

Chapter 1

Introduction

This chapter gives the preliminaries for comprehensive reading of the book: an intuitive introduction to quantum mechanics, a minimum necessary mathematics (of the Hilbert space), as well as some very basic facts about the human mind.

While appreciating the outstanding pioneering efforts of Werner Heisenberg and Erwin Schrödinger (and others), this book is mainly focused on quantum theory of Paul Dirac and Richard Feynman (with exception of the chapter four) and its applications to human mind and body. Note that Einstein's summation convention over repeated indices is assumed in the whole text. To make it more readable for physicists, the book is intentionally less rigorous than in similar texts on graduate-level mathematical physics. Also, every section begins more intuitively, and later develops more formally. For all additional explanations regarding the used mathematical formalisms, the reader is referred to our book *Applied Differential Geometry*, World Scientific, 2007 [Ivancevic and Ivancevic (2007b)].

1.1 Soft Introduction to Quantum Mechanics

According to quantum mechanics, *light* consists of particles called *photons*, and the Figure 1.1 shows a photon source which we assume emits photons one at a time. There are two slits, *A* and *B*, and a screen behind them. The photons arrive at the screen as individual events, where they are detected separately, just as if they were ordinary particles. The curious quantum behavior arise in the following way [Penrose (1997)]. If only slit *A* were open and the other closed, there would be many places on the screen which the photon could reach. If we now close the slit *A* and open the slit *B*, we may again find that the photon could reach the same spot on the screen.

However, if we open *both slits*, and if we have chosen the point on the screen carefully, we may now find that the photon cannot reach that spot, even though it could have done so if either slit alone were open. Somehow, the two possible things which the photon *might* do cancel each other out. This type of behavior does not take place in classical physics. Either one thing happens or another thing happens – we do not get two possible things which might happen, somehow conspiring to cancel each other out.

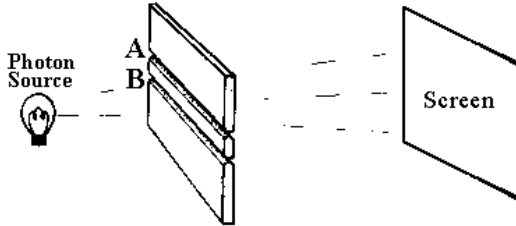


Fig. 1.1 The two-slit experiment, with individual photons of monochromatic light (see text for explanation).

The way we understand the outcome of this experiment in quantum mechanics is to say that when the photon is *en route* from the source to the screen, the state of the photon is not that of having gone through one slit or the other, but is some mysterious combination of the two, weighted by complex numbers [Penrose (1997)]. That is, we can write the state of the photon as a wave ψ -function,¹ which is the *linear superposition* of the two states, $|A\rangle$ and $|B\rangle$,² corresponding to the A -slot and B -slot alternatives,

$$|\psi\rangle = z_1|A\rangle + z_2|B\rangle,$$

where z_1 and z_2 are complex numbers (not both zero), while $|\cdot\rangle$ denotes the *quantum state ket-vector*.

Now, in quantum mechanics, we are not so interested in the sizes of the

¹In the *Schrödinger picture*, the *unitary evolution* U of a quantum system is described by the *Schrödinger equation*, which provides the time rate of change of the quantum state or wave function $\psi = \psi(t)$.

²We are using here the standard Dirac ‘bra-ket’ notation for quantum states. Paul Dirac was one of the outstanding physicists of the 20th century. Among his achievements was a general formulation of quantum mechanics (having Heisenberg matrix mechanics and Schrödinger wave mechanics as special cases) and also its relativistic generalization involving the ‘Dirac equation’, which he discovered, for the electron. He had an unusual ability to ‘smell out’ the truth, judging his equations, to a large degree, by their aesthetic qualities!

complex numbers z_1 and z_2 themselves as we are in their ratio – it is only the ratio of these numbers which has direct physical meaning (as multiplying a quantum state with a nonzero complex number does not change the physical situation). Recall that the *Riemann sphere* is a way of representing complex numbers (plus ∞) and their ratios on a sphere on unit radius, whose equatorial plane is the complex-plane, whose center is the origin of that plane and the equator of this sphere is the unit circle in the complex-plane. We can project each point on the equatorial complex-plane onto the Riemann sphere, projecting from its south pole S , which corresponds to the *point at infinity* in the complex-plane. To represent a particular complex ratio, say $u = z/w$ (with $w \neq 0$), we take the stereographic projection from the sphere onto the plane.

The Riemann sphere plays a fundamental role in the quantum picture of two-state systems [Penrose (1994)]. If we have a spin- $\frac{1}{2}$ particle, such as an electron, a proton, or a neutron, then the various combinations of their spin states can be realised geometrically on the Riemann sphere. Spin $-\frac{1}{2}$ particles can have two spin states: (i) spin-up (with the rotation vector pointing upwards), and (ii) spin-down (with the rotation vector pointing downwards). The superposition of the two spin-states can be represented symbolically as

$$|\nearrow\rangle = w|\uparrow\rangle + z|\downarrow\rangle.$$

Different combinations of these spin states give us rotation about some other axis and, if we want to know where that axis is, we take the ratio of complex numbers $u = z/w$. We place this new complex number u on the Riemann sphere and the direction of u from the center is the direction of the spin axis (see Figure 1.2).

More general quantum state vectors might have a form such as [Penrose (1994)]:

$$|\psi\rangle = z_1|A_1\rangle + z_2|A_2\rangle + \dots + z_n|A_n\rangle,$$

where $z_1 \dots z_n$ are complex numbers (not all zero) and the state vectors $|A_1\rangle, \dots, |A_n\rangle$ might represent various possible locations for a particle (or perhaps some other property of a particle, such as its state of spin). Even more generally, infinite sums would be allowed for a wave ψ -function or quantum state vector.

Now, the most basic feature of unitary quantum evolution U^3 is that it

³Recall that unitary quantum evolution U is governed by the time-dependent

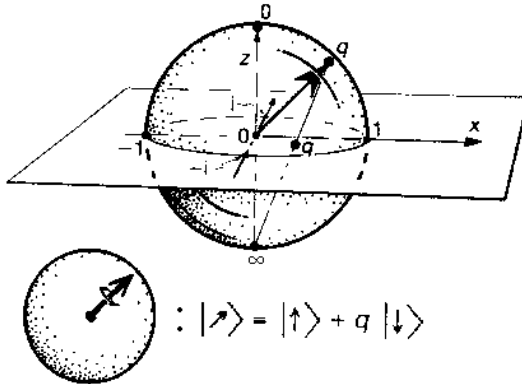


Fig. 1.2 The quantum Riemann sphere, represented as the space of physically distinct spin-states of a spin- $\frac{1}{2}$ particle (e.g., electron, proton, neutron): $|\nearrow\rangle = |\uparrow\rangle + q|\downarrow\rangle$. The sphere is projected stereographically from its south pole (∞) to the complex-plane through its equator (see text for explanation).

is *linear*. This means that, if we have two states, say $|\psi\rangle$ and $|\phi\rangle$, and if the Schrödinger equation would tell us that, after some time t , the states $|\psi\rangle$ and $|\phi\rangle$ would each individually evolve to new states $|\psi'\rangle$ and $|\phi'\rangle$, respectively then any linear superposition $z_1|\psi\rangle + z_2|\phi\rangle$, must evolve, after some time t , to the corresponding superposition $z_1|\psi'\rangle + z_2|\phi'\rangle$. Let us use the symbol \rightsquigarrow to denote the evolution after time t , Then linearity asserts that if

$$|\psi\rangle \rightsquigarrow |\psi'\rangle \quad \text{and} \quad |\phi\rangle \rightsquigarrow |\phi'\rangle,$$

then the evolution

$$z_1|\psi\rangle + z_2|\phi\rangle \rightsquigarrow z_1|\psi'\rangle + z_2|\phi'\rangle$$

would also hold. This would consequently apply also to linear superpositions of more than two individual quantum states. For example, Schrödinger equation,

$$i\hbar \partial_t |\psi(t)\rangle = H |\psi(t)\rangle,$$

where $\partial_t \equiv \partial/\partial t$, \hbar is the *Planck's constant*, and H is the Hamiltonian (total energy) operator. Given the quantum state $|\psi(t)\rangle$ at some initial time ($t = 0$), we can integrate the Schrödinger equation to get the state at any subsequent time. In particular, if H is independent of time, then

$$|\psi(t)\rangle = \exp\left(-\frac{iHt}{\hbar}\right) |\psi(0)\rangle.$$

$z_1|\psi\rangle + z_2|\phi\rangle + z_3|\chi\rangle$ would evolve, after time t , to $z_1|\psi'\rangle + z_2|\phi'\rangle + z_3|\chi'\rangle$, if $|\psi\rangle$, $|\phi\rangle$, and $|\chi\rangle$ would each individually evolve to $|\psi'\rangle$, $|\phi'\rangle$, and $|\chi'\rangle$, respectively. Thus, the evolution always proceeds as though each different component of a superposition were oblivious to the presence of the others.

As a second experiment, consider a situation in which light impinges on a half-silvered mirror, that is a semi-transparent mirror that reflects just half the light (composed of a *stream of photons*) falling upon it and transmits the remaining half [Penrose (1994)]. We might well have imagined that for a stream of photons impinging on our half-silvered mirror, half the photons would be reflected and half would be transmitted. Not so! Quantum theory tells us that, instead, each individual photon, as it impinges on the mirror, is separately put into a superposed state of reflection and transmission. If the photon before its encounter with the mirror is in state $|A\rangle$, then afterwards it evolves according to U to become a state that can be written $|B\rangle + i|C\rangle$, where $|B\rangle$ represents the state in which the photon is transmitted through the mirror and $|C\rangle$ the state where the photon is reflected from it (see Figure 1.3). Let us write this as

$$|A\rangle \rightsquigarrow |B\rangle + i|C\rangle.$$

The imaginary factor ‘ i ’ arises here because of a net phase shift by a quarter of a wavelength (see [Klein and Furtak (1986)]), which occurs between the reflected and transmitted beams at such a mirror.

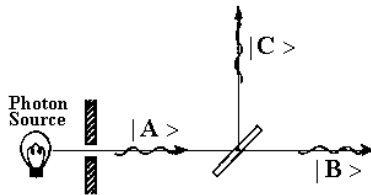


Fig. 1.3 A photon in state $|A\rangle$ impinges on a half-silvered mirror and its state evolves according to U into a superposition $|B\rangle + i|C\rangle$ (see text for explanation).

Although, from the classical picture of a particle, we would have to imagine that $|B\rangle$ and $|C\rangle$ just represent alternative things that the photon *might* do, in quantum mechanics we have to try to believe that the photon is now actually doing *both things at once* in this strange, complex superposition. To see that it cannot just be a matter of classical probability-weighted alternatives, let us take this example a little further and try to bring the

two parts of the photon state, i.e., the two photon beams, back together again [Penrose (1994)]. We can do this by first reflecting each beam with a fully silvered mirror. After reflection, the photon state $|B\rangle$ would evolve according to U , into another state $i|D\rangle$, whilst $|C\rangle$ would evolve into $i|E\rangle$,

$$|B\rangle \rightsquigarrow i|D\rangle \quad \text{and} \quad |C\rangle \rightsquigarrow i|E\rangle.$$

Thus the entire state $|B\rangle + i|C\rangle$ evolves by U into

$$|B\rangle + i|C\rangle \rightsquigarrow i|D\rangle + i(i|E\rangle) = i|D\rangle - |E\rangle,$$

since $i^2 = -1$. Now, suppose that these two beams come together at a fourth mirror, which is now half silvered (see Figure 1.4). The state $|D\rangle$ evolves into a combination $|G\rangle + i|F\rangle$, where $|G\rangle$ represents the transmitted state and $|F\rangle$ the reflected one. Similarly, $|E\rangle$ evolves into $|F\rangle + i|G\rangle$, since it is now the state $|F\rangle$ that is the transmitted state and $|G\rangle$ the reflected one,

$$|D\rangle \rightsquigarrow |G\rangle + i|F\rangle \quad \text{and} \quad |E\rangle \rightsquigarrow |F\rangle + i|G\rangle.$$

Our entire state $i|D\rangle - |E\rangle$ is now seen (because of the linearity of U) to evolve as:

$$\begin{aligned} i|D\rangle - |E\rangle &\rightsquigarrow i(|G\rangle + i|F\rangle) - (|F\rangle + i|G\rangle) \\ &= i|G\rangle - |F\rangle - |F\rangle - i|G\rangle = -2|F\rangle. \end{aligned}$$

As mentioned above, the multiplying factor -2 appearing here plays no physical role, thus we see that the possibility $|G\rangle$ is *not* open to the photon; the two beams together combine to produce just a *single* possibility $|F\rangle$. This curious outcome arises because *both* beams are present *simultaneously* in the physical state of the photon, between its encounters with the first and last mirrors. We say that the two beams *interfere* with one another.⁴

⁴This is a property of single photons: each individual photon must be considered to feel out both routes that are open to it, but it remains one photon; it does not split into two photons in the intermediate stage, but its location undergoes the strange kind of complex-number-weighted *co-existence of alternatives* that is characteristic of quantum theory.

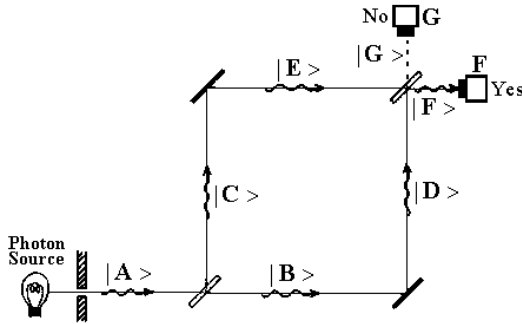


Fig. 1.4 *Mach-Zehnder interferometer*: the two parts of the photon state are brought together by two fully silvered mirrors (black), so as to encounter each other at a final half-silvered mirror (white). They interfere in such a way that the entire state emerges in state $|F\rangle$, and the detector at G cannot receive the photon (see text for explanation).

1.2 Hilbert Space

1.2.1 Quantum Hilbert Space

The family of all possible states ($|\psi\rangle, |\phi\rangle$, etc.) of a quantum system constitute what is known as a *Hilbert space*. It is a *complex vector space*, which means that it can perform the complex-number-weighted combinations that we considered before for quantum states. If $|\psi\rangle$ and $|\phi\rangle$ are both elements of the Hilbert space, then so also is $w|\psi\rangle + z|\phi\rangle$, for any pair of complex numbers w and z . Here, we even allow $w = z = 0$, to give the element $\mathbf{0}$ of the Hilbert space, which does not represent a possible physical state. We have the normal algebraic rules for a vector space:

$$\begin{aligned}
 |\psi\rangle + |\phi\rangle &= |\phi\rangle + |\psi\rangle, \\
 |\psi\rangle + (|\phi\rangle + |\chi\rangle) &= (|\psi\rangle + |\phi\rangle) + |\chi\rangle, \\
 w(z|\psi\rangle) &= (wz)|\psi\rangle, \\
 (w + z)|\psi\rangle &= w|\psi\rangle + z|\psi\rangle, \\
 z(|\psi\rangle + |\phi\rangle) &= z|\psi\rangle + z|\phi\rangle \\
 0|\psi\rangle &= \mathbf{0}, \quad z\mathbf{0} = \mathbf{0}.
 \end{aligned}$$

A Hilbert space can sometimes have a finite number of dimensions, as in the case of the spin states of a particle. For spin $\frac{1}{2}$, the Hilbert space is just 2D, its elements being the complex linear combinations of the two states $|\uparrow\rangle$ and $|\downarrow\rangle$. For spin $\frac{1}{2}n$, the Hilbert space is $(n+1)D$. However, sometimes the

Hilbert space can have an infinite number of dimensions, as e.g., the states of position or momentum of a particle. Here, each alternative position (or momentum) that the particle might have counts as providing a separate dimension for the Hilbert space. The general state describing the quantum location (or momentum) of the particle is a complex-number superposition of all these different individual positions (or momenta), which is the wave ψ -function for the particle.

Another property of the Hilbert space, crucial for quantum mechanics, is the *Hermitian inner (scalar) product*, which can be applied to any pair of Hilbert-space vectors to produce a single complex number. To understand how important the Hermitian inner product is for quantum mechanics, recall that the Dirac's 'bra-ket' notation is formulated on the its basis. If we have the two quantum states (i.e., Hilbert-space vectors) are $|\psi\rangle$ and $|\phi\rangle$, then their Hermitian scalar product is denoted $\langle\psi|\phi\rangle$, and it satisfies a number of simple algebraic properties:

$$\begin{aligned}\overline{\langle\psi|\phi\rangle} &= \langle\phi|\psi\rangle, & (\text{bar denotes complex-conjugate}) \\ \langle\psi|(|\phi\rangle + |\chi\rangle) &= \langle\psi|\phi\rangle + \langle\psi|\chi\rangle, \\ (z\langle\psi|)|\phi\rangle &= z\langle\psi|\phi\rangle, \\ \langle\psi|\phi\rangle &\geq 0, & \langle\psi|\phi\rangle = 0 \quad \text{if } |\psi\rangle = \mathbf{0}.\end{aligned}$$

For example, probability of finding a quantum particle at a given location is a *squared length* $|\psi|^2$ of a Hilbert-space position vector $|\psi\rangle$, which is the scalar product $\langle\psi|\psi\rangle$ of the vector $|\psi\rangle$ with itself. A *normalized state* is given by a Hilbert-space vector whose squared length is *unity*.

The second important thing that the Hermitian scalar product gives us is the notion of *orthogonality* between Hilbert-space vectors, which occurs when the scalar product of the two vectors is *zero*. In ordinary terms, orthogonal states are things that are independent of one another. The importance of this concept for quantum physics is that the different alternative outcomes of any measurement are always orthogonal to each other. For example, states $|\uparrow\rangle$ and $|\downarrow\rangle$ are mutually orthogonal. Also, orthogonal are *all* different possible *positions* that a quantum particle might be located in [Penrose (1994)].

1.2.2 Formal Hilbert Space

A *norm* on a complex vector space \mathcal{H} is a mapping from \mathcal{H} into the complex numbers, $\|\cdot\| : \mathcal{H} \rightarrow \mathbb{C}$; $h \mapsto \|h\|$, such that the following set of norm-axioms

hold:

(N1) $\|h\| \geq 0$ for all $h \in \mathcal{H}$ and $\|h\| = 0$ implies $h = 0$ (positive definiteness);

(N2) $\|\lambda h\| = |\lambda| \|h\|$ for all $h \in \mathcal{H}$ and $\lambda \in \mathbb{C}$ (homogeneity); and

(N3) $\|h_1 + h_2\| \leq \|h_1\| + \|h_2\|$ for all $h_1, h_2 \in \mathcal{H}$ (triangle inequality).

The pair $(\mathcal{H}, \|\cdot\|)$ is called a *normed space*.

A *Hermitian inner product* on a complex vector space \mathcal{H} is a mapping $\langle \cdot, \cdot \rangle : \mathcal{H} \times \mathcal{H} \rightarrow \mathbb{C}$ such that the following set of inner-product-axioms hold:

(IP1) $\langle h, h_1 + h_2 \rangle = \langle h, h_1 \rangle + \langle h, h_2 \rangle$;

(IP2) $\langle \alpha h, h_1 \rangle = \alpha \langle h, h_1 \rangle$;

(IP3) $\langle h_1, h_2 \rangle = \overline{\langle h_1, h_2 \rangle}$ (so $\langle h, h \rangle$ is real);

(IP4) $\langle h, h \rangle \geq 0$ and $\langle h, h \rangle = 0$ provided $h = 0$.

These properties are to hold for all $h, h_1, h_2 \in \mathcal{H}$ and $\alpha \in \mathbb{C}$; \bar{z} denotes the complex conjugate of the complex number z . (IP2) and (IP3) imply that $\langle \alpha h, h_1 \rangle = \bar{\alpha} \langle h_1, h_2 \rangle$. As is customary, for a complex number z we shall denote by $\operatorname{Re} z = \frac{z+\bar{z}}{2}$, $\operatorname{Im} z = \frac{z-\bar{z}}{2}$, $|z| = (z\bar{z})^{1/2}$ its real and imaginary parts and its absolute value.

The *standard inner product* on the product space $\mathbb{C}^n = \mathbb{C} \times \cdots \times \mathbb{C}$ is defined by $\langle z, w \rangle = \sum_{i=1}^n z_i w_i$, and axioms (IP1)–(IP4) are readily checked. Also \mathbb{C}^n is a normed space with $\|z\|^2 = \sum_{i=1}^n |z_i|^2$.

The pair $(\mathcal{H}, \langle \cdot, \cdot \rangle)$ is called an *inner product space*.

In an inner product space \mathcal{H} , two vectors $h_1, h_2 \in \mathcal{H}$ are called orthogonal, and we write $h_1 \perp h_2$, provided $\langle h_1, h_2 \rangle = 0$. For a subset

In an inner product space \mathcal{H} , two vectors $h_1, h_2 \in \mathcal{H}$ are called orthogonal, and we write $h_1 \perp h_2$, provided $\langle h_1, h_2 \rangle = 0$. For a subset $A \subset \mathcal{H}$, the set A^\perp defined by $A^\perp = \{h \in \mathcal{H} \mid \langle h, x \rangle = 0 \text{ for all } x \in A\}$ is called the orthogonal complement of A .

In an inner product space \mathcal{H} the *Cauchy–Schwartz inequality* holds: $|\langle h_1, h_2 \rangle| \leq \langle h_1, h_2 \rangle^{1/2} \langle h_1, h_2 \rangle^{1/2}$. Here, equality holds provided h_1, h_2 are linearly dependent.

Let $(\mathcal{H}, \|\cdot\|)$ be an inner product space and set $\|h\| = \langle h, h \rangle^{1/2}$. Then $(\mathcal{H}, \|\cdot\|)$ is a normed space.

Let $(\mathcal{H}, \langle \cdot, \cdot \rangle)$ be an inner product space and $\|\cdot\|$ the corresponding norm. Then we have

(1) *Polarization law*:

$$4 \langle h_1, h_2 \rangle = \|h_1 + h_2\|^2 - \|h_1 - h_2\|^2 + i \|h_1 + i h_2\|^2 - i \|h_1 - i h_2\|^2,$$

and

(2) *Parallelogram law*:

$$2 \|h_1\|^2 + 2 \|h_2\|^2 = \|h_1 + h_2\|^2 - \|h_1 - h_2\|^2.$$

Let $(\mathcal{H}, \|\cdot\|)$ be a normed space and define $d(h_1, h_2) = \|h_1 - h_2\|$. Then (\mathcal{H}, d) is a metric space.

Let $(\mathcal{H}, \|\cdot\|)$ be a normed space. If the corresponding metric d is complete, we say $(\mathcal{H}, \|\cdot\|)$ is a Banach space. If $(\mathcal{H}, \|\cdot\|)$ is an inner product space whose corresponding metric is complete, we say $(\mathcal{H}, \|\cdot\|)$ is a *Hilbert space* (see, e.g., [Abraham *et al.* (1988)]).

If \mathcal{H} is a Hilbert space and F its closed subspace, then \mathcal{H} splits into two mutually orthogonal subspaces, $\mathcal{H} = F \oplus F^\perp$, where \oplus denotes the *orthogonal sum*. Thus every closed subspace of a Hilbert space splits.

Let \mathcal{H} be a Hilbert space. A set $\{h_i\}_{i \in I}$ is called *orthonormal* if $\langle h_i, h_j \rangle = \delta_{ij}$, the Kronecker delta. An orthonormal set $\{h_i\}_{i \in I}$ is a *Hilbert basis* if $\text{closure}(\text{span}\{h_i\}_{i \in I}) = \mathcal{H}$. Any Hilbert space has a Hilbert basis.

In the finite dimensional case equivalence and completeness are automatic. Let \mathcal{H} be a finite-dimensional vector space. Then (i) there is a norm on \mathcal{H} ; (ii) all norms on \mathcal{H} are equivalent; (iii) all norms on \mathcal{H} are complete.

Consider the space $L^2([a, b], \mathbb{C})$ of *square-Lebesgue-integrable* complex-valued functions defined on an interval $[a, b] \subset \mathbb{C}$, that is, functions f that satisfy $\int_a^b |f(x)|^2 dx < \infty$. It is a Banach space with the norm defined by $\|f\| = \left(\int_a^b |f(x)|^2 dx\right)^{1/2}$, and a Hilbert space with the inner product defined by $\langle f, g \rangle = \int_a^b f(x) \overline{g(x)} dx$.

Recall from elementary linear algebra that the dual space of a finite dimensional vector space of dimension n also has dimension n and so the space and its dual are isomorphic. It is also true for Hilbert space.

Riesz Representation Theorem. Let \mathcal{H} be a real (resp., complex) Hilbert space. The map $h \mapsto \langle \cdot, h \rangle$ is a linear (resp., antilinear) norm-preserving isomorphism of \mathcal{H} with \mathcal{H}^* ; for short, $\mathcal{H} \cong \mathcal{H}^*$. (A map $A : \mathcal{H} \rightarrow F$ between complex vector spaces is called *antilinear* if we have the identities $A(h + h') = Ae + Ae'$, and $A(\alpha h) = \bar{\alpha} Ae$.)

Let \mathcal{H} and F be Banach spaces. We say \mathcal{H} and F are in *strong duality* if there is a non-degenerate continuous bilinear functional $\langle \cdot, \cdot \rangle : \mathcal{H} \times F \rightarrow \mathbb{R}$, also called a *pairing* of \mathcal{H} with F . Now, let $\mathcal{H} = F$ and $\langle \cdot, \cdot \rangle : \mathcal{H} \times \mathcal{H} \rightarrow \mathbb{R}$ be an inner product on \mathcal{H} . If \mathcal{H} is a Hilbert space, then $\langle \cdot, \cdot \rangle$ is a *strongly non-degenerate pairing* by the Riesz representation Theorem.

1.3 Human Intelligence, Mind and Reason

Recall that the word *intelligence* (plural *intelligences*) comes from Latin *intellegentia*.⁵ It is a property of *human mind* that encompasses many related *mental abilities*, such as the capacities to *reason*, *plan*, solve problems, think abstractly, comprehend ideas and *language*, and learn. Although many regard the concept of intelligence as having a much broader scope, for example in *cognitive science* and *computer science*, in some schools of *psychology*,⁶ the study of intelligence generally regards this trait as distinct from *creativity*, *personality*, *character*, or *wisdom*.

Briefly, the word *intelligence* has five common meanings:

- (1) Capacity of human mind, especially to understand principles, truths, concepts, facts or meanings, acquire knowledge, and apply it to practise; the ability to learn and comprehend.
- (2) A form of life that has such capacities.
- (3) Information, usually secret, about the enemy or about hostile activities.
- (4) A political or military department, agency or unit designed to gather such information.
- (5) Biological intelligent behavior represents animal's ability to make productive decisions for a specific task, given a root objective; this decision is based on learning which requires the ability to hold onto results from previous tasks, as well as being able to analyze the situation; the root objective for living organisms is simply survival; the 'specific task' could

⁵*Intellegentia* is a combination of Latin *inter* = *between* and *legere* = *choose, pick out, read*. Inter-lege-nt-ia, literally means 'choosing between.'

Also, note that there is a scientific journal titled 'Intelligence', dealing with intelligence and psychometrics. It was founded in 1977 by Douglas K. Detterman of Case Western Reserve University. It is currently published by Elsevier and is the official journal of the International Society for Intelligence Research.

⁶Recall that *psychology* is an academic and applied field involving the study of the human mind, brain, and behavior. Psychology also refers to the application of such knowledge to various spheres of human activity, including problems of individuals' daily lives and the treatment of mental illness.

Psychology differs from anthropology, economics, political science, and sociology in seeking to explain the mental processes and behavior of individuals. Psychology differs from biology and neuroscience in that it is primarily concerned with the interaction of mental processes and behavior, and of the overall processes of a system, and not simply the biological or neural processes themselves, though the subfield of neuropsychology combines the study of the actual neural processes with the study of the mental effects they have subjectively produced.

The word psychology comes from the ancient Greek 'psyche', which means 'soul' or 'mind' and 'ology', which means 'study'.

be a choice of food, i.e., one that provides long steady supply of energy as it could be a long while before the next mealtime; this is in perfect harmony with the root biological objective – survival.

According to Encyclopedia Britannica, *intelligence* is the *ability to adapt effectively to the environment, either by making a change in oneself or by changing the environment or finding a new one*. Different investigators have emphasized different aspects of intelligence in their definitions. For example, in a 1921 symposium on the definition of intelligence, the American psychologist Lewis Terman emphasized the *ability to think abstractly*, while another American psychologist, Edward Thorndike, emphasized *learning* and the ability to give good responses to questions. In a similar 1986 symposium, however, psychologists generally agreed on the importance of adaptation to the environment as the key to understanding both what intelligence is and what it does. Such adaptation may occur in a variety of environmental situations. For example, a student in school learns the material that is required to pass or do well in a course; a physician treating a patient with an unfamiliar disease adapts by learning about the disease; an artist reworks a painting in order to make it convey a more harmonious impression. For the most part, adapting involves making a change in oneself in order to cope more effectively, but sometimes effective adaptation involves either changing the environment or finding a new environment altogether. Effective adaptation draws upon a number of cognitive processes, such as perception, learning, memory, reasoning, and problem solving. The main trend in defining intelligence, then, is that it is not itself a cognitive or mental process, but rather a selective combination of these processes purposively directed toward effective adaptation to the environment. For example, the physician noted above learning about a new disease adapts by perceiving material on the disease in medical literature, learning what the material contains, remembering crucial aspects of it that are needed to treat the patient, and then reasoning to solve the problem of how to apply the information to the needs of the patient. Intelligence, in sum, has come to be regarded as not a single ability but an effective drawing together of many abilities. This has not always been obvious to investigators of the subject, however, and, indeed, much of the history of the field revolves around arguments regarding the nature and abilities that constitute intelligence.

1.3.0.1 *Human Reason*

Recall that in the philosophy of arguments, *reason* is the ability of the human mind to form and operate on concepts in abstraction, in varied accordance with rationality and logic — terms with which reason shares heritage. Reason is thus a very important word in Western intellectual history, to describe a type or aspect of mental thought which has traditionally been claimed as distinctly human, and not to be found elsewhere in the animal world. Discussion and debate about the nature, limits and causes of reason could almost be said to define the main lines of historical philosophical discussion and debate. Discussion about reason especially concerns:

- (a) its relationship to several other related concepts: language, logic, consciousness etc,
- (b) its ability to help people decide what is true, and
- (c) its origin.

The concept of reason is connected to the concept of language, as reflected in the meanings of the Greek word ‘logos’, later to be translated by Latin ‘ratio’ and then French ‘raison’, from which the English word derived. As reason, rationality, and logic are all associated with the ability of the human mind to predict effects as based upon presumed causes, the word ‘reason’ also denotes a ground or basis for a particular argument, and hence is used synonymously with the word ‘cause’.

It is sometimes said that the contrast between reason and logic extends back to the time of Plato⁷ and Aristotle.⁸ Indeed, although they had no

⁷Plato (c. 427 — c. 347 BC) was an immensely influential ancient Greek philosopher, a student of Socrates, writer of philosophical dialogues, and founder of the Academy in Athens where Aristotle studied. Plato lectured extensively at the Academy, and wrote on many philosophical issues, dealing especially in politics, ethics, metaphysics, and epistemology. The most important writings of Plato are his dialogues, although some letters have come down to us under his name. It is believed that all of Plato’s authentic dialogues survive. However, some dialogues ascribed to Plato by the Greeks are now considered by the consensus of scholars to be either suspect (e.g., First Alcibiades, Clitophon) or probably spurious (such as Demodocus, or the Second Alcibiades). The letters are all considered to probably be spurious, with the possible exception of the Seventh Letter. Socrates is often a character in Plato’s dialogues. How much of the content and argument of any given dialogue is Socrates’ point of view, and how much of it is Plato’s, is heavily disputed, since Socrates himself did not write anything; this is often referred to as the ‘Socratic problem’. However, Plato was doubtless strongly influenced by Socrates’ teachings.

Platonism has traditionally been interpreted as a form of metaphysical dualism, sometimes referred to as Platonic realism, and is regarded as one of the earlier representatives of metaphysical objective idealism. According to this reading, Plato’s metaphysics

separate Greek word for logic as opposed to language and reason, Aristotle's *syillogism* (Greek 'syllogismos') identified logic clearly for the first time as a distinct field of study: the most peculiarly reasonable ('logikê') part of reasoning, so to speak.

No philosopher of any note has ever argued that logic is the same as reason. They are generally thought to be distinct, although logic is one important aspect of reason. But the tendency to the preference for 'hard logic', or 'solid logic', in modern times has incorrectly led to the two terms occasionally being seen as essentially synonymous or perhaps more often logic is seen as the defining and pure form of reason.

However machines and animals can unconsciously perform logical operations, and many animals (including humans) can unconsciously, associate different perceptions as causes and effects and then make decisions or even plans. Therefore, to have any distinct meaning at all, 'reason' must be the type of thinking which links language, consciousness and logic, and at this time, only humans are known to combine these things.

However, note that reasoning is defined very differently depending on the context of the understanding of reason as a form of knowledge. The logical definition is the act of using reason to derive a conclusion from certain premises using a given methodology, and the two most commonly used explicit methods to reach a conclusion are deductive reasoning and inductive reasoning. However, within idealist philosophical contexts, reasoning is the mental process which informs our imagination, perceptions, thoughts, and feelings with whatever intelligibility these appear to contain; and thus links our experience with universal meaning. The specifics of the methods of reasoning are of interest to such disciplines as philosophy, logic, psychology, and artificial intelligence.

In deductive reasoning, given true premises, the conclusion must follow and it cannot be false. In this type of reasoning, the conclusion is inherent in the premises. Deductive reasoning therefore does not increase one's knowledge base and is said to be non-ampliative. Classic examples of deductive reasoning are found in such syllogisms as the following:

divides the world into two distinct aspects: the *intelligible world* of 'forms', and the *perceptual world* we see around us. The perceptual world consists of imperfect copies of the intelligible forms or ideas. These forms are unchangeable and perfect, and are only comprehensible by the use of the intellect or understanding, that is, a capacity of the mind that does not include sense-perception or imagination. This division can also be found in Zoroastrian philosophy, in which the dichotomy is referenced as the *Minu* (intelligence) and *Giti* (perceptual) worlds. Currently, in the domain of mathematical physics, this view has been adopted by Sir Roger Penrose [Penrose (1967)].

- (1) One must exist/live to perform the act of thinking.
- (2) I think.
- (3) Therefore, I am.

⁸Aristotle (384 BC — March 7, 322 BC) was an ancient Greek philosopher, a student of Plato and teacher of Alexander the Great. He wrote books on divers subjects, including physics, poetry, zoology, logic, rhetoric, government, and biology, none of which survive in their entirety. Aristotle, along with Plato and Socrates, is generally considered one of the most influential of ancient Greek philosophers. They transformed Presocratic Greek philosophy into the foundations of Western philosophy as we know it. The writings of Plato and Aristotle founded two of the most important schools of Ancient philosophy.

Aristotle valued knowledge gained from the senses and in modern terms would be classed among the modern empiricists. He also achieved a 'grounding' of dialectic in the *Topics* by allowing interlocutors to begin from commonly held beliefs (*Endoxa*), with his frequent aim being to progress from 'what is known to us' towards 'what is known in itself' (*Physics*). He set the stage for what would eventually develop into the empirical scientific method some two millennia later. Although he wrote dialogues early in his career, no more than fragments of these have survived. The works of Aristotle that still exist today are in treatise form and were, for the most part, unpublished texts. These were probably lecture notes or texts used by his students, and were almost certainly revised repeatedly over the course of years. As a result, these works tend to be eclectic, dense and difficult to read. Among the most important ones are *Physics*, *Metaphysics* (or *Ontology*), *Nicomachean Ethics*, *Politics*, *De Anima* (*On the Soul*) and *Poetics*. These works, although connected in many fundamental ways, are very different in both style and substance.

Aristotle is known for being one of the few figures in history who studied almost every subject possible at the time, probably being one of the first polymaths. In science, Aristotle studied anatomy, astronomy, economics, embryology, geography, geology, meteorology, physics, and zoology. In philosophy, Aristotle wrote on aesthetics, ethics, government, metaphysics, politics, psychology, rhetoric and theology. He also dealt with education, foreign customs, literature and poetry. His combined works practically constitute an encyclopedia of Greek knowledge.

According to Aristotle, everything is made out of the five basic elements:

- (1) Earth, which is cold and dry;
- (2) Water, which is cold and wet;
- (3) Fire, which is hot and dry;
- (4) Air, which is hot and wet; and
- (5) Aether, which is the divine substance that makes up the heavenly spheres and heavenly bodies (stars and planets).

Aristotle defines his philosophy in terms of essence, saying that philosophy is 'the science of the universal essence of that which is actual'. Plato had defined it as the 'science of the idea', meaning by idea what we should call the unconditional basis of phenomena. Both pupil and master regard philosophy as concerned with the universal; Aristotle, however, finds the universal in particular things, and called it the essence of things, while Plato finds that the universal exists apart from particular things, and is related to them as their prototype or exemplar. For Aristotle, therefore, philosophic

In inductive reasoning, on the other hand, when the premises are true, then the conclusion follows with some degree of *probability*.⁹ This method

method implies the ascent from the study of particular phenomena to the knowledge of essences, while for Plato philosophic method means the descent from a knowledge of universal ideas to a contemplation of particular imitations of those ideas. In a certain sense, Aristotle's method is both inductive and deductive, while Plato's is essentially deductive from *a priori* principles.

In the larger sense of the word, Aristotle makes philosophy coextensive with reasoning, which he also called 'science'. Note, however, that his use of the term science carries a different meaning than that which is covered by the scientific method. "All science (*dianoia*) is either practical, poetical or theoretical." By practical science he understands ethics and politics; by poetical, he means the study of poetry and the other fine arts; while by theoretical philosophy he means physics, mathematics, and metaphysics.

Aristotle's conception of logic was the dominant form of logic up until the advances in mathematical logic in the 19th century. Kant himself thought that Aristotle had done everything possible in terms of logic. The *Organon* is the name given by Aristotle's followers, the Peripatetics, for the standard collection of six of his works on logic. The system of logic described in two of these works, namely *On Interpretation* and the *Prior Analytics*, is often called Aristotelian logic.

Aristotle was the creator of syllogisms with modalities (modal logic). The word modal refers to the word 'modes', explaining the fact that modal logic deals with the modes of truth. Aristotle introduced the qualification of 'necessary' and 'possible' premises. He constructed a logic which helped in the evaluation of truth but which was difficult to interpret.

⁹Recall that the word *probability* derives from the Latin 'probare' (to prove, or to test). Informally, probable is one of several words applied to uncertain events or knowledge, being closely related in meaning to likely, risky, hazardous, and doubtful. Chance, odds, and bet are other words expressing similar notions. Just as the theory of mechanics assigns precise definitions to such everyday terms as work and force, the theory of probability attempts to quantify the notion of probable.

The scientific study of probability is a modern development. Gambling shows that there has been an interest in quantifying the ideas of probability for millennia, but exact mathematical descriptions of use in those problems only arose much later. The doctrine of probabilities dates to the correspondence of Pierre de Fermat and Blaise Pascal (1654). Christiaan Huygens (1657) gave the earliest known scientific treatment of the subject. Jakob Bernoulli's 'Ars Conjectandi' (posthumous, 1713) and Abraham de Moivre's 'Doctrine of Chances' (1718) treated the subject as a branch of mathematics.

Pierre-Simon Laplace (1774) made the first attempt to deduce a rule for the combination of observations from the principles of the theory of probabilities. He represented the law of probability of errors by a curve $y = \varphi(x)$, x being any error and y its probability, and laid down three properties of this curve: (i) it is symmetric as to the y -axis; (ii) the x -axis is an asymptote, the probability of the error being 0; (iii) the area enclosed is 1, it being certain that an error exists. He deduced a formula for the *mean* of three observations. He also gave (1781) a formula for the law of facility of error (a term due to Lagrange, 1774), but one which led to unmanageable equations. Daniel Bernoulli (1778) introduced the principle of the maximum product of the probabilities of a system of concurrent errors.

The *method of least squares* is due to Adrien-Marie Legendre (1805), who introduced it in his 'Nouvelles méthodes pour la détermination des orbites des comètes' (New Methods

of reasoning is ampliative, as it gives more information than what was contained in the premises themselves. A classical example comes from David Hume:¹⁰

- (1) The sun rose in the east every morning up until now.
- (2) Therefore the sun will also rise in the east tomorrow.

A third method of reasoning is called *abductive reasoning*, or inference to the best explanation. This method is more complex in its structure and can involve both inductive and deductive arguments. The main characteristic of abduction is that it is an attempt to favor one conclusion above others by either attempting to falsify alternative explanations, or showing the likelihood of the favored conclusion given a set of more or less disputable assumptions.

A fourth method of reasoning is analogy. Reasoning by analogy goes from a particular to another particular. The conclusion of an analogy is only plausible. Analogical reasoning is very frequent in common sense, science, philosophy and the humanities, but sometimes it is accepted only as an auxiliary method. A refined approach is *case-based reasoning*.

for Determining the Orbits of Comets). In ignorance of Legendre's contribution, an Irish-American writer, Robert Adrain, editor of 'The Analyst' (1808), first deduced the law of facility of error,

$$\phi(x) = ce^{-h^2 x^2}$$

where c and h are constants depending on precision of observation. He gave two proofs, the second being essentially the same as John Herschel's (1850). Carl Friedrich Gauss gave the first proof which seems to have been known in Europe (the third after Adrain's) in 1809. Further proofs were given by Laplace (1810, 1812), Gauss (1823), James Ivory (1825, 1826), Hagen (1837), Friedrich Bessel (1838), W. F. Donkin (1844, 1856), and Morgan Crofton (1870).

¹⁰David Hume (April 26, 1711 – August 25, 1776)[1] was a Scottish philosopher, economist, and historian, as well as an important figure of Western philosophy and of the Scottish Enlightenment.