Section 0

Introduction

Introductory Remark 2009

The intemperate young man who in 1972 wrote the Introduction below has vanished beyond recall. His successor has made numerous corrections, mostly typographical, some mathematical. The sweeping assertions and dubious jokes remain. Altering them would have been unjust to the author of long ago.

This book was written to answer one question: "Does a recursion theorist dare to write a book on model theory?" Consequently there are some observations scattered through it without proof concerning the absoluteness (in the sense of Gödel [Gö1]) of model theoretic notions and the ordinals needed to define them. For example Morley's notion of total transcendentality is absolute, and the only ordinals needed to decide the total transcendentality of a theory T are the ordinals recursive in T. Part of the blame belongs to B. Dreben who once asked with characteristic sweetness: "Does model theory have anything to do with logic?" It is true that model theory bears a disheartening resemblance to set theory, a fascinating branch of mathematics with little to say about fundamental logical questions. But the resemblance is more of manners than of ideas, because the central notions of model theory are absolute, and absoluteness, unlike cardinality, is a logical concept. That is why model theory does not founder on that rock of undecidability, the generalized continuum hypothesis, and why the Łos conjecture is decidable: A theory T is κ-categorical if all models of T of cardinality κ are isomorphic. Łos conjectured and Morley proved (Theorem 37.4) that if a countable theory is κ-categorical for some uncountable κ, then it is κ-categorical for every uncountable κ. The property "T is κ-categorical for every uncountable κ" is of course an absolute property of T.

The notion of rank of 1-types was invented by Morley to prove Łos's conjecture. There are proofs of it that make no mention of rank, but they leave one ill prepared to prove Shelah's uniqueness theorem (Sec. 36). I have made rank a central idea of the book, because it is the central idea of current model theory. The assignment of rank to the 1-types realized by elements of structures makes it possible to prove theorems about structures by induction on rank. Not all 1-types associated with substructures of models of a theory T need have a rank; if they do, then T is said to be totally transcendental. Morley' s notion of rank was inspired by the Cantor–Bendixson differentiation of a closed subset of a compact Hausdorff space; however, the Morley derivative differs from the Cantor–Bendixson derivative in that the former commutes with the inverse limit operation. The Morley derivative is expounded in Sec. 29 as a transformation which acts on functors. Section 25 reviews the apparatus of category theory needed in Sec. 29.

The title of this book is a misnomer. The coverage of saturated structures is far from complete: ultraproducts, a kind of canonical saturated extension of considerable importance, are not discussed. The title signifies a preference for the sort of model theory that minimizes syntactical questions. The book is briefer than it appears to be. The number of pages may be large, but the content of any one page is small because of the large size of type employed. A great deal of model theory has been left untouched, partly to achieve brevity, and partly to reflect the prejudices of the author. My mathematical taste favors new constructions and techniques, so I felt no urge to include important theorems whose proofs fail to be novel. Of course the limitations of my taste did not prevent me from repeating several of my favorite constructions.

It is no accident that the book suffers from a shortage of examples. As a rule examples are presented by authors in the hope of clarifying universal concepts, but all examples of the universal, since they must of necessity be particular and so partake of the individual, are misleading.

The least misleading example of a totally transcendental theory is the theory of differentially closed fields of characteristic 0 (DCF_0). Sections 40 and 41 are devoted to L. Blum's applications of Morley rank to DCF_0. There are many notable applications of model theory to algebra, and above all to theories of fields, but Blum was the first to apply something more than the compactness theorem (Corollary 7.2). (One of the most typical and influential uses of compactness in field theory is due to A. Robinson: Suppose F is a first order sentence (in the language of the theory of fields) that is true in every field of characteristic 0. Then there exists an integer n such that F is true in every field of characteristic $p \geq n$.) Blum showed every differential field of characteristic 0 has a prime differential closure. Her theorem follows from a general result of Morley (Theorem 32.4) which holds for all totally transcendental theories. An equally general result of Shelah implies the uniqueness of the prime differential closure (Theorem 41.4).

I am not a historian of model theory, so it is likely I have failed to assign credit justly to many who have contributed to the subject. Names have been attached parenthetically to most of the theorems, but it is morally certain that many of them were discovered independently by several persons (not all of whom were known to me), since it is rare that an inviting idea is the sole property of one mind. I hope no one will construe my ignorance as malice. It is not necessary to be a historian of model theory to realize that the subject owes its existence to the efforts of one man, Alfred Tarski.

Some precautions have been taken to make this book accessible to those with little logic. Definitions of "structures" and "sentences" are given in the early sections, and the commonsensical properties of "logical consequence" are sketched in enough detail in Sec. 7 to make the proof of the fundamental existence theorem (7.1) readable by all.

This book follows closely a course given at Yale University in the Fall of 1970, that course based on notes prepared by S. Simpson, those notes derived from a course given at the Massachusetts Institute of Technology in the Spring of 1969. A large debt — fortunately of the sort that never falls due — is owed to my students in both courses, who insisted relentlessly but rarely successfully that all proofs be complete and correct, to Jane MacIntyre who proved to be a very patient typist, and to my fellow model theorists, among them L. Blum, H. J. Keisler, G. Kreisel, A. H. Lachlan, M. Morley, A. Robinson, F. Rowbottom and S. Shelah, whose generous explanations opened me, contrary to my initial will, to a truly fascinating subject.

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