

Preface

The purpose of this book is to take the reader on a mathematical journey by which the meaning of mathematics at undergraduate level is better understood. It is a transitional work, designed to help bridge the gap between the mathematics which is encountered at high school and the mathematics which is encountered at university.

This book is not a traditional university textbook, nor is it of the popular genre. Textbooks for undergraduates in mathematics tend to be dense, formal and compact affairs, often ill suited for anything except dedicated study and reference. Popular mathematics books tend to focus in mathematically superficial, entertaining ways on the cutting edge, advanced research topics in mathematics, of little direct relevance to the vast majority of mathematicians. Whilst both types of books, of which there are undoubtedly many brilliant examples, have their essential place in a good mathematical education, this author felt that there was a need for a hybrid work which presented correct and full mathematical detail in the inspiring style of a popular book. This book is the result. It is a transitional work in which mathematics comes to life. Mathematics is simultaneously an art, a process and a way of thinking; here these aspects, often neglected in textbooks, are given equal emphasis to the mathematical detail.

Rather than aiming to explain the most difficult, leading edge research mathematics, the book covers the majority of the main ideas covered in a first year of university-level mathematical study, with the occasional foray into more advanced, yet accessible, material. It is the mathematics that, largely speaking, all mathematicians will meet and digest as core elements of their learning; it is the mathematics that all mathematicians will need to struggle to understand as a part of their development. However, this

material is not presented in a dry and formal way. The narrative takes time to be conversational, intuitive and descriptive, spending as much time on the why and the how as on the what. Although much is defined and many results are proved, you will not find the material covered in a traditional textbook format; rather, results are discovered naturally as the book progresses and the formality applied naturally as appropriate.

It is hoped that several categories of reader might enjoy and benefit from this book. Some readers might be mathematically rather advanced, fully fluent in undergraduate mathematics and beyond; it is hoped that they will gain insights from an alternative, discursive presentation of material which they have long dismissed as ‘standard’. Standard material can still be highly exciting; a fresh encounter with this material, it is hoped, will re-energise some aspects of the expert’s mathematical thought, perhaps to help enrich their own teaching. Although in some senses elementary, the material covered is very broad in scope; perhaps the mathematical expert might rediscover areas of mathematics long forgotten in the pursuit of their mathematical specialities?

Other readers might have studied mathematics or a mathematically rich subject at some time in the past. This book collects the highlights of the basic parts of university study together into a coherent whole. By dipping into the book at various places, long-forgotten gems might be uncovered and insights into mathematical activity gleaned. For the some-time mathematician this might be a process of rediscovery and remembrance; for the scientist, engineer or other user of mathematics this might serve to shed light on the place of their mathematical tools in the wider mathematical panorama.

The main set of readers, however, is to whom the book is principally aimed: the student of mathematics who is actively engaged with the transition from school mathematics to university mathematics. Such a student might be in his or her final year at school and studying hard to win a place at university to study mathematics or to succeed in special mathematics examinations or competitions. It is assumed that such readers will be mathematically very able, although not necessarily very well trained in the art of mathematics. Most or all of the material will be new to these readers; however, each section of the book is relatively self-contained and each chapter starts with material very accessible to the patient reader. Even reading a few sections will provide invaluable insight into undergraduate study and higher mathematics. For students already at university, the challenges in-

volved in the transition from school to university mathematics will already be apparent. They will have started to become familiar with some of the technical concepts discussed in the book, but they might still simply feel just technical. The presentation of the material in lectures is likely to be very different from the presentation given here; it has carefully been collected so as to emphasise the natural inevitability of mathematics. Far from being a collection of disjoint and diverse lemmas, theorems and tools, mathematics forms a vibrant, exciting whole. Students of mathematics will eventually come to this realisation; it is hoped that by reading the various sections relevant to their current topic of study students might accelerate and illuminate this process, thereby gaining the most from their time of study.

Many worthwhile endeavours require effort and this book is no exception. A wealth of ideas are covered and a high degree of mental effort will be required to read the text. A basic level of knowledge or familiarity with the mathematics detailed in the appendices will be required at various points. However, the book is very conversational and episodic, and may be read with varying levels of depth, from just noting the key ideas to analysing each proof in detail. Moreover, the dependence of one topic on the next is kept to a minimum. Wherever possible, each new section starts from the very beginning, so if one area is becoming too difficult or does not interest you, move on to the next. In addition, in order not to disrupt the flow of the text or lose sight of the key underlying mathematical ideas, at some points certain more technical details have been glossed over. Hopefully these points have been clearly indicated in the footnotes, and these omissions should not concern the majority of readers.

Mathematics is a rich and exciting art-form. It is hoped that this book passes on to you some appreciation of the real meaning of mathematics.

Stephen Hewson, July 2008