Jordan canonical form matrix.
$\{f_n\}$	Set of basis vectors in $\mathbb{R}^N$. Not necessarily same as $\{e_n\}$
K	Matrix of eigenvectors and/or generalized eigenvector chains.
n, m	Counters, e.g. on vector or matrix elements or on finite sums. In general, $n = 1..N$ and $m = 1..M$.
N, M	Upper limits on counters n and m.
s	Dummy variable, such as dummy variable of integration or parameter in a parametric representation.
t	Free variable in ordinary differential equations. This usage works fine in differential equations that are models of time dependent phenomena, but may give rise to confusion in differential equations that are typically associated with variation along a spacial coordinate.
v, w	vectors in finite dimensional vector spaces. Specific elements are indicated v_n and w_n.
$\tilde{v}_n$	Eigenvectors or rows of the eigenvalue λ_n. Or, generalized eigenvecor of rank n if the eigenvalue is understood.
$\tilde{v}_{n,m}$	Generalized eigenvector of rank m of λ_n.
V, W	Finite dimensional vector spaces.

Table 0.3 Greek letters.

Symbol	Common use
Λ	Diagonal matrix of eigenvalues.
λ	Eigenvalue.
ϕ	Longitudinal coordinate in spherical system.
τ	Integer counter in difference equations.
θ	Angular coordinate in cylindrical coordinate system or latitudinal coordinate in spherical system or any angle.
$\theta_{n,m}$	Direction cosines.
Θ	Rotation matrix.

Table 0.4 Other math notation.

Symbol	Common use
$\tilde{}$	Eigenvector or generalized eigenvector
$\hat{}$	Indicates either a different coordinate representation or parametric representation of an object. Thus v and $\hat{v}$ indicate the coordinate representation of the same vector and t and $\hat{t}$ indicate the parameter in different parametric representation of a vector space.
$\overline{C}$	Complex conjugate of C.
$*$	Adjoint operator.
$\mathbb{C}$	The set of complex numbers
$\mathbb{F}$	Algebraic field
i	$\sqrt{-1}$
$\mathbb{Q}$	The set of rational numbers
$\mathbb{R}$	The set of real numbers
$\mathbb{Z}$	The set of integers.
$\forall$	Short for "For all".
$\exists$	Short for "There exists".
$\ni$	Short for "Such that".