

Preface

The objective of writing this book is to bring together in a coherent fashion the foundations of and recent developments in finite temperature quantum field theory and its applications to physical problems. The basic tool of our presentation is symmetry.

Symmetry is the cornerstone of contemporary physics. It has played a fundamental role in our understanding, in particular, of elementary particles and their interactions. This concept has been useful not only in *ab initio* theories, but also in heuristic formulations, such as thermodynamics. In order to appreciate this point, let us trace some elements of temperature dependent phenomena. Thermodynamics is a theory describing macroscopic properties of matter without any microscopic dynamic input. For this reason, it was considered by many as a way of systematizing the properties of measurements. However, it is quite amazing that the description of entropy, free energy, specific heat and several types of phase transitions can be related and some conceptual progress has developed. In these developments, a crucial step was taken by Landau, guided by concepts of symmetry, who set forth his theory of first and second-order phase transitions. This introduced notions such as the order parameter and spontaneous symmetry breaking. In order to advance with the Landau theory describing properties of critical phenomena in matter, the methodology of the quantum field theory was borrowed by thermodynamics. This was a two-way street. The quantum field theory, describing elementary particles, adopted the concept of spontaneous symmetry breaking to justify, for instance, the origin of mass, superconducting transitions and many other phenomena.

As the understanding of the microscopic nature of matter developed, it became imperative to introduce dynamics to describe the thermodynamic properties in terms of forces and a consistent theoretical structure. But the large number of particles made it prohibitive to carry out a real microscopic calculation. This led to ideas of statistical mechanics, starting with Boltzmann and Maxwell, and finding a synthesis by Gibbs with the ensemble theory.

The last century saw a rapid growth of ideas and sophisticated methods to treat microscopic systems. The developments of both quantum theory and relativity led to the quantum field theory that incorporated these two ideas. Then this in turn

provided results such as the spin-statistics theorem. These formulations, initially considered to be useful in developing a theory for elementary particles, proved in fact highly beneficial to an understanding of the properties and phenomena in many-body physics. However, it still lacked the notion of temperature. Part of this theoretical apparatus, such as the partition function in statistical mechanics and Green function in quantum physics, was converted into a proper microscopic theory at finite temperature by Matsubara in 1955 using the expedient of imaginary-time.

At about that time, there was a tremendous development of quantum field theory with methods due to Feynman, Tomonaga and Schwinger, among others. There were two other fundamental achievements. One was carried out by Wigner who, in the late nineteen thirties, when studying representations of the Lorentz group, found a way to classify elementary particles. The other step was due to Yang and Mills, that extended the notion of gauge symmetry (due to Weyl) to describe the basic interactions in nature. These findings of a theory at zero-temperature were enough motivation for researchers to look for an extension of the Matsubara method. This was carried out by Ezawa, Tomozawa and Umezawa, in order to describe processes in relativistic physics, settling then a strong proximation of two different areas: statistical mechanics and quantum field theory. The consequence was a diversity of developments of practical and formal interest, such as the periodicity in time described by the KMS (Kubo-Martin-Schwinger) conditions, with topological implications, and the spontaneous symmetry breaking in particle physics by Dolan and Jackiw. It is also important to state that many findings, first introduced in the zero-temperature theories, were brought to the finite temperature theory. For example, this is the case for the Ward-Takahashi (W-T) relations, where using symmetry provides a way to carry out consistent perturbative calculations. Furthermore, the W-T relations present the only non-perturbative method in quantum field theory, at both zero and finite temperature.

The imaginary-time formalism is basically a theory for thermal equilibrium. However, time is a crucial ingredient in many processes in relativistic and in many-body physics. This led Schwinger, followed by Keldysh, to propose a method using elements of the imaginary-time formalism with real time. A decade later, while studying superconductivity, Umezawa and coworkers found that to transpose zero-temperature methods to imaginary-time problems with field operators and their products was difficult and cumbersome. The attempt to solve this led Takahashi and Umezawa to propose a real-time operator field theory at finite temperature, thermofield dynamics (TFD). This required the Hilbert space to be doubled. The second Hilbert space was eventually related to the heat bath as the old thermodynamics required a heat bath to have a system at constant temperature. TFD brought to the realm of thermal theories two elements. One was the Bogoliubov transformation, describing the temperature effect as a condensate of field in the vacuum, and it was well-known for superconducting phase transitions. The other was a Hilbert space structure associated to the thermal state. The latter is, in turn,

connected to the concept of c^* -algebras, the formal structure of statistical physics. As a consequence, representations of symmetry groups for thermal theories could be explored. In addition, these algebraic elements were combined with the KMS conditions giving rise to representations of quantum fields in topological manifolds, say $S^1 \times \mathbb{R}^3$, describing a system in compactified regions of imaginary-time, where the dimensions of the compactification, represented by the circle S^1 , is the temperature. In this book, we explore in detail this symbiosis of symmetry and topology.

The book is divided into five parts. The first part treats fundamental principles. We start, for the sake of completeness, by considering the status of thermodynamics. Our goal is to show its connection with the elements of field theory by discussing the Landau phenomenological description of first and second-order phase transitions. Then, in the second chapter, basic elements of statistical mechanics are presented. The Louiville-von Neumann equation and the von Neumann entropy are used to arrive at the Gibbs ensembles using the variational principle. Starting again from the Louiville-von Neumann equation, the Wigner function formalism is introduced. In the third chapter the notion of partition function is explored, leading us to consider the idea of a generating functional that has proved so very useful in the perturbative approach for quantum systems; in particular the path integral method, including gauges fields. Chapter 4 deals with the theory of interacting fields at zero-temperature. Examples of scalar field and Yang-Mills theory are presented. It provides a brief look at the set-up of the canonical theory and the perturbative approach. In this brief review of so many topics, compactified in four chapters in Part One, we have focussed on concepts that will be explored and developed at finite temperature in the rest of the book.

Part Two deals with the thermal field theory. We start with some basic notions of thermofield dynamics and statistical physics to introduce the concept of representations of symmetries associated with thermal phenomena. This is called the thermo-algebra. These ideas are illustrated with examples, by considering oscillators for bosons and fermions. The doubling of operators is interpreted physically. It is followed by considering thermal groups based on kinematic symmetries: Poincaré and Galilei, leading us to thermal Lagrangians. The representations of the kinematic groups provide, in particular, relativistic Liouville-von Neumann equations for fermions and bosons. The relationship among TFD, Matsubara and Keldysh-Schwinger formalisms is discussed in terms of symmetry, providing a unified view of these diverse techniques. Finally, the path integral approach at finite temperature is introduced. This is followed with some examples of calculating decay rates and cross sections at finite temperature and these are compared with those at zero temperature. We close this Part with a discussion of topics on renormalization and Ward-Takahashi relations at finite temperature.

Part Three contains applications to quantum optics. Exploring thermal representations in TFD, various thermal states of a field mode are introduced and defined consistently. In order to study the physical nature of such states, the sta-

tistical properties are analyzed by considering the Mandel factor and Wigner and P-functions. Finally bipartite states are introduced and their entanglement is considered, taking the TFD structure as a guide.

Part Four deals with compactified fields and their application. The basic idea is to explore the topology associated with the KMS conditions to treat fields in confined spatial regions at finite temperature. Starting with topological arguments, we discuss how to generalize the Bogoliubov transformations and the Matsubara imaginary time to allow a study of systems confined to finite regions, linear, surface and volume, consistent with topologies $\Gamma_D^d = (\mathbb{S}^1)^d \times \mathbb{R}^{D-d}$. Due to the close association of the Bogoliubov transformation and the imaginary-time method, the process of space compactification may be understood physically, in one case as in the other, as a process of condensation of the field in the vacuum. The first example is the Casimir effect for the electromagnetic field between plates, and in a parallelepiped box. The generalized discussion implies that the Casimir effect may be viewed as a condensation of the electromagnetic field in the vacuum. Then we consider the example of fermions in different configurations. Some results for a simplified QCD Lagrangian are obtained. This is followed by studying the case of the compactified $\lambda\phi^4$ theory and an analysis of spontaneous symmetry breaking for spatially confined systems at finite temperature. Then the examples of phase transitions in superconducting material in films, wires and grains are considered. This is to analyze the role of topology that may change the transition temperatures in these confined systems. The case of first order phase transition in superconducting films is also taken up. All these examples provide a strong support for the ideas, using symmetry representations with topological ingredients, to treat compactified fields.

In Part Five, first we analyze representations of thermo-algebras in the phase space. The Wigner function is then derived for relativistic and non-relativistic fields, followed by a study of the classical version of TFD. These results give us elements of kinetic theory and stochastic processes from a perspective of the representation theory of the kinematic groups, Poincaré and Galilei. Further chapters provide some ideas about open systems, exploring methods of quantum field theory. Systems in nonequilibrium states are considered only in a simple manner. An exhaustive analysis of such a problem would require a separate monograph.

The book is structured in such a way that some topics can be studied independently. For instance, a reader sufficiently trained in quantum field theory can start from Chapter 5. Another reader interested in optics can start from reviewing some basic concepts in Chapters 1 and 2, and then jump to Chapters 5, 6, 12, 13 and 14. Beyond Part One, other basic elements are introduced throughout the book to make it convenient for beginners and students to study.

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