Chapter 1

Introduction

At the end of the 19th century, one could take pride in the fact that the laws of physics were now understood. With newtonian mechanics, the statistical analysis of Boltzmann, and Maxwell's equations for electromagnetism, one had an excellent description of the world we see around us. At that point, a series of experiments were performed whose results simply could not be understood within this classical framework. These experiments probed a microscopic, or high-velocity, world that went far beyond our everyday perception. The resolution of these paradoxes led to the theories of quantum mechanics and special relativity, which provide the foundation of modern physics. Atomic, nuclear, particle, and condensed-matter physics are all built on this foundation. Einstein's extension of his theory of special relativity to general relativity provides the current basis for our understanding of gravitation and cosmology—physics to the farthest reaches of the universe. The theories of quantum mechanics, special relativity, and general relativity have been remarkably successful. All current experiments can be understood within this framework. The goal of this book is to introduce a reader, with an assumed knowledge of classical physics, to modern physics.[1]

It is assumed that the reader has had a good one-year, calculus-based freshman physics course, along with a good one-year course in calculus. A sufficient number of appendices are included to bring the reader up to speed on any additional mathematics required at the outset. Over 175 problems are included in the book, some for each chapter. While there are many problems that directly amplify the material in the text, there are also a great number of them that will take dedicated readers just as far as they want to go in modern physics. Although the book is designed so

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[1] There is a nice irony in the parallel statement, "At the end of the 20th century, one could take pride in the fact that the laws of physics were now understood."

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that one can, in principle, read and follow the text without doing any of the problems, the reader is strongly urged to attempt as many of them as possible in order to obtain some confidence in his or her understanding of the basics of modern physics. With very few exceptions, the reader should find that the text, appendices, and problems form a self-contained volume.

Chapter 2 reviews the essential elements of classical physics. We start with Newton's laws for point mechanics and then consider the continuum mechanics of the planar oscillations of a stretched string, an analysis that provides the basis for much of what is done in this book. An appendix summarizes the main features of the resulting Fourier series and Fourier integrals. With the Boltzmann distribution as starting point, the classical equipartition theorem is derived, which describes the equilibrium thermal energy in each normal-mode oscillation of a system. Two appendices present the essentials of thermodynamics and derive the Boltzmann distribution from Boltzmann's more general statistical assumptions. The essentials of electromagnetism are described and Maxwell's equations presented, with some applications to electromagnetic waves. An appendix derives two theorems of vector calculus that convert Maxwell's equations to the integral form in which they are customarily introduced.

Chapter 3 presents the key experiments whose results could not be understood in classical terms. These include the vanishing of the specific heat of a solid at low temperature, the spectrum of black-body radiation in a cavity, the photoelectric effect, Compton scattering of light from an electron, and the discrete spectral lines observed in atomic radiation. The resolution of the first two problems was obtained through Planck's thermal distribution, which reduces to the equipartition result at high temperature, but indicates a discrete excitation energy $h\nu$ of an oscillator of frequency ν at low temperature where $k_{\rm B}T/h\nu \ll 1$; here $k_{\rm B}$ is Boltzmann's constant, and h is the new Planck's constant. Einstein resolved the paradox in the photoelectric effect by assuming that the energy in a light wave actually comes in discrete packets of energy, called *photons*, each with energy $h\nu$. The Compton effect was then understood through the conservation of energy and momentum in the collision of a photon with an electron. The discrete nature of atomic radiation was explained through Bohr's revolutionary assumptions that atomic systems possess discrete stationary states in which they do not radiate, and they radiate a photon of energy $h\nu = E_i - E_f$ when making a transition between those states. His simple model of the atom involving the quantization of the angular momentum in circular orbits explained the main features of the spectrum of hydrogen.

De Broglie argued that if waves (electromagnetic radiation) exhibit particle properties (photons) then perhaps particles should manifest wave behavior. In chapter 4, the essential elements of quantum mechanics are developed starting from de Broglie's relation for matter, and culminating in the Schrödinger equation, a differential equation describing the space-time propagation of a wave associated with a particle. The absolute square of this wave function is given a probability interpretation, which together with the corresponding probability current, provides a connection with physical measurements. Many elementary one-dimensional problems are solved and interpreted, including the free particle, particle in a box, particle incident on a potential barrier, and simple harmonic oscillator. The analysis is extended to a three-dimensional box, and then angular momentum and spin are introduced. The quantum mechanics of many identical integer-spin bosons and half-integral-spin fermions is analyzed. Enough of the essential aspects of quantum mechanics is developed in the text for all of these applications. Several additional homework problems then take readers just as far as they want to go in further developing the theory. Two appendices derive the "Golden Rule" for the transition rate and solve the two-level problem in quantum mechanics.

The discussion then proceeds to further applications of the basic principles of quantum mechanics. Chapter 5 is concerned with atomic physics, where the vector model of angular momentum is developed, the Zeeman effect and spin-orbit interaction analyzed, and the Thomas-Fermi statistical model of the structure of many-electron atoms presented. The latter finds a more basic formulation in terms of the Hartree mean-field approximation, which provides valuable insight into the behavior of quantum many-body systems throughout the book. At this point, one has enough background to understand the periodic system of the elements and the chemical behavior of its various families.

Chapter 6 is concerned with nuclear physics. The concepts of baryons (neutrons and protons), mean lives, and atomic masses are presented, and β-decay described. The two-body problem is solved with a finite square-well potential, and the observed spectra of the lightest nuclei discussed. The semi-empirical mass formula, which gives the average binding energy of nuclei throughout the periodic table, is developed. It is shown how electron scattering provides a microscope for actually seeing the nucleus. As with the periodic system of the elements, the Hartree approximation provides a basis for understanding the nuclear shell model. With the introduction of a strong spin-orbit force, the shell model puts one in a position to predict properties of nuclei throughout the periodic table.

Applications to particle physics are covered in chapter 7. The relevant forces are categorized, and the concept of an antiparticle is introduced. Leptons, which have only weak and electromagnetic interactions, are discussed. The strongly interacting hadrons (mesons and baryons) are described, together with their properties of isospin, strangeness, and charm. The Yukawa interaction between two baryons, arising from the interaction with a meson field, is derived. The observed hadron multiplets are exhibited, and it is shown how hadron structure can be understood in terms of an underlying substructure of quarks. An argument is given as to why single quarks are not observed as free particles.

Feynman diagrams, together with associated Feynman rules for the scattering S-matrix, provide the language of particle physics. It is shown how one goes from the S-matrix to a transition rate and then to a cross section. Several examples are presented: quantum electrodynamics (QED) with virtual photon exchange; quantum chromodynamics (QCD) with the exchange of gluons between quarks, and gluon self-couplings; and the standard model of electroweak interactions involving the exchange of heavy weak vector bosons, both charged and neutral.

Special relativity is the topic in chapter 8, starting with a description of the Michelson-Morley experiment, which ultimately demonstrated that the speed of light is the same in any inertial frame. It was Einstein's genius to give the Lorentz transformation, which leaves the form of the wave equation for light invariant, a physical interpretation as the actual relation between space-time coordinates $(\mathbf{x}, t)$ in two different inertial frames. The consequences are profound, as time and space now become relative coordinates, changing from inertial frame to inertial frame. Both time dilation and Lorentz contraction are analyzed. Special relativity is formulated in terms of rotations in a four-dimensional Minkowski space, whose fourth component $x_4 = ict$ is imaginary. Several applications of special relativity are discussed, including an analysis of the structure of white-dwarf stars.

Chapter 9 discusses the union of the two underpinnings of modern physics, quantum mechanics and special relativity. The Dirac equation for spin-1/2 fermions is derived from general assumptions, and its implications are examined in detail. Knowledge of properties of the Dirac equation allows us to actually construct the vertices in the Feynman diagrams for QED, QCD, and the standard model of electroweak interactions.

A point particle moving without friction on a two-dimensional surface of arbitrary shape forms a paradigm for the introduction of Einstein's theory of general relativity in chapter 10. Einstein's theory is introduced through a set of three assumptions on the structure of space-time and the corresponding particle motion. The Schwarzschild solution to the Einstein field equations outside of a spherically symmetric source is introduced, and its physical implications analyzed, including time dilation and radial Lorentz contraction. The properties of the Robertson-Walker metric (with $k = 0$), corresponding to a uniform mass density throughout all space are examined as an introduction to cosmology. The leading order cosmological redshift is derived and the concept of the horizon introduced.

Quantum fluids are macroscopic systems of many identical particles whose behavior reflects the underlying quantum mechanics. In chapter 11 the properties of two such systems are analyzed. The first is superfluid ^{4}He, whose atoms are spin-zero bosons and whose properties reflect Bose condensation. The velocity field is related to the phase of the single-particle ground-state wave function, leading to the quantization of the circulation about a vortex in the superfluid. The Hartree approximation helps one to understand the interacting system. That analysis is extended through the introduction of the Gross-Pitaevskii equation, which is solved to obtain the spatial structure of a vortex. Landau's argument on the relation of the quasiparticle excitation spectrum to superfluidity is presented. The second quantum fluid is that of electrons in a superconductor, whose empirical properties are summarized. It is argued that bound pairs of electrons will yield properties similar to that of Bose systems, in particular superfluid flow. The Bethe-Goldstone equation for a pair of fermions interacting in the presence of a filled Fermi sea is derived. It is shown how an attractive interaction between pairs of particles near the Fermi surface, with opposite momenta and spins, will actually lead to such a bound state. These are known as Cooper pairs. Phonon exchange in metals provides the attractive interaction between electrons, and Cooper pairs form the underlying basis for the very successful BCS theory of superconductivity. In analogy with the quantization of circulation in a superfluid Bose system, a derivation is given of the quantization of flux in the magnetic flux tube that penetrates a type-II superconductor.

Chapter 12 introduces the notion of a quantum field, which underlies most of modern theoretical physics. To do this, we return to the starting point of the book, the analysis of the planar transverse oscillation of a string. An expansion of the displacement of the string $q(x, t)$ in normal modes reduces the energy of the string to the sum of contributions of uncoupled simple harmonic oscillators, one for each normal mode. These oscillators are readily quantized, and the quanta of the normal modes are identified as phonons. The displacement now becomes a local quantum field operator $\hat{q}(x,t)$, where the normal-mode amplitudes are the phonon creation and destruction operators. The electromagnetic field is quantized in an exactly analogous fashion. Here the quantum field operator is the Coulomb-gauge vector potential $\hat{\mathbf{A}}(\mathbf{x},t)$, and the quanta of the normal-mode excitations of the field are the transverse photons. The concept of stimulated emission into a mode where photons are already present now follows immediately. The general expression for the photon radiation rate for any quantum mechanical system is derived in the problems. The quantum field analysis is extended to that of a Dirac field, where the Fermi statistics necessitates the introduction of anticommutation relations for the creation and destruction operators. Finally, the utility of quantum fields in analyzing the behavior of many identical non-relativistic interacting particles is demonstrated.

It is worth re-emphasizing that physics is an *experimental science*, and a course such as this *must* be accompanied by a good, thorough laboratory course in experimental modern physics.

There are many good, existing books that develop the experimental and theoretical aspects of modern physics, although the theory is generally covered in less depth: for example, [Eisberg and Resnick (1985); Resnick and Halliday (1992); Ohanian (1995); Krane (1996); Serway, Moses, and Moyer (1997); Bernstein, Fishbane, and Gasiorowicz (2000); Beiser (2002); Taylor, Zafiratos, and Dobson (2003)]. It is assumed that readers will study at least one such book in parallel with this one, in order to add sufficient breadth to their knowledge of modern physics.

Progress in physics, as in all of science, does not take place in a linear fashion. For every one of the concepts and results described in this book there have been tens, if not hundreds, of wrong turns and blind alleys. On the other hand, progress does occur by building on the foundation laid by previous scientists. The goal of this book is to provide a tour of the physics foundation established in the twentieth-century. It is hoped that the reader will both enjoy and benefit from the journey. Let us begin.