

Contents

1	Nonlinear and Chaotic Maps	1
1.1	One-Dimensional Maps	1
1.1.1	Exact and Numerical Trajectories	3
1.1.2	Fixed Points and Stability	14
1.1.3	Invariant Density	16
1.1.4	Liapunov Exponent	20
1.1.5	Autocorrelation Function	23
1.1.6	Discrete Fourier Transform	25
1.1.7	Fast Fourier Transform	28
1.1.8	Logistic Map and Liapunov Exponent for $r \in [3, 4]$	33
1.1.9	Logistic Map and Bifurcation Diagram	34
1.1.10	Random Number Map and Invariant Density	36
1.1.11	Random Number Map and Random Integration	38
1.1.12	Circle Map and Rotation Number	40
1.1.13	Newton Method	41
1.1.14	Feigenbaum's Constant	43
1.1.15	Symbolic Dynamics	45
1.1.16	Chaotic Repeller	47
1.1.17	Chaos and Encoding	48
1.2	Two-Dimensional Maps	54
1.2.1	Introduction	54
1.2.2	Phase Portrait	57
1.2.3	Fixed Points and Stability	64
1.2.4	Liapunov Exponents	65
1.2.5	Correlation Integral	67
1.2.6	Capacity	68
1.2.7	Hyperchaos	70
1.2.8	Domain of Attraction	74
1.2.9	Newton Method in the Complex Domain	75
1.2.10	Newton Method in Higher Dimensions	77
1.2.11	Ruelle-Takens-Newhouse Scenario	78
1.2.12	Periodic Orbits and Topological Degree	80
1.2.13	JPEG file	82

2	Time Series Analysis	85
2.1	Introduction	85
2.2	Correlation Coefficient	86
2.3	Liapunov Exponent from Time Series	87
2.3.1	Jacobian Matrix Estimation Algorithm	88
2.3.2	Direct Method	89
2.4	Hurst Exponent	96
2.4.1	Introduction	96
2.4.2	Implementation for the Hurst Exponent	98
2.4.3	Random Walk	102
2.5	Higuchi's Algorithm	106
2.6	Complexity	107
3	Autonomous Systems in the Plane	111
3.1	Classification of Fixed Points	111
3.2	Homoclinic Orbit	113
3.3	Pendulum	114
3.4	Limit Cycle Systems	116
3.5	Lotka-Volterra Systems	119
4	Nonlinear Hamilton Systems	123
4.1	Hamilton Equations of Motion	123
4.1.1	Hamilton System and Variational Equation	126
4.2	Integrable Hamilton Systems	127
4.2.1	Hamilton Systems and First Integrals	127
4.2.2	Lax Pair and Hamilton Systems	128
4.2.3	Floquet Theory	130
4.3	Chaotic Hamilton Systems	133
4.3.1	Hénon-Heiles Hamilton Function and Trajectories	133
4.3.2	Hénon Heiles and Surface of Section Method	135
4.3.3	Quartic Potential and Surface of Section Technique	136
5	Nonlinear Dissipative Systems	139
5.1	Fixed Points and Stability	139
5.2	Trajectories	144
5.3	Phase Portrait	148
5.4	Liapunov Exponents	150
5.5	Generalized Lotka-Volterra Model	153
5.6	Hyperchaotic Systems	155
5.7	Hopf Bifurcation	158
5.8	Time-Dependent First Integrals	161
6	Nonlinear Driven Systems	163
6.1	Introduction	163
6.2	Driven Anharmonic Systems	166

6.2.1	Phase Portrait	166
6.2.2	Poincaré Section	167
6.2.3	Liapunov Exponent	169
6.2.4	Autocorrelation Function	170
6.2.5	Power Spectral Density	173
6.3	Driven Pendulum	174
6.3.1	Phase Portrait	174
6.3.2	Poincaré Section	176
6.4	Parametrically Driven Pendulum	178
6.4.1	Phase Portrait	178
6.4.2	Poincaré Section	179
6.5	Driven Van der Pol Equation	181
6.5.1	Phase Portrait	181
6.5.2	Liapunov Exponent	183
6.6	Parametrically and Externally Driven Pendulum	185
6.7	Torsion Numbers	187
7	Controlling of Chaos	191
7.1	Introduction	191
7.2	Ott-Yorke-Grebogi Method	191
7.2.1	One-Dimensional Maps	191
7.2.2	Systems of Difference Equations	195
7.3	Small Periodic Perturbation	199
7.4	Resonant Perturbation and Control	201
8	Synchronization of Chaos	203
8.1	Introduction	203
8.2	Synchronization of Chaos	203
8.2.1	Synchronization Using Control	203
8.2.2	Synchronizing Subsystems	206
8.3	Synchronization of Coupled Dynamors	209
8.4	Phase Coupled Systems	211
9	Fractals	217
9.1	Introduction	217
9.2	Iterated Function System	219
9.2.1	Introduction	219
9.2.2	Cantor Set	220
9.2.3	Heighway's Dragon	223
9.2.4	Sierpinski Gasket	225
9.2.5	Koch Curve	227
9.2.6	Fern	229
9.2.7	Grey Level Maps	231
9.3	Mandelbrot Set	232
9.4	Julia Set	234

9.5	Fractals and Kronecker Product	236
9.6	Lindenmayer Systems and Fractals	240
9.7	Weierstrass Function	243
10	Cellular Automata	245
10.1	Introduction	245
10.2	One-Dimensional Cellular Automata	248
10.3	Sznajd Model	249
10.4	Conservation Laws	252
10.5	Two-Dimensional Cellular Automata	253
10.6	Button Game	257
11	Solving Differential Equations	261
11.1	Introduction	261
11.2	Euler Method	262
11.3	Lie Series Technique	264
11.4	Runge-Kutta-Fehlberg Technique	268
11.5	Ghost Solutions	269
11.6	Symplectic Integration	272
11.7	Verlet Method	277
11.8	Störmer Method	279
11.9	Invisible Chaos	280
11.10	First Integrals and Numerical Integration	281
12	Neural Networks	283
12.1	Introduction	283
12.2	Hopfield Model	287
12.2.1	Introduction	287
12.2.2	Synchronous Operations	289
12.2.3	Energy Function	291
12.2.4	Basins and Radii of Attraction	293
12.2.5	Spurious Attractors	293
12.2.6	Hebb's Law	294
12.2.7	Hopfield Example	296
12.2.8	Hopfield C++ Program	298
12.2.9	Asynchronous Operation	302
12.2.10	Translation Invariant Pattern Recognition	303
12.3	Similarity Metrics	305
12.4	Kohonen Network	309
12.4.1	Introduction	309
12.4.2	Kohonen Algorithm	310
12.4.3	Kohonen Example	312
12.4.4	Traveling Salesman Problem	318
12.5	Perceptron	322
12.5.1	Introduction	322

12.5.2	Boolean Functions	324
12.5.3	Linearly Separable Sets	325
12.5.4	Perceptron Learning	326
12.5.5	Perceptron Learning Algorithm	330
12.5.6	One and Two Layered Networks	333
12.5.7	XOR Problem and Two-Layered Networks	335
12.6	Multilayer Perceptrons	339
12.6.1	Introduction	339
12.6.2	Cybenko's Theorem	340
12.6.3	Back-Propagation Algorithm	340
12.6.4	Recursive Deterministic Perceptron Neural Networks	348
12.7	Chaotic Neural Networks	350
12.8	Neuronal-Oscillator Models	351
12.9	Radial Basis Function Networks	353
12.10	Neural Network, Matrices and Eigenvalues	355
13	Genetic Algorithms	357
13.1	Introduction	357
13.2	Sequential Genetic Algorithm	358
13.3	Schemata Theorem	362
13.4	Bitwise Operations	364
13.4.1	Introduction	364
13.4.2	Assembly Language	367
13.4.3	Floating Point Numbers and Bitwise Operations	369
13.4.4	Java Bitset Class	370
13.4.5	C++ bitset Class	371
13.5	Bit Vector Class	373
13.6	Penna Bit-String Model	376
13.7	Maximum of One-Dimensional Maps	378
13.8	Maximum of Two-Dimensional Maps	384
13.9	Finding a Fitness Function	392
13.10	Problems with Constraints	398
13.10.1	Introduction	398
13.10.2	Knapsack Problem	399
13.10.3	Traveling Salesman Problem	404
13.11	Simulated Annealing	412
14	Gene Expression Programming	415
14.1	Introduction	415
14.2	Example	418
14.3	Numerical-Symbolic Manipulation	430
14.4	Multi Expression Programming	435

15 Optimization	441
15.1 Lagrange Multiplier Method	441
15.2 Karush-Kuhn-Tucker Conditions	449
15.3 Support Vector Machine	453
15.3.1 Introduction	453
15.3.2 Linear Decision Boundaries	453
15.3.3 Nonlinear Decision Boundaries	457
15.3.4 Kernel Fisher Discriminant	461
16 Discrete Wavelets	465
16.1 Introduction	465
16.2 Multiresolution Analysis	468
16.3 Pyramid Algorithm	470
16.4 Biorthogonal Wavelets	475
16.5 Two-Dimensional Wavelets	480
17 Discrete Hidden Markov Processes	483
17.1 Introduction	483
17.2 Markov Chains	485
17.3 Discrete Hidden Markov Processes	489
17.4 Forward-Backward Algorithm	493
17.5 Viterbi Algorithm	496
17.6 Baum-Welch Algorithm	497
17.7 Distances between HMMs	498
17.8 Application of HMMs	499
17.9 C++ Program	502
18 Fuzzy Sets and Fuzzy Logic	513
18.1 Introduction	513
18.2 Operators for Fuzzy Sets	521
18.2.1 Logical Operators	521
18.2.2 Algebraic Operators	524
18.2.3 Defuzzification Operators	525
18.2.4 Fuzzy Concepts as Fuzzy Sets	527
18.2.5 Hedging	528
18.2.6 Quantifying Fuzzyness	529
18.2.7 C++ Implementation of Discrete Fuzzy Sets	530
18.2.8 Applications: Simple Decision-Making Problems	549
18.3 Fuzzy Numbers and Fuzzy Arithmetic	555
18.3.1 Introduction	555
18.3.2 Algebraic Operations	556
18.3.3 LR-Representations	559
18.3.4 Algebraic Operations on Fuzzy Numbers	562
18.3.5 C++ Implementation of Fuzzy Numbers	563
18.3.6 Applications	570

18.4 Fuzzy Rule-Based Systems	571
18.4.1 Introduction	571
18.4.2 Fuzzy If-Then Rules	574
18.4.3 Inverted Pendulum Control System	575
18.4.4 Fuzzy Controllers with B-Spline Models	577
18.4.5 Application	580
18.5 Fuzzy C-Means Clustering	582
18.6 fXOR Fuzzy Logic Networks	586
18.7 Fuzzy Hamming Distance	588
18.8 Fuzzy Truth Values and Probabilities	591
Bibliography	593
Index	601