

# Preface

Although every modern house has a TV set and a computer, one can still find here and there, a grandfather wall clock going tick-tock. The invention of this clock goes back to 1581, when the 17 year-old Galileo Galilei watched a suspended lamp swinging back and forth in the cathedral of Pisa, and found, to his surprise, that it took the same number of pulse beats for the chandelier to complete one swing, no matter how large the amplitude. The larger the swing, the faster the motion, but the time was the same. Therefore, time could be measured by the swing of a pendulum – the basis for the pendulum clock. In 1602, Galilei explained the isochronism of long pendulums in a letter to a friend, and a year later, another friend, Santorio Santorino, a physician in Venice, began using a short pendulum, which he called a “pulsilogium”, to measure the pulse of his patients. This discovery had important implications for the measurement of time intervals<sup>1</sup>. Following Galilei, a few other famous scientists continued the analysis of the pendulum. For example, the period of rotation and oscillation with finite amplitude were calculated by Huygens (1673) and by Euler (1736), respectively.

Although the legend of how Galilei discovered this property of the simple pendulum is probably apocryphal, the main idea of a pendulum is widely used today to describe the oscillation of prices in the stock market or the

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<sup>1</sup>As a matter of fact, one of the earliest uses of the pendulum was in the seismometer device of the Han Dynasty (202 BC – 220 AD) by the scientist and inventor Zhang Heng (78 – 139). Its function was to sway and activate a series of levers after being disturbed by the tremor of an earthquake. After being triggered, a small ball would fall out of the urn-shaped device into a metal toad’s mouth, signifying the cardinal direction where the earthquake was located (and where government aid and assistance should be swiftly sent). Also, an Arabian scholar, Ibn Vinus, is known to have described an early pendulum in the 10th century [1].

change of mood of our wives (husbands). However, scientists use the concept of a pendulum in a much more comprehensive sense, considering it as a model for a great diversity of phenomena in physics, chemistry, economics and communication theory.

A harmonic oscillator is the simplest linear model, but most, if not all physical processes are nonlinear, and the simplest model for their description is a pendulum. Tremendous effort has gone into the study of the pendulum. From the enormous literature it is worth mentioning the bibliographic article [2], the International Pendulum conference [3], and the recent book [4] written in a free, easy-going style, in which calculations alternate with historical remarks.

The aim of the present book is to give the “pendulum dictionary”, including recent (up to 2007) results. It can be useful for a wide group of researchers working in different fields where the pendulum model is applicable. We hope that teachers and students will find some useful material in this book. No preliminary knowledge is assumed except for undergraduate courses in mechanics and differential equations. For the underdamped pendulum driven by a periodic force, a new phenomenon - deterministic chaos - comes into play, and one has to take into account the delicate balance between this chaos and the influence of noise.

The organization of the book is as follows.

Part 1 is comprised of three sections. Sec. 1.1 contains the description and solution of the dynamic equation of motion for the simple mathematical pendulum oscillating in the field of gravity, which creates a torque. This equation is isomorphic to a model description of different phenomena, which are presented in Sec. 1.2. The description of noise as used in the book is given in Sec. 1.3.

Part 2 describes the properties of the overdamped pendulum. The simple overdamped case for which one neglects the inertial term is considered first, thereby reducing the second-order differential equation to one of first-order. The analytical solution of the overdamped deterministic pendulum is described in Sec. 2.1. The addition of additive and multiplicative noise, both white and dichotomous, provide the subject of Sec. 2.2. An overdamped pendulum subject to a periodic torque signal is described in Sec. 2.3, in particular, the analytic solution for a periodic force having the form of a pulsed signal.

Part 3 is devoted to the underdamped pendulum, including the effect of friction (Sec. 3.1), multiplicative noise (Sec. 3.2), additive noise (Sec. 3.3), the periodically driven pendulum (Sec. 3.4), overall forces (Sec. 3.5), and an

oscillating suspension point (Sec. 3.6). The spring pendulum is described in Sec. 3.7. Finally, resonance-type phenomena are considered in Sec. 3.8.

Part 4 is devoted to a description of the phenomenon of deterministic chaos. Deterministic chaos leads to “noise-like” solutions, which explains why this chapter appears in the noisy pendulum book. The general concepts are introduced in Sec. 4.1, and the transition to chaos for different cases provides the subject matter for Sec. 4.2. New phenomena (chaos control, erratic motion and vibrational resonance), which are caused by an addition of a second periodic force, are considered in Sec. 4.3.

Part 5 contains the analysis of the inverted pendulum. The inverted position can be stabilized either by periodic or random oscillations of the suspension axis or by the influence of a spring inserted into a rigid rod. These three cases are considered in Secs. 5.1–5.4, while the joint effect of these factors is described in Sec. 5.5. Finally, we present our conclusions.

Two comments are appropriate. First, in spite of the fact that the majority of results presented in this book are related to nonintegrable equations which demand numerical solutions, we are not discussing technical questions concerning the methods and accuracy of numerical calculations. Second, we consider only a single one-dimensional classical pendulum. The quantum pendulum and interactions between pendula are beyond the scope of this book. Even then, there are a tremendous number of published articles devoted to the pendulum (331 articles in the American Journal of Physics alone). In order not to increase unduly the size of this book and to keep it readable, I have not described all these articles. I ask the forgiveness of the authors whose publications remain beyond the scope of this book.