

MANABU HARADA - THE MAN AND HIS WORK

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At the Fifth China-Japan-Korea International Symposium on Ring Theory in Tokyo, we celebrated Professor Shaoxue Liu, Professor Hiroyuki Tachikawa and Professor Manabu Harada who had planned and originated the First Symposium at Guangxi in China in 1991. It was a great pleasure for all participants to celebrate these three eminent ring theorists.

In this paper, as one of Harada's students, I would like to briefly describe Professor Harada's career and work.

He was born in 1931 and raised in Osaka City. He spent his undergraduate and graduate years at Osaka City University from 1949 through to 1955. After he received his master's degree in 1955, he joined the staff of the Department of Mathematics of Osaka City University as an assistant professor. In those days, Keizo Asano was a professor and Hiroshi Nagao was an associate professor at the University and Harada was influenced by them. Professor Asano is well-known for pioneering classical quotient rings. He studied orders in simple rings in the 1950s, nowadays called "Asano orders" ([1]). On the other hand, Professor Nagao studied global dimensions of residue rings of hereditary semiprimary rings (with Eilenberg and Nakayama ([9])), and is known as the coauthor of the book [17] on the modular representation of finite groups.

Under such excellent circumstances, Harada began studying algebras and soon began writing a sequence of fundamental articles on ring theory. His first paper is:

M. Harada: *Note on the dimension of modules and algebras*, J. Inst. Polytechnics, Osaka City Univ., 1956.

In this paper, he showed that von Neumann regular rings can be characterized as those rings whose weak global dimensions are zero. This is a beautiful theory on von Neumann regular rings. After writing nine papers from 1956 through to 1960, he studied Asano orders and maximal orders

using homological algebra. In particular, he wrote four papers on orders in 1963, including the famous paper:

M. Harada: *Hereditary orders*, Trans. Amer. Math. Soc., 1963.

Hereditary orders are hereditary noetherian algebras. Consequently, these algebras encouraged the study of hereditary noetherian rings and so Harada is recognized as a pioneer in the study of these rings.

In the early 1960s, Goldie's article [13] on semi-prime rings with maximal conditions and Mitchell's book [15] on category theory were published.

By the Goldie Theorem, the study of hereditary orders was developed in a more general setting, that is, hereditary noetherian rings were studied as a branch of non-commutative noetherian rings. Chatters [5] showed that hereditary noetherian rings can be represented as direct sums of prime rings and artinian rings. Moreover, Harada completely determined hereditary artinian rings in the article:

M. Harada: *Hereditary semiprimary rings and triangular matrix rings*, Nagoya J. Math., 1966.

Eisenbud-Griffith-Robson ([10], [11]) showed that if R is a hereditary noetherian prime ring, then R/I is a Nakayama ring for every non-zero ideal I . Consequently, this theorem turned the spotlight on Nakayama rings.

During 1961-1963, Harada visited Brandeis University and Northwestern University in the U.S.. In those days, M. Auslander was a professor at Brandeis University and he and Goldman had studied maximal orders. In 1962, Harada received his Ph.D. from Brandeis under Auslander's supervision. After Harada returned to Japan in 1963, he also obtained his D.Sc. at Osaka City University in that year.

In the middle of 1960s, Harada began studying category theory. For this study, in 1967, he concentrated on reading Mitchell's book [15] with his student Sai. I can well remember those days, since Sai was one of my graduate classmates and I attended their seminar. Harada was strongly interested in the so-called factor category which is quite different from the quotient category introduced by Gabriel [12] and Serre [22]. For an additive category \mathbf{A} , an ideal of \mathbf{A} is defined by a subclass \mathbf{I} with certain conditions on the class of all morphisms of \mathbf{A} and the factor category \mathbf{A}/\mathbf{I} is introduced ([8] and [14]). Auslander had also used the factor category for the representation of finite dimensional algebras ([2]).

Soon, Harada began writing papers on factor categories. For a family \mathbf{A} of R -modules, Harada considered the full sub-additive category \mathbf{A} (of the category of all R -modules) whose objects are modules in \mathbf{A} , and de-

defined the “Jacobson radical $J(\mathbf{A})$ ” of \mathbf{A} , and introduced the factor category $\mathbf{A}/J(\mathbf{A})$. Taking good families \mathbf{A} (injective modules, projective modules, completely indecomposable modules, etc.), he wrote about 30 papers on the factor category and its applications from 1967 through to 1978. His work on this topic is remarkable in both the quality and quantity. Well-known in particular are the following papers written jointly with his students Y. Sai and H. Kanbara:

M. Harada and Y. Sai: *On categories of indecomposable modules*, I, Osaka J. Math., 1970.

M. Harada and H. Kanbara: *On categories of projective modules*, Osaka J. Math., 1971.

In these papers, the famous Harada-Sai Lemma appeared, and the Krull-Remak-Schmidt-Azumaya Theorem was studied by making use of his factor category, and the final version of Krull-Remak-Schmidt-Azumaya’s Theorem was completed by introducing the very useful $LsTn$ (local semi- T -nilpotency) condition.

In 1970, Harada became a professor at Osaka City University, and after he wrote various papers on the factor category, his concern turned to other topics, and from the end of 1970s, he produced numerous new important concepts on modules, such as small modules, cosmall modules, simple injective modules, mini-injective modules, extending modules and lifting modules, etc.. In 1978, he introduced two new important artinian rings concerning non-small modules and non-cosmall modules in the paper:

M. Harada: *Non-small modules and non-cosmall modules*, in “Ring Theory. Proceedings of 1978 Antwerp Conference” Dekker, New York, 1979.

This paper considered artinian rings with the condition

(\star): *Every non-small module contains a non-zero injective summand,*

and also artinian rings with the condition

(\star) * : *Every non-cosmall module contains a non-zero projective summand.*

These artinian rings are generalizations of QF-rings and Nakayama rings. Several years later, however, it was shown that these two classes of artinian rings R coincide, since (\star) holds for left R -modules if and only if (\star) * holds for right R -modules (Oshiro [21]). These new artinian rings are now named “Harada rings” in his honour, and the structure theory of Harada rings

was established in connection with QF-rings and Nakayama rings (Oshiro [19]-[20]). Actually, Harada rings R have the frame QF-subrings $F(R)$ from which R can be constructed as an upper staircase factor ring of a suitable block extension of $F(R)$. Furthermore, the structure of Harada rings gives much information about the structure of Nakayama rings. Therefore, Harada rings lead us to study QF-rings and Nakayama rings from a new point of view. Through Harada rings, there is keen interest in Nakayama permutations and Nakayama automorphisms on QF-rings, constructions of local QF-rings, skew-matrix rings, structures of Nakayama rings, Fuller's theorem on i -pairs, artinian rings with self-duality, lifting modules, extending modules, and Nakayama QF-group algebras, etc.. By the study on Harada rings, many new theorems and facts were produced; for example, a complete classification of Nakayama rings, the existence of QF-rings without Nakayama automorphisms, and new artinian rings with self-duality, etc. (cf. Baba-Oshiro [4]).

Furthermore, it is worthwhile to note that lifting modules, extending modules and simple injectivity have been extensively studied by many people. These fields were witnessed in the following books: Mohamed-Müller [16]: *Continuous Modules and Discrete Modules* (1990), Dung-Huynh-Smith-Wisbauer [7]: *Extending Modules* (1994), Yousif-Nicholson [18]: *Quasi-Frobenius Rings* (2003) and Clark-Lomp-Vanaja-Wisbauer [6]: *Lifting modules* (2007).

Harada retired from Osaka City University in 1994. During 1956-1994, he had written approximately 90 articles and published three books, including the book:

M. Harada: *Factor Categories with Applications to Direct Decomposition of Modules*, Lecture Notes in Pure and Applied Mathematics, 88, Marcel Dekker, 1983.

As witnessed above, Harada's works are outstanding and important in the progress of the ring theory. After retiring, he also completely retired from mathematics and enjoys his happy retirement as an Emeritus Professor at Osaka City University.

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