

Contents

<i>Preface</i>	v
1. Formulation of the Problem	1
1.1 Mathematical pendulum	1
1.2 Isomorphic models	10
1.2.1 Brownian motion in a periodic potential	10
1.2.2 Josephson junction	10
1.2.3 Fluxon motion in superconductors	11
1.2.4 Charge density waves (CDWs)	11
1.2.5 Laser gyroscope	12
1.2.6 Synchronization phenomena	12
1.2.7 Parametric resonance in anisotropic systems	12
1.2.8 The Frenkel-Kontorova model (FK)	13
1.2.9 Solitons in optical lattices	13
1.3 Noise	13
1.3.1 White noise and colored noise	13
1.3.2 Dichotomous noise	15
1.3.3 Langevin and Fokker-Planck equations	15
2. Overdamped Pendulum	19
2.1 Deterministic motion	19
2.2 Influence of noise	20
2.2.1 Additive white noise	21
2.2.2 Additive and multiplicative white noise	23
2.2.3 Additive dichotomous noise	30
2.2.4 Multiplicative dichotomous noise	33

2.2.5	Joint action of multiplicative noise and additive noise	34
2.2.6	Correlated additive noise and multiplicative noise	37
2.3	Periodic driven force	39
2.3.1	Deterministic equation	39
2.3.2	Influence of noise	40
2.3.3	Deterministic telegraph signal	41
3.	Underdamped Pendulum	43
3.1	Pendulum with constant torque	43
3.2	Pendulum with multiplicative noise	45
3.3	Pendulum with additive noise	46
3.3.1	Damped pendulum subject to additive noise	46
3.3.2	Damped pendulum subject to constant torque and noise	47
3.4	Periodically driven pendulum	49
3.5	Damped pendulum subject to constant torque, periodic force and noise	52
3.6	Pendulum with oscillating suspension point	53
3.6.1	Vertical oscillations	53
3.6.2	Horizontal oscillations	57
3.6.3	Pendulum with parametric damping	57
3.7	Spring pendulum	60
3.8	Resonance-type phenomena	66
3.8.1	Stochastic resonance (SR)	66
3.8.2	Absolute negative mobility (ANM)	68
3.8.3	Ratchets	68
3.8.4	Resonance activation (RA) and noise enhanced stability (NES)	70
4.	Deterministic Chaos	71
4.1	General concepts	71
4.1.1	Poincare sections and strange attractors	72
4.1.2	Lyapunov exponent	73
4.1.3	Correlation function	73
4.1.4	Spectral analysis	73
4.1.5	Period doubling and intermittency	74
4.2	Transition to chaos	75

4.2.1	Damped, periodically driven pendulum	75
4.2.2	Driven pendulum subject to a periodic and constant torque	79
4.2.3	Pendulum with vertically oscillating suspension point	80
4.2.4	Pendulum with horizontally oscillating suspension point	81
4.2.5	Pendulum with applied periodic force	84
4.2.6	Spring pendulum	86
4.3	Pendulum subject to two periodic fields	87
4.3.1	Controlling chaos	87
4.3.2	Erratic motion	88
4.3.3	Vibrational resonance	90
5.	Inverted Pendulum	93
5.1	Oscillations of the suspension axis	93
5.2	The tilted parametric pendulum	95
5.3	Random vibrations of the suspension axis	98
5.4	Spring pendulum	100
5.5	Spring pendulum driven by a periodic force	101
6.	Conclusions	109
	<i>Bibliography</i>	113
	<i>Index</i>	119