

# Introduction

The problems of research on turbulent and hydrodynamic instabilities occupy the scientific minds for many years. The creation of highly efficient computational technique and its usage gave a new impulse to the study of such complicated phenomenon, which are described by “multi-dimensional nonlinear equations in partial derivatives with complicated internal structure.” Many ideas developed in this monograph have been published earlier in Russia and elsewhere, but this book is the most complete one, in a certain way it synthesizes the ideology of computational turbulence, suggested by the author and his followers.

The history of the turbulence theory begins from the classical researches of Reynolds and Richardson. In the middle of 1920s, Keller and Fridman proposed the idea of stochastic description of turbulent processes that had invaluable principal significance. A number of important achievements were made (for instance, Kolmogorov–Obukhov spectrum). Nevertheless, the physical principles of turbulence development remain unclear (for instance, what is the source of energy of chaotic motion). Currently, the experimental data point clearly to the existence of the large-scale “coherent” structures (especially, for fully developed turbulence), where the main part of energy transferred is disposed. Let us note that, *for high Reynolds number (Re), the energetic part of spectrum is far from the dissipative one in terms of wave numbers.*

The last decades have been marked by a new approach<sup>1,2</sup> in the study of turbulence, namely, by the direct numerical simulation of the processes of hydrodynamic flow on the basis of the solution of hydrodynamic and kinetic equations. The results of such an approach demonstrate an important difference from the traditional statistical methods. *For instance, the significant role of large-scale “ordered” structures in the turbulence development was shown.*

The advent of the highly efficient computational technique made it possible to realize the simulation of the quite complicated phenomena and processes in mechanics, physics, and even in . . . medicine. Generally speaking, here we have to do with multi-dimensional (spatial) nonstationary problem, whose original mathematical formulation is rather confusing. The solution to such problems is carried out, as a rule, within the frame of a **computational experiment**, when the systematical computations might be realized with the variations of the original problem setting, so that the eventual results correspond both to the criteria prescribed and (within certain limits) to the initial physical process. Moreover, if one takes into account the need to the development of the effective “rational” numerical methods (which would permit to bring about their realization using the “admissible” computer time), then it becomes quite clear that the organization of computational cycle on the whole demands an utmost skill.

Quite evidently, such a class of problems belong to those of a numerical simulation of the phenomena of turbulence and of the instabilities’ development. Taking into account the *structural* character of turbulence (large-scale “coherent” structures, statistical background, laminar–turbulent transition, etc.), which has, generally speaking, various mechanisms of interaction, it seems necessary to consider also various original formulations (sets of equations), which would be the most adequate to the processes under investigation. Only by taking into account all the factors indicated it might prove possible to accomplish effectively the numerical realization of the wide spectrum of the problems of turbulence.

Treated in Chapter 1 is one of the complicated types of fully developed shear turbulent flows, namely, the phenomena related to the high Reynolds number within the wake (both the close-range and long-range ones) behind the moving body, oceanic flows, etc. In this case apart from the three dimensionality and unsteadiness, one takes into consideration the medium’s compressibility and the effect of viscosity (where the prevailing role is played by the molecular mechanism of interaction) as well. Thus, all the observations, as a whole, form the basis for the “rational” approach, which is not quite ordinary by the numerical modeling of structural turbulence.

## Main Ideology

1. To put it briefly, the main features of our approach might be described as follows. For the wide class of phenomena of that type, by the high

Reynolds numbers within the low-frequency and inertial intervals of turbulent motion, the effect of molecular viscosity and of the small flow elements in the largest part of perturbation domain is not practically essential neither for the general characteristics of macroscopic structures of the flow developed, nor for the flow pattern in general. This makes it possible to disregard the effects of molecular viscosity when studying the dynamics of large vortices, and to implement their study on the basis of the models of the ideal gas — “discrete” Euler equations for compressible gas (using the methods of “rational” averaging, but without applying any explicit subgrid approximation and semiempirical models of turbulence). The main ideology of “rational” approach for direct numerical simulation (DNS) of the characteristics of the fully developed shear turbulence by high  $Re$  — the investigation of the large-ordered structure (LOS) and small-scale stochastic turbulence (ST) is based on the two hypotheses: the “independence of LOS and ST” and the “weak influence of molecular viscosity (or, more generally, the mechanism of dissipation) on properties of LOS”.<sup>1,2</sup> Eventually, Kolmogorov has received the form of a spectrum in an inertial interval assuming nothing in general about a kind of dissipative mechanism. Among the problems to be studied here are those of the jet-type flow in the wake behind the body, the motions of ship frames with stern shearing, the formation of anterior stalling zones by the flow about blunted bodies with jets or needles directed to meet the flow, etc.

2. At the same time, the properties of flows within the boundary layers and within the thin layers of mixing, at the viscous interval of turbulence, as well as those of flows by the moderate Reynolds numbers and in the domain of laminar–turbulent transition, are primarily determined by the molecular diffusion, and for these flows it is necessary to consider Navier–Stokes models.
3. The pulsational motions in turbulence are of chaotic type and have an unstable, irregular character, thus constituting a stochastic process. In this respect, one can speak here only about obtaining the mean characteristics of that type of motion (like the moments of various orders) by means of the statistical processing of the results using kinetic Monte Carlo approaches.

The studies of various kinds of instabilities (like, for example, **Richtmyer–Meshkov**, **Rayleigh–Taylor**, **Kelvin–Helmholtz instabilities**) are of high interest, especially for the calculations extended to

the large temporal intervals, up to the turbulent stage. The main difficulty, which one encounters in considering the above-indicated class of problems, consists in the development of the **general concept** of the creation of constructive numerical models of turbulence, of the instabilities' development, and of the transition to chaos.

When studying complicated physical phenomena that take place either on the galactic scale, for example, in astrophysical investigations of the substance-mixing processes in supernovas, or on the microscale inherent to nuclear physics, for example, in realizing the inertial confinement fusion, there arises the necessity to analyze physical mechanisms and their adequate description for various hydrodynamic instabilities. These are the Rayleigh–Taylor, Kelvin–Helmholtz, and Richtmyer–Meshkov instabilities. As numerous experimental and theoretical investigations show, the change of dimensional characteristics of phenomena, namely, the transition from two-dimensional to three-dimensional flows is accompanied by the appearance of new physical effects. In lower dimension problems, these effects are either absent or manifest themselves to the degree inaccessible for observation. Among the hydrodynamic instabilities, the three-dimensional flows formed when developing the Richtmyer–Meshkov instability are the least studied because, aiming to realize their numerical simulation, it is necessary, not only to have an extended difference grid, but also higher-quality algorithms for considering strong discontinuities, in particular, shock waves and their interactions. It is this fact that substantially complicates experimental diagnostic investigations. In the numerical simulation of Rayleigh–Taylor and Kelvin–Helmholtz instabilities, it was established that, for the identical initial amplitudes of perturbations and wavelengths, the growth rate for perturbations is higher in the three-dimensional case as compared to the two-dimensional one, while the process of formation of mushroom-shaped structures proceeds more slowly. Similar results for the Richtmyer–Meshkov instability were obtained by other experiments. In this connection, two types of questions arise. They are associated, first, with studying the physical mechanisms that lead to the observable phenomena, and, second, with establishing the relationship between the growth rates for perturbation amplitude and a number of geometrical and physical quantities. These are the amplitude and the duration of the perturbation, its shape, the Mach and Atwood numbers, the thermodynamic properties of substances etc. which require systematic investigations (both theoretical and experimental).

Presented in Chapter 2 are the results of comparative calculation studies of the development of Richtmyer–Meshkov instability (RMI) for the two-dimensional Cartesian case and for the corresponding axially symmetrical and three-dimensional versions.

The investigation of Rayleigh–Taylor instability (RTI) is of considerable practical interest at present. A numerical experiment is an effective means of investigation. The solutions to the problem of a nonlinear phase of spatially three-dimensional perturbations have been obtained through it, and the spatially two-dimensional RTI problem has been investigated in detail. There is an urgent need to study RTI because it must be taken into account in analyzing problems of practical importance, concerning the instability of the boundary between detonation products and a gas in an explosion, the compression of spherical laser targets, etc.

In the analysis of the actual RTI processes, it is necessary to investigate not the individual perturbations with a fixed wavelength but the development of a collection of perturbations with different wavelengths and the interaction of these perturbations.

Chapter 3 represents the results of the numerical experiment carried out by the various methods. A complete vortical system of Euler equations of the compressible medium motion is solved. We consider the influence of the interaction of two or more waves on the evolution of the contact surface in an RTI for the two- and three-dimensional cases.

Finally, Chapter 4 is devoted to the investigation of the turbulence problems, using the statistical Monte Carlo method. The approach of that kind proved to be highly effective when considering the problems of rarefied gas-dynamics. However, as concerns the turbulent motions, the possibility of the application of Monte-Carlo methods is still evaluated on a comparatively small scale.

There are also five appendices in the book.

Appendix A is a reprint of my paper<sup>1</sup> dated 1976, where the main principles of our ideology were published at first. I also included in the book two papers — “Formation of large-scale structures in the gap between rotating cylinders: the Rayleigh–Zeldovich problem” (Appendix B) and “Universal technology of parallel computations for the problems described by systems of the equations of hyperbolic type: a step to supersolver” (Appendix C) published in the last two years. Appendix D presents itself my scientific report “Mathematical modeling using supercomputers with parallel architecture” given on October 15, 2003, at the meeting of Presidium of Russian Academy of Sciences.

And at last in Appendix E, “On nuts and bolts of structural turbulence and hydrodynamic instabilities”, are formulated the main positions for the analyses of so complicated and multifunctional problems, that it appeared as the result of many discussions (Academicians O. M. Belotserkovskii, Dr. A. M. Oparin, Dr. O. V. Troshkin, and Dr. V. M. Chechetkin). This part can be used as a program for the further researches, which is opened for talks over.

The classical fundamental results of research in the area of turbulence and instabilities are well known. One might cite the monographs and papers by G. I. Taylor, G. K. Batchelor, H. Lamb, A. N. Kolmogorov, A. M. Obukhov, F. Harlow, J. O. Hinze, W. Heisenberg, L. D. Landau and E. M. Lifshitz, A. S. Monin and A. M. Yaglom, A. Einstein, C. C. Lin, A. A. Townsend, V. M. Ievlev, and many others. Actually, following these works, the elaboration of the general numerical methods for solution of nonlinear problems of aerodynamics, turbulence, and plasma physics was carried out.

The approaches which are mainly discussed further on, are those which use (for the description of the free-developed turbulent flow for extended temporal intervals) the **complete** (and **closed**) set of the dynamical equations for true values of velocities and pressure, as well as the statistical methods. The combined application of both these approaches (based on the use of hydrodynamic equations and on the statistical Monte Carlo method) permits to understand in more detail the structures of turbulence, and to determine the rational ways of constructing the corresponding mathematical models. The series research in this field was started from Karman’s Lecture<sup>1</sup> given by O. M. Belotserkovskii in March 15–19, 1976. This new approach to the study of turbulence was published (in rather detailed form) in a paper.<sup>2</sup>

One should admit that some postulates of this approach have a heuristic and intuitive character but the results obtained speak for themselves. The overviews of these approaches were presented in different periods at the seminars of Academicians P. L. Kapitza, N. N. Yanenko, V. V. Struminskii, A. M. Obukhov, A. S. Monin, and, also, G. Batchelor, H. Daiguji *et al.* The detailed discussion was held in 1994–1995 in Los Alamos (USA) in collaboration with Dr. F. Harlow and his colleagues, where the author spent about 4 months. As a rule, the concepts proposed were apprehended quite adequately. One might think that a sufficiently minute account on the approach developed would be of an indubitable interest both from the purely academic point of view and for the practical experts in the corresponding field.

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I will also be grateful to the readers for any remarks and suggestions, which I take into account in the next work.

I invite readers to the mutual cooperation!

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## References

1. O. M. Belotserkovskii, Computational experiment: direct numerical simulation of complex gas-dynamics flows on the basis of Euler, Navier–Stokes, and Boltzmann models, Karman’s Lecture, von Karman Institute for Fluid Dynamics, 15–19 March 1976, in *Numerical Methods in Fluid Dynamics*, eds. H. J. Wirz and J. J. Smolderen (Hemisphere, Washington, London, 1978), pp. 339–387.
2. O. M. Belotserkovskii, Direct numerical modeling of free induced turbulence, *Zh. Vychisl. Mat. Mat. Fiz.* (in Russian) **25**(12), 1857–1883 (1985) [translated in *J. Comput. Math. Math. Phys.* **25**(6), 166–183 (1985)].