

Erratum II: Lecture Notes in Applied Differential Equations of Mathematical Physics - Some misprints corrections

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2) “Lecture Notes in Applied Differential Equations of Mathematical Physics”,

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Pag. 4 - Eq.(1.15), read:

$$\left\{ 4\pi \int_0^\varepsilon dv \cdot v^2 \cdot \frac{1}{v^2} \right\}^{1/2}.$$

Pag. 7 - Eq.(1.26), read:

$$\lim_{N \rightarrow \infty} \left\{ \frac{\Gamma(\frac{2}{2})}{2(\sqrt{\pi})^2} \cdot \frac{1}{N-2} e^{(2-N)\lg(\bar{r}-r')} - \frac{1}{N-2} \right\}.$$

Pag. 23 - Eq.(1.93), read:

$$\|(TV)(\vec{r})\|_{L^2} \leq (4\pi \text{diam}(\Omega))^{1/2} \|U\|_{L^2}.$$

$$|\mu_r| \leq (4\pi \text{diam}(\Omega))^{1/2}.$$

Pag. 60 - below Eq.(3.5), read: “...given by the vector (φ, π) such that $(A\varphi, A\varphi)^{(0)} < \infty$ and $(A\pi, \pi)^{(0)} < \infty$.”.

Pag. 73 - Eq.(3.79), read:

$$\frac{\partial^2 U}{\partial^2 t} = -AU + B_1 \frac{\partial U}{\partial t} + B_2 U + F(U, t).$$

Pag. 74 - Eq.(3.84), read:

$$2V_2 = +B_1 \pm (B_1^2 - 4B_2)^{1/2}.$$

Pag. 111 - Eq.(5.29), read:

$$\overbrace{\left[\bigcup_{N=1}^{\infty} L^2(\mathbb{R}^N, d\mu(h_1, \dots, h_N)) \right]}^{\text{Topological closure}} = L^1(L^2(\mathbb{R}^2), d\mu(h)).$$

Pag. 124 - Eq.(5.50a) - Eq.(5.50b); read correctly

$$(-\Delta)_\varepsilon^\alpha = P_\Omega [(-\Delta)^\alpha + \varepsilon_{IR}]^{-1} P_\Omega$$

a) $P_\Omega: L^2(\mathbb{R}^N) \rightarrow L^2(\Omega)$ is the usual cut off finite volume projection

$$\left(P_\Omega(f) = \frac{f \cdot \chi_\Omega(x)}{\text{vol}_N(\Omega)} \right)$$

b) $\varepsilon_{IR} = \inf \text{spectrum}(-\Delta_\Omega)$, here $-\Delta_\Omega$ is the Laplacean acting on $L^2(\Omega)$ endowed with Dirichlet Boundary conditions.

c) See further comments on arXiv: 0902.3041.

Pag. 125 - Eq.(5.53), read:

$$\exp \left\{ -g_{bare}(\alpha) \int d^2x V(\varphi(x)) \right\} = \lim_{N \rightarrow \infty} \{ \dots \}.$$

Pag. 126 - Eq.(5.56), read:

$$G_\alpha(x_i, x_j) = +\frac{1}{2\pi} |x_i - x_j|^{2(\alpha-1)} \times \frac{\Gamma(1-\alpha)}{\Gamma(\alpha)} [2^{1-2\alpha} (1 + (-1)^{2(\alpha-1)})] \quad \text{for } \alpha < 1.$$

Line below eq.(5.74), read: ... in the case $n = 1$, $k < \frac{1}{2}$, and $\nu = 1$.

Pag. 134 - Eq.(5.94) and Eq.(5.95), change

$$\int_{\mathcal{H}} d_A(\varphi) \rightarrow \int_{\mathcal{H}^{alg}} d_A \mu(\varphi)$$

Pag. 134 - Eq.(5.95)

$$Tr_{L^2(R^N)}[P_\Omega \exp(-t(-\Delta + m^2)P_\Omega)]$$

Pag. 139 - Eq.(5.118), read:

$$\frac{\Gamma(\frac{n-2}{2})}{2^2 \pi^{n/2} |x-y|^{n-2}}.$$

Pag. 151 - Eq.(C-5), read:

$$\frac{d}{d\sigma} [\log \det_f(A(\sigma))] = \dots$$

Pag. 163 - Eq.(D-18), read:

$$Tr(e^{-tA}) \stackrel{t \rightarrow 0^+}{=} \sum_{j=0}^{\infty} \left(\frac{1}{2\pi}\right)^2 \left\{ \int_{R^2} d^2\xi d^2x \sigma(e^{-tA})(x, \xi) \right\} = \dots$$

Pag. 179 - Eqs.(6.49), (6.50):

The correct assertive is the following (below Eq.(6.48))

“..., the translational-invariance of the measure is full insured only if h is such that $\langle Ah, h \rangle < \infty$...”.

Pag. 180, Correctly, one should read:

$$\begin{aligned} \mu(\chi_{Dom(A)}(\varphi)) &= \lim_{\alpha \rightarrow 0} \left\{ \lim_{N \rightarrow \infty} \int_{\mathcal{H}} d_{A^{-1}} \mu(\varphi) e^{-\alpha \langle AP_N \varphi, AP_N \varphi \rangle} \right. \\ &= \lim_{\alpha \rightarrow 0} \lim_{N \rightarrow \infty} \left(\prod_{n=1}^N (1 + \alpha \lambda_n)^{-\frac{1}{2}} \right) = \lim_{\alpha \rightarrow 0^+} \\ &\quad \times \det(1 + \alpha A) = 1. \end{aligned}$$

In the case of $\mathcal{H} = (L^2(R^N), d^N x)$, one can show that the functional measure of the smooth field configurations defined as “ $\lim_{s \rightarrow \infty} \mathcal{H}^S(R^N)$ ”, has zero measure of the characteristic functional $Z(j)$ has a bound of the form below in each cylinder $P_N: \mathcal{H} \rightarrow R^N$;

$$|Z(P_N j)| \leq Q^{(N)}(j_1, \dots, j_N) \exp \left(- \sum_{s,p=1}^N j_s k_{sp} j_p \right)$$

with Q denoting a polynomial of N -variables, namely

$$\lim_{s \rightarrow \infty} \mu(\mathcal{H}^s(R^N)) = 0$$

Pag. 184, below Eq.(6.70), read: correct to "... to our class of cut-off datum

$$f(x)\chi_{W\{Q, \{\delta_k\}\}}$$

with

$$W\{Q, \{\delta_k\}\} = \{x \in \mathcal{H} \mid (\langle x, e_k \rangle)^2 = x_k^2 \geq \delta_k + 2\lambda_k \text{ with } \Sigma \delta_k = \bar{\delta} < \infty\}.$$

Pag. 185 - Eq.(6.72), read:

$$\begin{aligned} & \int_4 d_Q \mu(x) \cdot g^2(x) \chi_{W\{Q, \{\delta_k\}\}}(x) \\ & \leq \frac{4Tr(Q^2)}{\delta} \int_4 d_Q \mu(x) \langle Dg, Dg \rangle(x). \end{aligned}$$

Pag. 215 - above Eq.(7.22), read:

$$(\text{ with } G'(x) = F(x); F(0) = 0 = G'(0)).$$

Pag. 219 - Eq.(7.35), read:

$$\begin{aligned} & \det^{-\frac{1}{2}}[1 + A\lambda M(Q^{(\varepsilon)})] = \dots \\ & = \exp \left\{ -\frac{A}{2} Tr_{L^2(\Omega)} \int_0^\lambda d\lambda' R(\lambda') \right\}. \end{aligned}$$

and

$$Q_{ij}^{(\varepsilon)}(\sigma, \sigma') = \frac{1}{\pi\varepsilon} \left[\exp \left(- \left(\frac{(\sigma - \sigma')^2}{\varepsilon^2} \right) \right) \right] \cdot \delta_{ij} \quad \text{Eq.(7.33).}$$

Pag. 267 - Eq.(A-7), read:

$$\frac{1}{\pi} \|S_n(f)(x) - f(x)\|_{L^1} \leq \sqrt{2\pi} [(M_A + M_B)n^{-\varepsilon}]^{1/2}.$$

Pag. 274 - Eq.(B-6), read:

$$\sup_{z \in H^+} \left\{ \int_{\Delta} |f e^{-f}|^z(m) d\mu(m) \right\} \leq \mu(X) < \infty.$$

Pag. 285 - Eq.(D-6), read:

$$\begin{aligned} \lim_{z \rightarrow z_k} \left[\left(\frac{d}{dz} \right)^p \left\{ \frac{h'(z)}{h(z)} \right\} + \frac{p!}{(z_k - z)^{p+1}} \right] \\ = -p! \left(\sum_{\substack{n \neq k \\ n-1}}^{\infty} \frac{1}{(\bar{z}_\mu - \bar{z}_k)^{p+1}} \right). \end{aligned}$$

Pag. 292 - Eq.(10.23a), read:

$$\int_{R^2} dx \int_0^\infty d\xi \left(\frac{\partial v^i}{\partial \xi} \Delta_x v_i \right) (x, \xi).$$