

# Contents

|  |     |
|--|-----|
| Preface .....  | vii |
| List of Tables .....                                   | xv  |
| List of Figures .....                                  | xvi |
| Chapter 1: Introduction .....                          | 1   |
| Chapter 2: Basic Knowledge on Classical Sets .....     | 4   |
| 2.1 Classical Sets and Set Inclusion .....             | 4   |
| 2.2 Set Operations .....                               | 7   |
| 2.3 Set Sequences and Set Classes .....                | 10  |
| 2.4 Set Classes Closed Under Set Operations .....      | 13  |
| 2.5 Relations, Posets, and Lattices .....              | 17  |
| 2.6 The Supremum and Infimum of Real Number Sets ..... | 20  |
| Exercises .....  | 22  |
| Chapter 3: Fuzzy Sets .....                            | 24  |
| 3.1 The Membership Functions of Fuzzy Sets .....       | 24  |
| 3.2 Inclusion and Operations of Fuzzy Sets .....       | 27  |
| 3.3 $\alpha$ -Cuts .....                               | 33  |
| 3.4 Convex Fuzzy Sets .....                            | 36  |
| 3.5 Decomposition Theorems .....                       | 37  |
| 3.6 The Extension Principle .....                      | 40  |
| 3.7 Interval Numbers .....                             | 42  |
| 3.8 Fuzzy Numbers and Linguistic Attribute .....       | 45  |
| 3.9 Binary Operations for Fuzzy Numbers .....          | 51  |
| 3.10 Fuzzy Integers .....                              | 58  |
| Exercises .....  | 59  |
| Chapter 4: Set Functions .....                         | 62  |
| 4.1 Weights and Classical Measures .....               | 63  |
| 4.2 Extension of Measures .....                        | 66  |
| 4.3 Monotone Measures .....                            | 69  |
| 4.4 $\lambda$ -Measures .....                          | 74  |

|   |     |
|---|-----|
| 4.5 Quasi-Measures.....   | 82  |
| 4.6 Möbius and Zeta Transformations .....   | 87  |
| 4.7 Belief Measures and Plausibility Measures.....  | 91  |
| 4.8 Necessity Measures and Possibility Measures .....   | 102 |
| 4.9 $k$ -Interactive Measures .....   | 107 |
| 4.10 Efficiency Measures and Signed Efficiency Measures.....  | 108 |
| Exercises .....   | 112 |
| Chapter 5: Integrations.....  | 115 |
| 5.1 Measurable Functions .....  | 115 |
| 5.2 The Riemann Integral.....   | 123 |
| 5.3 The Lebesgue-Like Integral .....  | 128 |
| 5.4 The Choquet Integral.....   | 133 |
| 5.5 Upper and Lower Integrals.....  | 153 |
| 5.6 $r$ -Integrals on Finite Spaces.....  | 162 |
| Exercises .....   | 174 |
| Chapter 6: Information Fusion.....  | 177 |
| 6.1 Information Sources and Observations.....   | 177 |
| 6.2 Integrals Used as Aggregation Tools .....   | 181 |
| 6.3 Uncertainty Associated with Set Functions.....  | 186 |
| 6.4 The Inverse Problem of Information Fusion .....   | 190 |
| Chapter 7: Optimization and Soft Computing.....   | 193 |
| 7.1 Basic Concepts of Optimization.....   | 193 |
| 7.2 Genetic Algorithms .....  | 195 |
| 7.3 Pseudo Gradient Search .....  | 199 |
| 7.4 A Hybrid Search Method .....  | 202 |
| Chapter 8: Identification of Set Functions .....  | 204 |
| 8.1 Identification of $\lambda$ -Measures .....   | 204 |
| 8.2 Identification of Belief Measures .....   | 206 |
| 8.3 Identification of Monotone Measures.....  | 207 |
| 8.3.1 Main algorithm.....   | 210 |
| 8.3.2 Reordering algorithm .....  | 211 |
| 8.4 Identification of Signed Efficiency Measures by a Genetic Algorithm.....                                  | 213 |
| 8.5 Identification of Signed Efficiency Measures by the Pseudo Gradient Search.....                           | 215 |
| 8.6 Identification of Signed Efficiency Measures Based on the Choquet Integral<br>by an Algebraic Method..... | 217 |
| 8.7 Identification of Monotone Measures Based on $r$ -Integrals by a Genetic<br>Algorithm.....                | 219 |
| Chapter 9: Multiregression Based on Nonlinear Integrals .....   | 221 |
| 9.1 Linear Multiregression .....  | 221 |

|   |     |
|---|-----|
| 9.2 Nonlinear Multiregression Based on the Choquet Integral.....  | 226 |
| 9.3 A Nonlinear Multiregression Model Accommodating Both Categorical<br>and Numerical Predictive Attributes ..... | 232 |
| 9.4 Advanced Consideration on the Multiregression Involving Nonlinear<br>Integrals .....                          | 234 |
| 9.4.1 Nonlinear multiregressions based on the Choquet integral with<br>quadratic core.....                        | 234 |
| 9.4.2 Nonlinear multiregressions based on the Choquet integral involving<br>unknown periodic variation .....      | 235 |
| 9.4.3 Nonlinear multiregressions based on upper and lower integrals .....   | 236 |
| Chapter 10: Classifications Based on Nonlinear Integrals .....  | 238 |
| 10.1 Classification by an Integral Projection.....  | 238 |
| 10.2 Nonlinear Classification by Weighted Choquet Integrals .....   | 242 |
| 10.3 An Example of Nonlinear Classification in a Three-Dimensional Sample<br>Space.....                           | 250 |
| 10.4 The Uniqueness Problem of the Classification by the Choquet Integral<br>with a Linear Core .....             | 263 |
| 10.5 Advanced Consideration on the Nonlinear Classification Involving the<br>Choquet Integral .....               | 267 |
| 10.5.1 Classification by the Choquet integral with the widest gap between<br>classes .....                        | 267 |
| 10.5.2 Classification by cross-oriented projection pursuit .....  | 268 |
| 10.5.3 Classification by the Choquet integral with quadratic core.....  | 270 |
| Chapter 11: Data Mining with Fuzzy Data .....   | 272 |
| 11.1 Defuzzified Choquet Integral with Fuzzy-Valued Integrand (DCIFI).....  | 273 |
| 11.1.1 The $\alpha$ -level set of a fuzzy-valued function.....  | 274 |
| 11.1.2 The Choquet extension of $\mu$ .....   | 275 |
| 11.1.3 Calculation of DCIFI .....   | 277 |
| 11.2 Classification Model Based on the DCIFI.....   | 282 |
| 11.2.1 Fuzzy data classification by the DCIFI .....   | 283 |
| 11.2.2 GA-based adaptive classifier-learning algorithm via DCIFI<br>projection pursuit .....                      | 286 |
| 11.2.3 Examples of the classification problems solved by the DCIFI<br>projection classifier.....                  | 290 |
| 11.3 Fuzzified Choquet Integral with Fuzzy-Valued Integrand (FCIFI) .....   | 300 |
| 11.3.1 Definition of the FCIFI .....  | 300 |
| 11.3.2 The FCIFI with respect to monotone measures.....   | 303 |
| 11.3.3 The FCIFI with respect to signed efficiency measures.....  | 306 |
| 11.3.4 GA-based optimization algorithm for the FCIFI with respect to<br>signed efficiency measures .....          | 309 |

|   |     |
|---|-----|
| 11.4 Regression Model Based on the CIII.....  | 319 |
| 11.4.1 CIII regression model.....             | 319 |
| 11.4.2 Double-GA optimization algorithm ..... | 321 |
| 11.4.3 Explanatory examples .....             | 324 |
| Bibliography .....                            | 329 |
| Index .....                                   | 337 |