Preface

This volume is dedicated to the presentation and discussion of state of the art studies in Mathematical Programming and Game Theory for decision making problem in the form of twenty five papers. It is a peer reviewed volume under the Platinum Jubilee Volume Series of Indian Statistical Institute. The topics of this volume include the application of the theory and methods of mathematical programming to problems in statistics, finance, electrical networks and game theory. Mathematical programming comprises a variety of paradigms (theoretical frameworks) tailored to different kinds of problems and it is extremely useful to problems in strategic decision making. Support vector machines, bilevel programming, neural network models, cooperative games, non-cooperative games and stochastic games appear in this volume. It is hoped that the research articles of this volume will significantly aid in the dissemination of research efforts in these areas. In this volume some pioneers of the field, as well as some prominent younger researchers have contributed articles which are briefly mentioned below.

Mathematical programming has long been recognized as a vital modelling approach to solve optimization problems. In Chapter 1, Lyn C Thomas presents a review on some of the applications of mathematical programming in finance which includes prominent and well documented applications in long-term financial planning and portfolio problems. This includes asset-liability management for pension plans and insurance companies, integrated risk management for intermediaries, and long-term planning for individuals. In this chapter, it is discussed how one can use linear programming to estimate the term structure of interest rates for the prices of bonds.

Even though several anti-cycling pivot selection rules exist for the sim-

plex method for a general linear program (LP), none is known to avoid stalling (an exponential sequence of degenerate pivots). Santosh N. Kabadi and Abraham P. Punnen discuss an anti-stalling pivot rule for linear programs with totally unimodular coefficient matrix in Chapter 2. For an LP with m constraints and totally unimodular coefficient matrix, pivot selection rule presented in this chapter guarantees that the simplex method performs at most m consecutive degenerate pivots or declares that the current solution is optimal.

Katta G. Murty developed a new interior point method for linear programming, based on a new centering strategy that moves any interior feasible solution x^0 to the center of the intersection of the feasible region with the objective hyperplane through x^0, before beginning the descent moves. Using this centering strategy, that method obtains an optimum solution for an LP by a very efficient descent method that uses no matrix inversions. In Chapter 3, he extended this method into a descent method for solving quadratic programs (QP). Compared to other existing methods for QP, the new method is able to handle it with minimal matrix inversion computations.

Chapter 4 by Richard Caron and Tim Traynor is about the analysis of sets of constraints, with no explicit assumptions. The relationship between the minimal representation problem and a certain set covering problem of Boneh is explored. This provides a framework that shows the connection between minimal representations, irreducible infeasible systems, minimal infeasibility sets, as well as other attributes of the preprocessing of mathematical programs.

Most research on algorithms for combinatorial optimization uses the costs of the elements in the ground set for making decisions about the solutions that the algorithms would output. For traveling salesman problems, this implies that algorithms generally use arc lengths to decide on whether an arc is included in a partial solution or not. In Chapter 5, Diptesh Ghosh, Boris Goldengorin, Gregory Gutin and Gerold Jäger study the effect of using element tolerances for making these decisions and several greedy algorithms for it based on tolerances are proposed for traveling salesman problem.

In Chapter 6, T. S. Arthanari studies the membership problem for the pedigree polytope. In this chapter, it is shown that a necessary condition for membership in the pedigree polytope is the existence of a multicommodity flow with value equal to unity in a layered network.

Many real life scheduling problems involve the use of a graph coloring

problem where the vertices of a graph $G(V, E)$ are colored such that the coloured graph satisfies certain desired properties. Nirmala Achuthan, N. R. Achuthan and R. Collinson discuss one such graph coloring problem in Chapter 7. The $k-defective\ chromatic\ number\ \chi_k(G)$ of a graph G is the least positive integer m for which G is $(m, k) - colorable$. In this chapter, exact algorithms based on partial enumeration methods to determine the one defective chromatic number $\chi_1(G)$, of a graph G are developed.

The vertical block matrix arises naturally in the literature of stochastic games where the states are represented by the columns and actions in each state are represented by rows in a particular block. S. K. Neogy, A. K. Das and P. Das present some results related to complementarty problem involving vertical block matrices in Chapter 8. A neural network algorithm for solving a vertical linear complementarity problem is also discussed.

In Chapter 9, Reshma Khemchandani, Jayadeva and Suresh Chandra present a fuzzy extension to twin support vector machines for binary data classification. The approach can be used to obtain an improved classification when one has an estimate of the fuzziness of samples in either class.

Except for constrained least squares, seldom is linear regression by least squares presented as an optimization problem whereas regression by minimum sum of absolute errors (MSAE) regression is always framed as a linear optimization problem. However, most students of statistics are unfamiliar with methods of mathematical programming. Given the dearth of treatment to regression by MSAE in textbooks, literature reviews and updates to MSAE regression such as Chapter 10 by Subhash C. Narula and John F. Wellington becomes an important learning resources to the student, researcher, and practitioner.

Consider a stochastic securities market model with a finite state space and a finite number of trading dates. In Chapter 11, Stephen A. Clark and Cidambi Srinivasan discuss how arbitrage price theory is modified by a no short-selling constraint. The principle of No Arbitrage is characterized by the existence of an equivalent supermartingale measure. In this chapter, it is shown that the Law of One Price holds for marketed claims if and only if there exists an equivalent martingale measure. Given that the Law of One Price prevails, then a contingent claim has a unique fundamental value if and only if it is the difference of two marketed claims. The main tool for arbitrage analysis in this essay is finite-dimensional LP duality theory.

In Chapter 12, H. Narayanan discusses about solving min cost flow problems approximately by transforming them to network analysis prob-

lems. This Chapter provides a relook at commonly used algorithms in computational linear algebra by associating an electrical network with the linear equations.

In Chapter 13, A. K. Bardhan and Udayan Chanda present optimal control policies of quality level and price for the introduction of a new product with two competing technology generations in a dynamic environment and also proposes a new model in this regard. The proposed model in this chapter is a combination of diffusion models and the cost function, which is capable of estimating the future profit trends.

Katta G. Murty presents a simple and easy method to implement nonparametric technique to forecast the demand distribution based on statistical learning, and ordering policies in Chapter 14. An application of this nonparametric forecasting method to portfolio management is also presented.

Chapter 15 by S. Dempe, J. Dutta and B. S. Mordukhovich is devoted to an application of advanced tools of modern variational analysis and generalized differentiation to problems of optimistic bilevel programming. Some new necessary optimality conditions are derived for two major classes of bilevel programs: those with partially convex and with fully convex lower-level problems.

Chapter 16 contains a summary of the talk by R. J. Aumann, Nobel Laureate, which contains a discussion on Game Engineering.

In Chapter 17, Pradeep Dubey and Rahul Garg consider a communications network in which users transmit beneficial information to each other at a cost. Conditions under which the induced cooperative game is supermodular (convex) is presented. This analysis is in a lattice-theoretic framework, which is at once simple and able to encompass a wide variety of seemingly disparate models.

Magnus Hennlock define a robust feedback Nash equilibrium in Chapter 18 and solve analytically in a differential climate model with N regions based on an approach of IPCC 2001 scientific report for calculating radiative forcing due to anthropogenic CO_2 emissions. In addition, uncertainty is introduced by perturbing the climate change dynamics such that future radiative forcing and global mean temperature will have unknown outcomes and probability distributions. There are n asymmetric investors, each investing in a portfolio containing N regional capital stocks used in production that generates CO_2 emissions. In each region there is one policy maker, acting as a regional social planner, that chooses regionally optimal abatement policies. Dynamic maximin decision criteria are applied for the

policy makers in a robust feedback Nash equilibrium for N policy makers' abatement strategies and n investors' investment strategies.

In Chapter 19, Haruo Imai and Katsuhiko Yonezaki consider a multi-person bargaining problem where players interests are correlated. This chapter investigates the limit outcomes of the stationary subgame perfect equilibrium outcomes of the sequential bargaining game with a coalition under two different bargaining protocols and correlation of interests are found within each coalition. Here limit means the case where the interval between the two consecutive offers vanishes. The result shows that an endogenous delegation occurs in each coalition to its toughest member. The outcome exhibits a sharp distinction that under the fixed order rule, the size of coalition does not matter, while under the predetermined proposer rule, it matters.

Chapter 20 by Dawidson Razafimahatolotra investigates stability properties of effectivity functions. The Bargaining Set in effectivity function generalizes the concept of cycles and connects it with the well known stability notion of bargaining sets. The first part devotes to the study of relations between cycles and implement a class of effectivity functions for which theses cycles are equivalent. Part two of this Chapter is devoted to analyze the stability of the bargaining sets and give relations between them. Bargaining sets considered are by Zhou, the Mass-Colell and the Aumann Davis Maschler's bargaining sets.

In Chapter 21, Agnieszka Wiszniewska-Matyszkiel considers a game modelling a market consisting of two firms with market power and a continuum of consumers. A specific feature of a market for toys is considered with each firm producing two kinds of distinguishable goods. The problem of finding a Nash equilibrium implies firms' optimal advertising and production plans over time, where the aggregate of demands of consumers may depend on firms' past decisions. Equilibria at this market may have strange properties, like oscillatory production and advertising strategies.

R. B. Bapat introduces two classes of games in Chapter 22 and shows that they are balanced. In regression games, the observations in a regression model are controlled by players, and the worth of a coalition is inversely proportional to the variance of the estimate of the regression parameter. In connectivity games the players control the edges of a graph and the worth of a coalition is directly proportional to the degree of connectivity of the subgraph formed by the corresponding edges.

Chapter 23 by Somdeb Lahiri presents the concept of the induced combinatorial auction of a nonnegative TU game and shows that the existence

of market equilibrium of the induced combinatorial auction implies the existence of a possibly different market equilibrium as well, which corresponds very naturally to an outcome in the matching core of the TU game. In this Chapter, it is shown that the matching core of the nonnegative TU game is non-empty if and only if the induced combinatorial auction has a market equilibrium.

Arrow formulated an important conceptual framework enabling one to discuss various collective decision making problems in an axiomatic fashion. There is, nevertheless, no topological structure given in Arrow's social choice framework to make it possible to discuss continuity of social welfare functions. In the turn of 1980s Chichilnisky had a systematic framework to discuss continuity of certain type of social welfare functions. In Chapter 24 by Kari Saukkonen, it is explained what continuity of a social welfare function is for Chichilnisky. It is then pointed out that there are difficulties, if this viewpoint is extended to cover continuity of Arrovian social welfare function, because of too specific assumption about the topological structure and dimension of the state sets. The discussion suggests that Chichilnisky's framework is not of much help in formulating appropriate topological foundations for the Arrovian social choice theory conceptualizing, for example, the workings of capitalistic democracy.

Finally in Chapter 25, S. K. Neogy, A. K. Das, S. Sinha and A. Gupta consider a mixture class of zero-sum stochastic game in which the set of states are partitioned into sets S_1, S_2 and S_3 so that the law of motion is controlled by Player I alone when the game is played in S_1, Player II alone when the game is played in S_2 and in S_3 the reward and transition probabilities are additive. It is proved that the game with SC/AR-AT mixture has the ordered field property by showing that the problem of solving the value vector v_s^β and optimal stationary strategies $f^\beta(s)$ for Player I and $g^\beta(s)$ for Player II for such a mixture type of game can be formulated as a complementarity problem. This gives an alternative proof of the ordered field property that holds for such a mixture type of game.

The 25 refereed articles contained in this volume are selected from 43 papers presented in International Symposium on Mathematical Programming for Decision Making: Theory and Applications which was organized as a part of the Platinum Jubilee Celebrations of the Indian Statistical Institute during January 10-11, 2007 at Indian Statistical Institute, Delhi Centre. The symposium was inaugurated by Professor Robert J. Aumann who delivered the inaugural talk on Game Engineering. The welcome address was delivered by Professor S. K. Pal, Director, Indian Statistical Institute. This

symposium provided a forum for national and international academicians, researchers and practitioners to exchange ideas and approaches, to present research findings and state-of-the-art solutions, to discuss new developments in the theory and applications of mathematical programming to the problems in business and industries. A session titled S. R. Mohan Memorial Session was arranged to recall the memory of our colleague Professor S. R. Mohan (who passed away in October 2005) and his contribution in the area of Mathematical Programming and Game Theory. In fact, some of the papers are dedicated to the memory of Professor S. R. Mohan. It is the hope of the editors that the majority of the papers will simulate questions and possible solutions that are of interest to researchers of these areas.

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(Editors)