

CHAPTER 1

THE SUSTAINED GROWTH AND ITS RELATION TO THE INITIAL CONDITIONS

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1.1. Introduction

In recent years, deep interest has been shown not only by economic historians and policy makers but also by scholars of economic theory in rapid economic developments that have been accomplished by Asian economies with Japan as a leading country of the industrialization during the latter half of the 20th century in the long-term process of world economic development.

The accelerated rapidity of growth of these economies is presented statistically in Table 1.1, where changes in GDP per capita are compared among four typical countries (UK, USA, Japan, and South Korea) which have achieved their industrialization one after another for these 300 years. In order to compare the rapidity of economic growth among them, it is considered to the point to see the changes after they accomplished take-off for their industrialization. As regards South Korea, the last runner of the take-off among four, Rostow (1983) has indicated that the economy entered the take-off stage in and has completed it for subsequent sustained growth until 1968. With his suggestion being applied to the historical changes in GDP per capita of that country in Table 1.1, the advent of industrialization in South Korea is to have occurred when the economy reached around \$200 of GDP per capita.

Assuming that this is the case for the other three economies, we compare the number of years needed for each of them to reach \$500 of GDP per capita after they are in the take-off stage of \$200. Considering the year of start of its industrialization to be 1600, the UK took about 230 years. As for Japan, regarding the beginning of development policy taken by the Meiji Government in 1868 as the start of its

Table 1.1. Historical change in GDP per capita of four countries.

	1700	1800	1840	1871	1916	1940	1958	1967	1976	1989
UK	288	385	603	1015	1633	2004	2458	3092	3671	4933
USA		417	526	785	1869	2383	3565	4614	5412	7041
Japan				251	552	852	975	2203	3752	5075
S. Korea							144	185	543	1706

Sources: Maddison (1979) and UN-National Account Statistics.

Notes: Figures are represented in terms of PPP conversion US\$ in 1970 price.

Those for South Korea are in terms of market exchange rate conversion.

US\$ in the same fixed price (author's estimates).

industrialization, it took 48 years. In South Korea, it needed only eight years, taking 1968 as the end of its take-off stage according to Rostow. For the more advanced stage beyond \$1000 of GDP per capita, Japan took no more than eight years until the economy reached \$2000 from \$1000 and South Korea would be found to have needed at most four years for it if the data for recent years become available, whilst the UK took 70 years and the USA, 38 years, to double their GDP per capita from \$1000. The years when the economies have reached their advanced stage of the level of \$2000 of GDP per capita are 1940, 1923, 1967, and presumably 1991 for UK, US, Japan, and South Korea, respectively, showing that the two Western representatives of advanced industrialized countries, the UK and the USA, have already accomplished the level of \$2000 before the Second World War, whilst the other two North-East Asian countries, Japan and South Korea, have reached it in the process of industrialization after the War.

Such a rapid and sustained growth as accomplished by Japan and South Korea has been followed by other Asian economies like Hong Kong, and Taiwan, China and more recently it has extended to other Asian economies called Newly Industrializing Economies (NIEs) — Thailand, Malaysia, and Indonesia. It has been accompanied by great social transformation of each country such as urbanization, growing enrollment in general and higher education, and rise in living standards, among other things.

The problems of this sustained and high growth of these Asian economies have been tackled by a number of researchers and scholars theoretically for example, Krugman (1987), Lucas (1988, 1993), Matsuyama (1991), Murakami (1982), Romer (1986) and empirically for example, The World Bank (1991, 1993), Park and Kwon (1995), Muscatelli, Stevenson and Montagna (1995), Kim and Lau (1994), Young (1992, 1994), Edwards (1992), Chen (1979), Tsao (1985, 1986), Wong (1986), Lau and Wan, Jr. (1993), Amsden and Singh (1994) to explain the reasons, identify the sources, and recognize whether the growth will be able to be sustained.

As a matter of course, it is a common feature of the literature by these authors that special emphasis is placed on change in productivity of production processes and a technological factor in international trade. Traditional theories in international trade, however, pay little attention to a role which technology plays in changes in trade patterns, and, therefore, works have been done at first to accommodate a technological factor in trade theories in order to shed light on the "Asian miracle." The introduction of the technological effects in the traditional Ricardian comparative advantage theory was argued by Dornbusch, Fischer, and Samuelson (1977) and the role of technology was discussed by Jones (1970) in the conventional Heckscher–Ohlin's factor-proportions theory. In contrast to these standard static trade theories, Posner (1961) and Vernon (1966) presented a dynamic conception explaining changes in trade patterns through a life cycle of goods, stating that a commodity is initially invented and produced only in developed rich countries and part of the product is exported to developing countries, being followed by the mature stage when it can be produced by them with transferred technology and their advantage of lower labor costs, whilst the country which initially produced and exported it turns importer.

Each of these trade theories was separately contrived with special emphasis on an important but particular economic aspect in production and trade by each of the celebrated economists and has been extended so as to accommodate the effects of technological changes generally in a neoclassical framework by the followers to explain the patterns of trade flows. It seems, however, that the factor-proportions theory, to say the least of it, in the static models and the dynamic product-cycle theory could be incorporated into a single trade theory by explicitly introducing a technological factor in a theoretical framework: technical changes in production process of a commodity in the transition of maturity stages will alter the optimal factor-proportions for production, and the patterns of trade flows will undergo changes accordingly through a life cycle of a good. In this connection, special attention should be paid to policy efforts by the developing economies to introduce new goods and technology when we consider the sources of "Asian miracle" and analyze real features of that sustained high growth.

Although the Richard's and Heckacher–Ohlin's theories have been sophisticated in mathematical frameworks by many economists, Vernon's product-cycle theory remained a descriptive conceptual model. It has to be expressed in a model of some mathematical expression in order to be directed to rigid examination and analysis of economic growth of developing countries. The first achievement, to the best of my knowledge, of formulating the product-cycle model in a mathematical framework was made by Krugman (1979), who developed a simple general-equilibrium model to analyze North-South trade, attempting to explicitly introduce innovation in North and eventual transfer of that technology to South so as to determine the pattern of world trade and its changes over time. According to Krugman's model of the product cycle, it is neatly explained that the trade balance

of a developed country which initially introduced a new good changes from a surplus to a deficit in terms of that commodity after a certain lapse of time and that the balance gradually tends to zero as the technology gap reduces between the developed and developing countries.

Some empirical findings, however, indicate that the real behavior of trade patterns may differ from the results of the theoretical investigations. Gognon and Rose (1995), for instance, empirically examined with the use of disaggregated four-digit SITC level data of trade commodities whether most international trade flows change dynamically as the product-cycle theory states. Their investigation shows mainly “an extremely high degree of persistence” in patterns of international trade flows in terms of individual goods. As a matter of fact, there are some robust evidences that patterns of trade has changed significantly in these years in Asian countries like South Korea, and this will be explained compatibly by changes in factor endowments exerted by rapid industrialization as they state. In this discussion, too, emphasis should be received on the need that the traditional static Heckscher–Ohlin’s trade theory of factor-proportions is incorporated with the dynamic technological theory of product-cycle.

Another direction of researches on “Asian miracle” and North-South trade is the development or suggestion of new growth theories by Lucas (1988, 1993), Murakami (1982), Matsuyama (1991), Romer (1986), Stokey (1988, 1991a, 1991b), Grossman and Helpman (1991) and others, besides Krugman (1979) as mentioned above in relation with the product-cycle model, which explicitly take into consideration increasing returns to scale, accumulation of technological knowledge, or innovative and R&D activities in their theories. A characteristic common to this line of researches lies in constructing a model by paying an attention of great significance to the human capital or accumulation of technological knowledge by learning-by-doing, which may cause increasing returns to scale, of developing economies as an important source of high growth rate of per capita income in Asian countries.

This paper attempts to contribute to an understanding of the way in which the sustained high growth could be accomplished in not a few Asian economies during these two or three decades and to know the reasons why some can make such a “miracle” and some cannot in the same Asian region. Although it explicitly takes not only a technological factor but also a governmental policy aspect in consideration in order to analyze the problems of Asian economic growth in line with the actual environment of the economies, the model represents only the fundamental aspects of growth structure in a simple mathematical framework. Thus, it does not design to develop a full theory to integrate the standard static trade theories and the dynamic technological ones, which would require further considerable amount of work, but to be of some help to theoretically understand the “Asian miracle” and to have some insight into the future development of this region.

Following this introductory section, this paper refers in Section 1.2 to Krugman's analysis and opinion about the "Asian miracle." As mentioned above, Krugman is one of the scholars who have made most influential researches into North-South problems including the recent developments of Asian economies. With his deep academic studies and deliberate considerations as a backing, Krugman has recently documented his views about the "Asian miracle" and the future possibilities of the Asian economies in journals like *Harvard Business Review* (Krugman, 1994a) and *Foreign Affairs* (Krugman, 1994b). I briefly discuss his opinion in reference to Lucas's paper "Making a Miracle" (Lucas, 1993) and propose my own idea, which leads to a model for growth of the Asian economies.

Section 1.3 presents a simple growth model placing emphasis on accumulation of technological knowledge and governmental policy efforts associated with it. This is designed for explanation of the Asian developing economies and for consideration of the conditions for the sustained growth. According to this mathematical model, the possible growth paths are investigated, subsequently. In Section 1.4, the conditions for sustained growth are discussed in connection to structural and industrial policies of the developing economies. Differences in the policies of economic development among Asian economies (especially, between North-East and South-East Asian economies) and their changes during these two or three decades are taken into consideration in this discussion. The considerations and discussions of this paper conclude with Section 1.5 stating the implications of the whole analyses for the "Asian miracle."

1.2. Recent Discussions about "Asian Miracle"

There has appeared a great amount of literature, academic as well as general, on the Asian economies on account of their sustained high growth achieved in the last few decades of this century. Almost all the books and professional papers on them describe the remarkability of high and sustained growth of Japan, South Korea, and some other rapidly growing Asian economies as "Asia's miracle" and attempt to find out the sources or analyze its reasons. Some of these theoretical attempts are those which are mentioned in the previous section, proposing new growth theory by explicitly assuming production functions with increasing returns to scale, innovative activities, or technology transfer mechanisms.

It is the general tone of the argument in the literature about the Asian economies (for instance, the World Bank, 1993) that their growth can be positively appraised because it resulted in the rapid rise in per capita GDP with comparatively high equalization of income distribution and that the success was accomplished by social, industrial, and financial policies of making the fundamental conditions well-suited to domestic and international environments of each country and, at the same time, by the strong, sometimes almost coercive (Song (1990), p. 58), interventions

to the market and the industry. Amidst this general argument, Paul Krugman expresses a different view to the Asian economic growth in his recent article titled "The Myth of Asia's Miracle," which appeared in *Foreign Affairs*, saying that "Popular enthusiasm about Asia's boom deserves to have some cold water thrown on it. Rapid Asian growth is less of a model for the West than many writers claim, and the future prospects for that growth are more limited than almost anyone now imagines" (Krugman (1994b), p. 64). He first mentions the growth and slowdown of the Soviet economy in the following way. The growth of that economy was driven almost by increases in such inputs for production as employment (moving laborers from farms to cities, pushing women into the labor force, and people into longer working hours) and the stock of physical capital (machines and equipment, factories, roads, and other infrastructure) and the contribution of increased efficiency or technical progress in the broad sense of production and management processes, the other important element of growth, was "virtually nonexistent"; the increase in efficiency in using labor and capital in production is essential for continuity of economic growth because mere increases in inputs must eventually lead to diminishing returns, and thus it was inevitable that the growth of the Soviet economy had to come to an end.

Then, Krugman sees "surprising similarities" between the Asian economies of recent years and that of the Soviet Union of three decades ago. Referring to empirical researches about the newly industrialized East Asian economies conducted by Kim and Lau (1994) and Young (1992, 1994 a,b) finding that there is little evidence of improvements in efficiency, he says that rapid growth of Asian economies seems to be driven in large part by an astonishing mobilization of resources of labor and capital and not to be achieved by gains in efficiency, and thus remarks that "Asian economic growth, incredibly, ceases to be a mystery." Accordingly, he calls into question many writers' opinion that the technology gaps between the advanced industrialized countries and the rapidly growing ones will vanish owing to the diffusion of technology, which will undermine the technological advantages and industrial base of the former. Even Japan, he mentions from the recent observation of its rapid economic slow-down, may never overtake America in terms of the level of per capita income even if its technology is gaining on that of the United States because Japan shows a lack of continuous productivity growth comparable to the United States.

Krugman develops a consideration on this issue more deliberately by using models of South-North trade in his article "Does Third World Growth Hurt First World Prosperity?" published in *Harvard Business Review* (Krugman, 1994a). What he virtually wanted to say about the developing economies is not the manifestation of the everlasting supremacy of the Western society over the Third World in terms of economic efficiency, but the warning against disguised protectionism taken by the First World on the basis of the popular but questionable view that competition from the Third World is the biggest threat to the prosperity of the developed

countries and of the misguided belief that the First World will be allowed to take measures to protect their living standard. What he really meant is the hopes for realization of a decent living standard for a vast number of people of the Third World through widespread economic development accomplished by free economic activities. This is typically expressed in the last sentence of his article in *Harvard Business Review* above cited, saying that “Economic growth in the Third World is an opportunity, not a threat; it is our fear of Third World success, not that success itself, that is the real danger to the world economy.” This is rated as a highly convincing opinion for the development of world economy.

Krugman also extends his argument to the common assertion that the sophisticated industrial policies and selective interventions in economic activities by Asian governments have proved effective to the rapid growth. He says that if these strategic policies had really contributed to the growth, the benefits should have been measured as increases in efficiency in the researches of growth accounting for the Asian economies which have been conducted so far.

This recognition slightly differs from the results of the analysis made by the World Bank in *The East Asian Miracle Economic Growth and Public-Policy* (The World Bank, 1993). This analysis admits that efficiency growth is not the dominant factor to the East Asian success, but it positively appraises higher rise in productivity in this region than in other developing economies, explaining that it was accomplished “by the combination of unusual success at allocating capital to high-yielding investments and at catching up technologically to the industrial economies” (p. 8), which was driven by the policy efforts, that is, implementation of “industrial policy deliberate, government-sponsored interventions to alter industrial structure” (p. 259).

The study also mentions rapid accumulation of human capital as one of the fundamental ingredients of the high, sustained growth in the East Asian economies (pp. 192–203). The accumulation of human capital is effectuated in both quantity and quality aspects. Whilst its accumulation in terms of quantity means mobilization of a greater number of workers into production, human capital in terms of quality is augmented by higher and more expanded schooling in educational facilities among people and by greater acquisition of vocational skills and knowledge through various channels, for instance, firm-level training, on-the-job training, introduction of foreign techniques, and so on. The latter aspect of accumulation of human capital can be considered to be related to technical advance embodied in labor input. Krugman does refer to the remarkable upgrading of educational standards of work force in Asian countries, especially in Singapore. But, he regards it as a mere increase in labor input because he probably takes the rise in labor quality relying on a formal educational system not to last long.

This argument puts aside a question that advance in labor quality due to accumulation of knowledge other than school education will substantially contribute to technical advance in production and that the general upgrading of educational

standards of labor force through expanded schooling in the nation will provide the economy with a sound basis for the acquisition of best-practiced technology and management know-how causing the efficiency growth. In relation to this, Lucas constructs a theoretical model explicitly incorporating the role of the growth of human capital into an aggregate production function in order to explain, or to narrow the theoretical possibilities of the problem of, the growth miracles of East Asia (Lucas, 1993). After the discussions on this model with special attention to the on-the-job accumulation of human capital (learning by doing), he concludes that the main engine of growth is the accumulation of human capital (knowledge) and physical capital accumulation plays no more than a subsidiary role. In particular, learning on the job on a sustained basis seems to be by far the most central for the rapid and sustained rise in living standard of a nation.

This paper constructs a simple theoretical model to understand the rapid growth and its possible conditions of Asian economies, placing emphasis on two important ingredients: accumulation of knowledge and policy efforts of the government. Krugman has not fully investigated the role of the accumulation of knowledge and Lucas has not explicitly considered a process of the accumulation, either. The latter ingredient, governmental policy, has been ignored in the discussions of both the scholars, at least in their explicit theoretical frameworks. The analysis of growth paths follows proposition of the model and, subsequently, some policy implications for the possibilities of sustained growth for the developing economies are derived from it in reference to the observation of the realities of Asian economies.

1.3. A Theoretical Model and Growth Paths

Being grounded on the considerations in Section 1.2, I first present a simple growth model designed to represent the main characteristics of the development behavior of the Asian economies: high rates of physical capital accumulation, growing emphasis on accumulation of technological knowledge, and importance of policy efforts by the governments to realize these accumulation, for rapid economic growth.

In order to follow the growth path of per capita income of an economy, I consider an ordinary Cobb-Douglas type production relation that per capita income at t , $y(t)$, is represented by using the stock of physical capital per worker (capital-labor ratios), $k(t)$ and the accumulation of knowledge, $A(t)$, at that time in the following form:

$$y(t) = k(t)^\alpha A(t) \quad (1.1)$$

Here, α is a positive constant which is smaller than 1. It varies in the value from country to country according to the development stage and other economic

and social conditions. (As I do not pretend to take the traditional neoclassical assumptions of perfect competitiveness to apply in the markets of actual Asian economies, I steer clear of mention that it represents the share of capital.)

One of the essential efforts for developing economies to make take-off towards the sustained growth paths is admittedly the accumulation of knowledge in its broad sense. The knowledge includes general education for people as a fundamental condition necessary for a rise in quality of human capital and more specific vocational knowledge such as engineering and management techniques used in actual production activity. The acquisition of this knowledge is conducted through a variety of channels, but in the case of developing countries aiming to rapid catch-up to the level of advanced industrial countries, it is usually observed that governmental policies play an overwhelmingly important role for it. For the upgrading of the educational standards of the general public, it is inevitable to implement sophisticated social policies including establishment of the comprehensive educational system in the nation, and for the acquisition of new technology, governmental industrial policies have a dominant influence over the behavior of individual firms through subsidizing imports of capital goods embodying new technology, encouraging technology transfer in the form of licenses and foreign training, liberalizing capital movement to introduce direct foreign investment, financially supporting R&D activity, and so forth.

Although governmental policy interventions aiming to the accumulation of knowledge would take different forms depending on actual economic, social, and political circumstances of each country and might change forms according to the situations, the efforts in the aggregate will be measured by government expenses devoted to all what is related to the acquisition of knowledge by the people. What we have to take notice, here, is that the effect of the policy efforts is regarded to depend on the amount per capita rather than the total. (Consider the relation of the total amount of government budget for R&D to the effects between Singapore and China.)

Another thing to be considered in the process of knowledge accumulation is the influence of technological gap between the developing and the developed countries on the rates of the acquisition by the former. It is assumed that if a nation ceases to make efforts to add new technology to its stock, that level measured by a yardstick of the contemporaneous level of the world best-practiced technology decreases at a constant rate, ρ . This is caused by the birth of new technical knowledge in the advanced countries on one hand, and by the continuous decay of old one on the other hand. (Consider one typical example of technological transition from the punched card data processing system to the system of electronic computers which arose in the computer industry in the 1960s. Owing to this technological progress, the programming technique of back pannel wiring in Personal Computer Disk Player (PCDF) was reduced to complete obsolescence.)

From the above consideration, the process of accumulation of knowledge is represented in the following equation

$$\frac{dA(t)}{dt} = u(t)y(t) - \rho A(t), \quad (1.2)$$

where $u(t)$ means a fraction of national income devoted to the acquisition of knowledge, that is, a tax rate for that purpose.

The process of change in physical capital per worker, $k(t)$, can be constructed on the basis of the macroeconomic relation of gross physical investment being equal to national savings. That is, the net investment to physical capital at time t is written as

$$\frac{dk(t)}{dt} = s(1 - u(t))y(t) - \lambda k(t) \quad (1.3)$$

Here, s and λ denote national savings ratio and rate of depreciation, respectively, which are both assumed positive constants being smaller than 1. The first term of the right-hand side, $(1 - u(t))y(t)$, is the per capita disposable income net of tax for the acquisition of knowledge and thus the savings ratio has to be understood accordingly.

The rate of obsolescence of technical knowledge in the stock of the developing economies, ρ , is considered to correspond to the rate of technical progress of the world best-practiced technology. From Eqs. (1.1) and (1.3), we can derive the amount of policy efforts necessary for the developing economies not to widen the present technological gap anymore and to keep pace with the progressing level of world technology by using $dA(t)/dt = 0$:

$$u(t) = \frac{\rho}{k(t)^\alpha} \quad (1.4)$$

This shows that the developing economies having a smaller capital-labor ratio, have to continue the policy efforts more in order to keep pace with the technological standards of the advanced economies. The greater the pace, the more efforts are required, naturally.

In the same way, we can obtain the relation of the stock of knowledge $A(t)$ to the capital-labor ratio, $k(t)$ when $k(t)$ remains constant, $dk(t)/dt = 0$, in the equation

$$A(t) = \frac{\lambda k(t)^{1-\alpha}}{s(1 - u(t))} \quad (1.5)$$

The above model has the similar expressions to what Karl Shell (1966) presented in his study of relationship between inventive activity and economic growth, which was highly suggestive to my study. The similarity of models takes place from

a common nature intrinsic to the accumulation activity of knowledge. Special attention is paid in my analysis, however, to the policy efforts by the governments of developing economies to the growth through transfer of technology from the advanced countries and to the investigation of the initial conditions for take-off towards the sustained growth in relation to these efforts with the use of this model.

Hereafter, as the first step of the study, the policy efforts are assumed to be continued with a certain level by the governments, and thus,

$$u(t) = \bar{u} \tag{1.6}$$

Then, the values of $A(t)$ and $k(t)$ corresponding to $dA(t)/dt = 0$ and $dk(t)/dt = 0$ are determined to be A_* and K_* , respectively, irrelevant to time:

$$A_* = \frac{\lambda k_*^{1-\alpha}}{s(1-\bar{u})} \tag{1.7}$$

$$k_* = \frac{\rho}{\bar{u}} \tag{1.8}$$

As a starting step of the analysis, with the level of the policy efforts being taken to be a time-invariant constant, I attempt to derive the growth equations for $A(t)$ and $k(t)$ to know the behavior of these trajectories according to the initial conditions of the developing economies, and then examine the possibilities of take-off towards their sustained growth by changing the levels of policy efforts in connection to the initial situations.

In order to solve the differential Eqs. (1.2) and (1.3), I try to transform it to a linear system by the Taylor expansion about (A_*, k_*) , which makes it possible to analyze the behavior of the growth paths around that point and obtain the approximate expressions for the trajectories. The transformation practiced on Eqs. (1.2) and (1.3) gives the following non-homogeneous linear differential equation system:

$$\frac{dv(t)}{dt} = \lambda w(t) - \ln(k_*) \tag{1.9}$$

$$\frac{dw(t)}{dt} = \alpha \rho v(t) - \lambda(1-\alpha)w(t) - [\ln(A_*) - (1-\alpha)\ln(k_*)], \tag{1.10}$$

where $v(t) = (1/\alpha\rho) \ln(A(t))$, and $w(t) = (1/\lambda) \ln(k(t))$.

For the sake of obtaining a fundamental system of solution for the homogeneous part of the above equation system, I construct the characteristic equation:

$$\psi(\varphi) = \begin{bmatrix} 0 & \lambda \\ \alpha\rho & -\lambda(1-\alpha) \end{bmatrix} - \varphi E = \varphi^2 + \lambda(1-\alpha)\varphi - \alpha\rho\lambda = 0 \tag{1.11}$$

From this, the characteristic roots are obtained as:

$$\begin{aligned}\varphi_1 &= -\frac{1}{2}[\lambda(1-\alpha) + \sqrt{D}] = -\mu \quad (\mu > 0) \\ \varphi_2 &= \frac{1}{2}[-\lambda(1-\alpha) + \sqrt{D}] = \omega \quad (\omega > 0)\end{aligned}\quad (1.12)$$

$$(D = \lambda^2(1-\alpha)^2 + 4\alpha\rho\lambda).$$

Therefore, the characteristic equation of this system has real different roots of opposite signs, and thus the stationary point (A_*, k_*) is a local saddle point. Moreover, as the function $y(t)$ is certainly twice-continuously differentiable with respect to k , that point proves to be a global saddle point.

From the characteristic roots of φ_1 and φ_2 , a solution matrix is constructed as:

$$Y = \left(\begin{bmatrix} A_{11} \\ A_{21} \end{bmatrix} e^{\varphi_1 t} \quad \begin{bmatrix} A_{12} \\ A_{22} \end{bmatrix} e^{\varphi_2 t} \right), \quad (1.13)$$

where (A_{1i}, A_{2i}) is a characteristic vector corresponding to φ_i . Now, the Wronskian $|Y|$ is not zero for at least a certain value of t , and thus Y becomes the fundamental system of solution for the homogeneous part of the above differential equation system.

With the use of this fundamental system of solution, the general solution for the original non-homogeneous system is obtained in the form of:

$$\begin{bmatrix} v(t) \\ w(t) \end{bmatrix} = Y(t) \begin{bmatrix} c_1(t_0) \\ c_2(t_0) \end{bmatrix} + Y(t) \int_{t_0}^t Y^{-1}(\tau) \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} d\tau, \quad (1.14)$$

having arbitrary constants c_1 and c_2 at the initial stage t_0 . Putting the solution matrix $Y(t)$ in this relation, we obtain the explicit expressions for the general solution as:

$$\begin{aligned}v(t) &= A_{11}c_1(t_0)e^{\varphi_1 t} + A_{12}c_2(t_0)e^{\varphi_2 t} \\ &+ \frac{\lambda}{\varphi_1\sqrt{D}} \left[\ln(A_*) - \left(\frac{\varphi_2}{\lambda} + 1 - \alpha \right) \ln(k_*) \right] \cdot \{ \exp[\varphi_1(t-t_0)] - 1 \} \\ &+ \frac{\lambda}{\varphi_2\sqrt{D}} \left[\ln(A_*) - \left(\frac{\varphi_1}{\lambda} + 1 - \alpha \right) \ln(k_*) \right] \cdot \{ \exp[\varphi_2(t-t_0)] - 1 \}\end{aligned}\quad (1.15)$$

$$\begin{aligned}w(t) &= A_{21}c_1(t_0)e^{\varphi_1 t} + A_{22}c_2(t_0)e^{\varphi_2 t} \\ &+ \frac{\lambda}{\varphi_1\sqrt{D}} \left[\ln(A_*) - \left(\frac{\varphi_2}{\lambda} + 1 - \alpha \right) \ln(k_*) \right]\end{aligned}$$

$$\begin{aligned} & \times \left(\frac{A_{21}}{A_{11}} \right) \{ \exp[\varphi_1(t - t_0)] - 1 \} \\ & + \frac{\lambda}{\varphi_2 \sqrt{D}} \left[\ln(A_*) - \left(\frac{\varphi_1}{\lambda} + 1 - \alpha \right) \ln(k_*) \right] \\ & \times \left(\frac{A_{22}}{A_{12}} \right) \{ \exp[\varphi_2(t - t_0)] - 1 \}. \end{aligned} \tag{1.16}$$

Changing $v(t)$ and $w(t)$ into original variables, we have the final expressions for time paths of the stock of knowledge $A(t)$ the capital-labor ratio $k(t)$ in the following way:

$$A(t) = \exp\{\alpha\rho[(g_1 + h_1e^{-\varphi_1t_0})e^{\varphi_1t} + (g_2 + h_2e^{-\varphi_2t_0})e^{\varphi_2t} - h_1 - h_2]\} \tag{1.17}$$

$$k(t) = \exp\{\varphi_1(g_1 + h_1e^{-\varphi_1t_0})e^{\varphi_1t} + \varphi_2(g_2 + h_2e^{-\varphi_2t_0})e^{\varphi_2t} - \varphi_1h_1 - \varphi_2h_2\}, \tag{1.18}$$

where constant and coefficient terms g_1, g_2, h_1, h_2 are represented as

$$g_1 = \left[\frac{\varphi_2 \ln(A_0) - \alpha\rho \ln(k_0)}{\alpha\rho(\varphi_2 - \varphi_1)} \right] e^{-\varphi_1t_0} \tag{1.19}$$

$$g_2 = - \left[\frac{\varphi_1 \ln(A_0) - \alpha\rho \ln(k_0)}{\alpha\rho(\varphi_2 - \varphi_1)} \right] e^{-\varphi_2t_0} \tag{1.20}$$

$$h_1 = \frac{\lambda}{\varphi_1 \sqrt{D}} \left\{ \ln(A_*) - \left(\frac{\varphi_2}{\lambda} + 1 - \alpha \right) \ln(k_*) \right\} \tag{1.21}$$

$$h_2 = \frac{\lambda}{\varphi_2 \sqrt{D}} \left\{ -\ln(A_*) + \left(\frac{\varphi_1}{\lambda} + 1 - \alpha \right) \ln(k_*) \right\} \tag{1.22}$$

In the relations (1.19) and (1.20), A_0 and k_0 denote the initial values of $A(t)$ and $k(t)$ at $t = t_0$, respectively. These initial conditions are due to play a crucial role in the future growth paths of the economies, which will be investigated in the next section.

The time paths of $A(t)$ and $k(t)$ obtained here represent no more than the approximation to those implied by the original model postulated in (1.1)–(1.3), but they preserve the fundamental nature inherent in it. Thus, Eqs. (1.17) and (1.18) yield the expressions for the trajectories of growth in an (A, k) -plane by a process of elimination of time t from them in the following form:

$$\begin{aligned} & |\mu \ln[A(t)] + \alpha\rho \ln[k(t)] + \alpha\rho(\mu + \omega)h_2|^\mu \\ & \times |\omega \ln[A(t)] + \alpha\rho \ln[k(t)] + \alpha\rho(\mu + \omega)h_1|^\omega \\ & = |\alpha\rho(\mu + \omega)(g_2 + h_2e^{-\omega t_0})|^\mu \cdot |\alpha\rho(\mu + \omega)(g_1 + h_1e^{-\mu t_0})|^\omega. \end{aligned} \tag{1.23}$$

Some computational manipulations make it possible to rewrite this expression in the form

$$\left| \left(\frac{\mu}{\alpha\rho} \right) \ln \left(\frac{A(t)}{A_*} \right) + \ln \left(\frac{k(t)}{k_*} \right) \right|^\mu \cdot \left| \left(\frac{\omega}{\alpha\rho} \right) \ln \left(\frac{A(t)}{A_*} \right) + \ln \left(\frac{k(t)}{k_*} \right) \right|^\omega = |(\mu + \omega)A^{12}y_2(t_0)|^\mu \cdot |(\mu + \omega)A^{11}y_1(t_0)|^\omega, \quad (1.24)$$

where $A^{11}y_1(t_0)$ and $A^{12}y_2(t_0)$ are obtained from the initial conditions for $A(t)$ and $k(t)$ as expressed in the following way:

$$A^{11}y_1(t_0) = \frac{1}{\mu + \omega} \cdot \left\{ \frac{\omega}{\alpha\rho} \cdot \ln \left(\frac{A_0}{A_*} \right) - \ln \left(\frac{k_0}{k_*} \right) \right\} \quad (1.25)$$

$$A^{12}y_2(t_0) = \frac{1}{\mu + \omega} \cdot \left\{ \frac{\omega}{\alpha\rho} \cdot \ln \left(\frac{A_0}{A_*} \right) + \ln \left(\frac{k_0}{k_*} \right) \right\} \quad (1.26)$$

[See also a mathematical note in Appendix.]

In order to depict the trajectories of the growth paths, it is convenient to express at first the parts enclosed with the absolute value marks in the left-hand side of Eq. (1.24) in the linear form. This can be done by putting

$$x_1(t) = \left(\frac{1}{\alpha\rho} \right) \ln \left(\frac{A(t)}{A_*} \right) \quad \text{and} \quad x_2(t) = \left(\frac{1}{\lambda} \right) \ln \left(\frac{k(t)}{k_*} \right), \quad (1.27)$$

which gives

$$\begin{aligned} & |\mu x_1(t) + \lambda x_2(t)|^\mu \cdot |\omega x_1(t) - \lambda x_2(t)|^\omega \\ &= |(\mu + \omega)A^{12}y_2(t_0)|^\mu \cdot |(\mu + \omega)A^{11}y_1(t_0)|^\omega. \end{aligned} \quad (1.28)$$

The trajectories of $x_1(t)$ and $x_2(t)$ are shown in the phase plane of Fig. 1.1. In the laws of motion there, attention must be paid to the properties that the growth of $x_1(t)$ or $x_2(t)$ always becomes zero when a trajectory cuts across the lines

$$x_2(t) = 0 \quad \text{and} \quad \alpha\rho x_1(t) - \lambda(1 - \alpha)x_2(t) = 0.$$

The trajectories approach an asymptotic line,

$$\omega x_1(t) - \lambda x_2(t) = 0,$$

as time goes on, one group expanding and the other dampening. The origin corresponds to the point (k_*, A_*) .

After the law of motion in the phase diagram is understood, the growth trajectories of $A(t)$ and $k(t)$ are easily depicted. Since the slope of an asymptotic line extending northeastward is easily ascertained to be greater than that of the line signifying $dx_2(t)/dt = 0$ as shown in Fig. 1.1, the asymptotic curve

$$\left(\frac{\omega}{\alpha\rho} \right) \ln \left(\frac{A(t)}{A_*} \right) - \ln \left(\frac{k(t)}{k_*} \right) = 0 \quad (1.29)$$

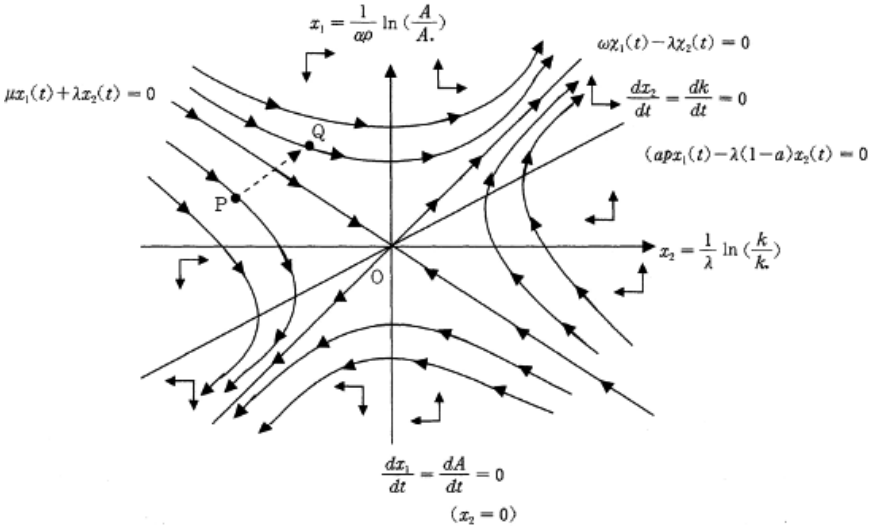


Fig. 1.1. Phase plane of $(x_1(t), x_2(t))$.

cuts the $dk(t)/dt = 0$ curve at the point $O(k_*, A_*)$ from below to above as it stretches to the northeast. And it has already been found that the crossing point is a saddle point.

In the plane of the growth trajectories, the point O moves according to the change in the value u , policy efforts. This means that policy efforts might switch some growth paths which are to dampen in the future to the other paths and make them go into the trajectories having expanding properties. This possibility is exemplified in Fig. 1.1 by a switch of a trajectory from P to Q . The effects of policy on the growth paths together with the initial conditions are considered in the next section, especially with the East Asian economies in mind.

1.4. Initial Conditions and Policy Efforts

Among the results derived from the above theoretical analysis based on a simple growth model, what seems to be of special significance from the point of view of economic development is the relation between the initial state of an economy as the fundamental conditions for the development and the effects of policy implementation on the possibility of growth.

As mathematical models invented by economists always do so, my model presented in this paper takes into account only the essential properties associated with the problems to be considered. Thus, the discussion here is not concerned with full

description about sophisticated aspects in the real economy, but it refers to those to the extent that the model is related to them. The purpose of model-building is to extract essential law of immanent behavior in the reality from its complexity for the cause of better understanding of the actual economy and better policy-making appropriate for it.

As regards the initial conditions of an economy aiming to the take-off for sustained growth, the model typically indicates what amounts of accumulated knowledge A and of physical capital stock per worker k are needed at $t = t_0$. Figure 1.1 shows that in order to get on the paths which are to be led to the sustained growth, the economy must be in the region above the line

$$\mu x_1(t) + \lambda x_2(t) = 0 \quad (1.30)$$

in the phase plane at the initial time $t = t_0$. This region is formed by an asymptotic curve equivalent to the equation which is given as

$$\frac{\mu}{\alpha\rho} \cdot \ln\left(\frac{A(t)}{A_*}\right) + \ln\left(\frac{k(t)}{k_*}\right) = 0 \quad (1.31)$$

in the trajectories diagram which is easily graphed out. The economies having the endowments of A and k at the initial time, $A(t_0)$ and $k(t_0)$, which are located in the region under this curve, are on the trajectories going eventually to the state at a low ebb.

This simple theoretical conclusion throws some light on the arguments concerning the growth of Asian economies. As mentioned in Section 1.2, Krugman poured cold water on the popular enthusiasm about Asian rapid economic growth, stating that Asian input-driven growth is an inherently limited process as far as it is not accompanied by an increase in efficiency with which inputs are used. With reference to the above conclusion seen in Fig. 1.1, it seems that Krugman's view is correct no matter how much physical capital is piled up, the economies cannot go into the trajectories eventually heading for the developed state as long as they stay in the region under the boundary curve marked by Eq. (1.31) owing to the absence of efficiency growth, that is, the low level of the accumulation of knowledge A . In that circumstance, it appears that the economies make rapid growth through high rates of the mobilization of resources into production, but as a matter of fact, they may simply move to one of the upper trajectories of the same dampening nature, and then the growth is not sustained actually. In a different situation, however, where the economies have already accumulated a considerable amount of knowledge whilst the stock of physical capital is at an extremely low level, the economies could find a way to go into a sustained growth path towards the developed state. This might be possible by only a small amount of efforts of accumulation of physical capital through the measures like receipt of foreign aids. Otherwise, those economies with a high level of knowledge might ever remain in the sustained growth region above the boundary curve even at a low level of capital stock. This is considered the case

with Japan's economy that was immediately after the end of the World War II. Japan had attained to the industrial state near the then Western economies, but the accumulated production facilities were almost completely destroyed by bombardment in wartime. No one can destroy knowledge dwelling in the nation, however, Japan embarked on the reconstruction of its economy after the war, keeping the technological knowledge associated to productive activities accumulated in peacetime before the war and augmented by the government-financed R&D mainly concerned with advanced strategic technology. In the process of the reconstruction, the lack of physical capital was the problem of vital importance for the government to deal with. In order to tackle this problem, it took a bold step called the "priority production system" that the limited amounts of Japan's scarce resources at that time were to be committed mainly to the construction of productive facilities of the fundamental industries such as iron and steel, shipbuilding, and electric power industries. In so doing, Japan's economy could go into the trajectory of rapid and self-sustaining growth in five years after the end of the war. (In addition to such policy efforts, it is commonly admitted that the Korean War in 1950 played a definite role as an engine in sending Japan's economy to the high-growth path.)

As far as the static initial conditions are concerned, Krugman's statement on the limitedness of the economic growth of the newly industrializing countries in the Pacific Rim seems to be correct on the basis of the analyses here. However, Krugman disregards the fact that the continuous efforts of augmenting knowledge or human skills in the nation could change the economy's nature from "One-time changes" by mobilization of inputs to vitalized growth by increases in efficiency. A continuous policy effort to augment knowledge, $u(t) = \bar{u}$, moves the point $O(k_*, A_*)$ as time goes on. If this point moves towards the origin in compliance with the efforts, the dampening region of the economy diminishes accordingly. The behavior of the point O in response to changes in the level of u is shown in Fig. 1.2. Recall that $u(t)$ denotes part of national income devoted to the acquisition of knowledge and is generally considered to be the tax rate for that purpose. However, the accumulation of knowledge is not always implemented by administrative measures based on governmental policies, but it could also be taken place in ordinary behavior of free enterprises or individuals being driven by market competition or a desire to improve oneself. The variable $u(t)$ includes all these activities related to the acquisition of knowledge by the nation.

When the value of u is so small as to be almost zero, the initial conditions for an economy to go along the developing trajectories are highly demanding: the economy has to be equipped with a vast amount of physical capital stock along with a fairly large accumulation of knowledge at the beginning of stage of development since k_* is of almost infinity and A_* is $(\lambda/s)k_*^{1-\alpha}$ at $u = 0$. This can be said to be a contradiction in terms of "development." As $u(t)$ takes a larger value starting from zero, the conditions for growth become less demanding. This stable (zero growth) point O approaches the origin accordingly and the required

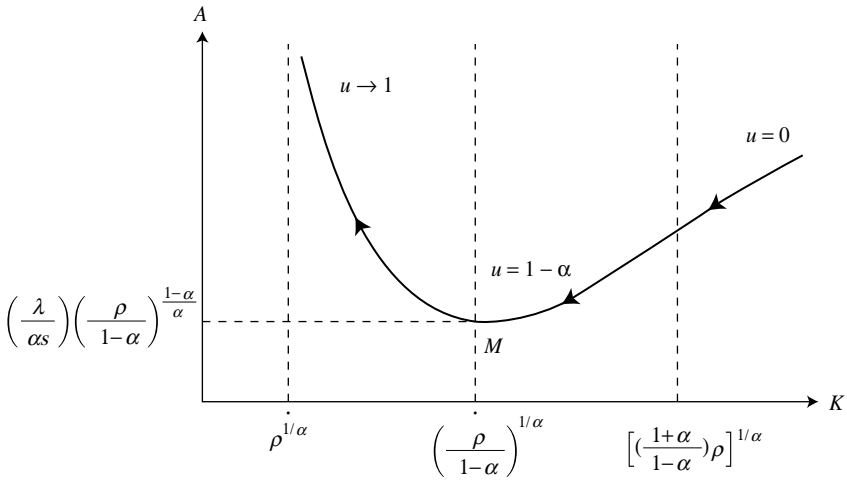


Fig. 1.2. Motion of stable (zero growth) point subject to policy efforts.

accumulation of knowledge corresponding to the smallest value of k_* reaches the minimum $(\lambda/\alpha s)[\rho/(1-\alpha)]^{1-\alpha}$. This point M is attained by the level of the policy efforts $u = 1 - \alpha$. After that, the required levels of knowledge at the initial state increase rapidly as u approaches 1. This is a broad outline of changes in the required initial conditions of the economies aiming at self-sustaining growth derived from the motion of the stable point caused by the changes in the level of policy efforts to acquire knowledge. These efforts are not limited to the activities of directly acquiring knowledge through, for instance, government-financed R&D or importation of foreign techniques. Technological knowledge could be accumulated through indirect channels, too, of importing capital goods furnished with highly advanced technology needed for the production of exports under the export promotion strategy.

The analysis shows that policy efforts by the nation could ease the initial conditions required to make the economy grow by changing the state in the poverty region (under the asymptotic curve given by the Eq. (1.31)) to that in the prosperity region (above the curve). This same effect produced by the policy efforts can be explained in the phase diagram of Fig. 1.1 by a switch of a declining path at a point P to a self-sustaining growth path at a point Q. I do not intend here to assert the advantages in economic policy of the selective interventions over the traditional laissez-faire approach. I only would like to indicate that continuous policy efforts can play an important role in improving the initial conditions of the economies which remain in poverty state and cannot find a way to get rid of it. It is taken for granted that whether an economy in the underdeveloped state can actually move

to the prosperity region, not taking only an upper growth path in the same poverty region by “one-time changes”, depends upon the extent to which the economy can achieve efficiency growth by the accumulation efforts of knowledge of the nation.

1.5. Concluding Remarks

I have tried in this paper to analyze the fundamental structure of the Asian economic growth and to bring to light the basic problem to be considered for the sake of the development of the East Asian economies in the hope of further promoting constructive arguments about them between the popular enthusiasm alleging the “Asia’s miracle” and the scholarly skepticism about the Asian supremacy of the growth asserting a lack of efficiency growth in these economies.

Economic growth, as an increase in per capita national income, is achieved through both quantitative and qualitative advances in economic activities: quantitative advances of the economy can be accomplished by mobilization of capital and labor inputs in production processes, while qualitative advances can be caused by efficiency growth in the economy typically through technological improvements. In the light of this consideration, I introduced a simple mathematical model to analyze the fundamental properties of growth paths in relation to the initial state of a country visualizing to enter the take-off stage followed by sustained growth. Growth equations derived from that model show that all the growth trajectories are divided into two groups according to the initial conditions, one of which consists of trajectories having the eventually expanding properties with the other being those of the eventually dampening properties. The phase plane accommodating all the possible growth paths is separated into two parts by an equation signifying an asymptotic curve. One is the area above the curve where all the eventually expanding trajectories exist and the other is the area below the curve having all the eventually dampening ones. I designated the former as the “prosperity region” and the latter as the “poverty region.” An economy lying in the poverty region at the initial state will be destined to stay in the situation of low standard of living, if it does not make any policy efforts to accumulate knowledge. On the contrary, the policy efforts could send the economy to one of the growth paths in the prosperity region. From the properties of the poverty region which enlarges or reduces according to the changes in the level of policy efforts, growth policy unaccompanied by increases in efficiency-mobilization of resources without augmentation of knowledge might give the economy only “one-time changes” of the state which move it into one of the upper trajectories in the same region. This may correspond to the case of an inherently limited input-driven growth mentioned by Krugman about Asian rapid economic growth. However, much attention must be paid to the possibility that continuous efforts to augment knowledge by the nation would lead the economy to a sheer self-sustaining growth path by moving it from the poverty

region to the prosperity region, or by switching the trajectories from the eventually dampening path to one of the eventually expanding paths. As a matter of fact, Japan has accomplished this switch in growth path through the implementation of sophisticated economic policies characterized by a series of “economic plans” which extended over five decades after the war. South Korea, as well, has made high and sustained growth accompanied by structural transformation through continuous policy efforts strongly led by the government in pursuit of the Japanese model to a great extent (Takashima, 1994).

An economy may differ with another on the forms of policy efforts to accumulate knowledge, depending on the differences of political and economic surroundings between them. In reality, the economies of South-East Asia — Singapore, Thailand, Malaysia, and Indonesia — have taken different strategies to acquire knowledge from those of the North-East Asian forerunners, Japan and South Korea, by opening their markets to foreign direct investment (The Economist, 1995), as compared with these East-Asian peers having striven against it.

It depends on a degree of increases in efficiency whether these East-Asian economies truly have made or can make a growth miracle. At the early-industrial stage when the economy is about to start its industrialization for growth, policy measures like selective interventions and coerced mobilization of resources might prove effective for structural transformation and produce a rapid growth. But the quantitative mobilization of resources inevitably reduces the economy to the state of decreasing returns. The sheer self-sustaining growth can be achieved only by continuous efficiency growth through accumulation of knowledge, typically speaking, technological advance in the broad sense of the word, which can realize increasing returns in production. And this cannot be made by government regulations and protectionism, but can be progressed only by continuous development of creativity inspired in the nation by the free competitive environment.

Mathematical Appendix

In this Appendix, I provide another method of calculations for the solutions of the non-homogeneous differential Eqs. (1.2) and (1.3), which proves to give the same result as (1.23).

I start with the non-homogeneous linear differential system, Eqs. (1.9) and (1.10), obtained by the Taylor expansion of the original system about zero-growth point (A_*, k_*) . This system can be reduced to the form of a linear homogeneous system of differential equations

$$\frac{dx_1}{dt} = \lambda x_2 \quad (\text{A.1})$$

$$\frac{dx_2}{dt} = \alpha \rho x_1 - \lambda(1 - \alpha)x_2 \quad (\text{A.2})$$

by replacing $v(t)$ and $w(t)$ with new variables $x_1(t)$ and $x_2(t)$ having the relations

$$\begin{aligned} x_1(t) &= v(t) - (1/\alpha\rho) \cdot \ln(A_*) \\ &= (1/\alpha\rho) \cdot \ln(A(t)/A_*) \end{aligned} \tag{A.3}$$

$$\begin{aligned} x_2(t) &= w(t) - (1/\lambda) \cdot \ln(k_*) \\ &= (1/\lambda) \cdot \ln(k(t)/k_*) \end{aligned} \tag{A.4}$$

Thus, the differential system can be written in vector form as follows

$$\begin{pmatrix} dx_1(t)/dt \\ dx_2(t)/dt \end{pmatrix} = \begin{bmatrix} 0 & \lambda \\ \alpha\rho & -\lambda(1-\alpha) \end{bmatrix} \cdot \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \tag{A.5}$$

Considering a certain linear transformation

$$y = A_1x_1 + A_2x_2 \quad \text{and} \quad \frac{dy}{dt} = \phi y, \tag{A.6}$$

next relation can be obtained from Eqs. (A.1) and (A.2):

$$A_1(\lambda x_2) + A_2[\alpha\rho x_1 - \lambda(1-\alpha)x_2] = \phi(A_1x_1 + A_2x_2) \tag{A.7}$$

(Here, notations y and A_i have no conceptual relation with the variables denoting per capita income and level of knowledge in the original model of the text, respectively.) Since the variables x_1 and x_2 are to take arbitrary values in Eq. (A.6), Eq. (A.7) yields

$$-\phi A_1 + \alpha\rho A_2 = 0 \tag{A.8}$$

$$\lambda A_1 + [-\lambda(1-\alpha) - \phi] A_2 = 0 \tag{A.9}$$

In these equations, the next relation must be established in order for the non-zero solutions for A_1 and A_2 to exist

$$\begin{vmatrix} -\phi & \alpha\rho \\ \lambda & -\lambda(1-\alpha) - \phi \end{vmatrix} = \begin{vmatrix} -\phi & \lambda \\ \alpha\rho & -\lambda(1-\alpha) - \phi \end{vmatrix} = 0 \tag{A.10}$$

This is nothing else but the characteristic equation for the coefficient matrix of the homogeneous differential system Eq. (A.5).

The characteristic roots ϕ_1 and ϕ_2 of this equation were given in the text as Eq. (1.12) and proved to be real and different, having opposite signs. Therefore, characteristic vectors (A_{i1}, A_{i2}) are obtained from Eq. (A.8) or Eq. (A.9) corresponding to $\phi_i (i = 1, 2)$. (One of the Eqs. (A.8) and (A.9) is derived from the other one along with the above characteristic equation which gives the existence conditions for the solution.) From $\phi_1 \neq \phi_2$ and the second relation of Eq. (A.6), y_1 and y_2 are found to form a system of linear independence.

With the characteristic roots ϕ_1 and ϕ_2 having real different values of opposite signs, the second relation of Eq. (A.6) is written as a system of two equations, each corresponding to one of the roots:

$$\begin{pmatrix} dy_1(t)/dt \\ dy_2(t)/dt \end{pmatrix} = \begin{bmatrix} \phi_1 & 0 \\ 0 & \phi_2 \end{bmatrix} \cdot \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} \quad (\text{A.11})$$

Thus, the system can easily be solved in the following way:

$$y_1(t) = y_1(t_0) \exp[\phi_1(t - t_0)] \quad (\text{A.12})$$

$$y_2(t) = y_2(t_0) \exp[\phi_2(t - t_0)], \quad (\text{A.13})$$

where negative root ϕ_1 and positive one ϕ_2 are rewritten with the use of positive notations and as μ and ω as $\phi_1 = -\mu$ and $\phi_2 = \omega$, respectively.

Using the two variables y_1 and y_2 corresponding to those two different characteristic roots, the full linear transformation Eq. (A.6) between x_i and y_i is written in the form

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad (\text{A.14})$$

Here, the first row in the matrix A_{ij} represents the characteristic vector of the coefficient matrix of the system Eq. (A.5) corresponding to the first characteristic root ϕ_1 and the second row is that corresponding to the second root ϕ_2 . Then, the solutions of Eqs. (A.12) and (A.13) can be transformed into those of $x_1(t)$ and $x_2(t)$ in the following expressions:

$$x_1(t) = A^{11} y_1(t_0) e^{-\mu(t-t_0)} + A^{12} y_2(t_0) e^{\omega(t-t_0)} \quad (\text{A.15})$$

$$x_2(t) = A^{21} y_1(t_0) e^{-\mu(t-t_0)} + A^{22} y_2(t_0) e^{\omega(t-t_0)}, \quad (\text{A.16})$$

where A^{ij} is an (i, j) element of the inverse matrix of (A_{ij}) in Eq. (A.14). The relations between A^{ij} s and the coefficients of the original system are obtained by putting the above solutions into Eqs. (A.1) and (A.2), which yields

$$\mu A^{11} + \lambda A^{21} = 0 \quad \text{and} \quad \omega A^{21} - \lambda A^{22} = 0 \quad (\text{A.17})$$

$$\alpha \rho A^{11} + [\mu - \lambda(1 - \alpha)] A^{21} = 0 \quad \text{and} \quad \alpha \rho A^{12} + [-\omega - \lambda(1 - \alpha)] A^{22} = 0. \quad (\text{A.18})$$

Equations (A.17) and (A.18) are not independent because the first relation of Eq. (A.18) can be derived from the first relation of Eq. (A.17) with the help of the relation expressed in the characteristic equation and the second relation of Eq. (A.18) can be obtained from the second relation of Eq. (A.17) exactly the same way.

Through a process of elimination of t , Eqs. (A.15) and (A.16) yield the general equation of trajectories of $(x_1(t), x_2(t))$ in the following form

$$\begin{aligned} & |\mu x_1(t) + \lambda x_2(t)|^\mu \cdot |\omega x_1(t) - \lambda x_2(t)|^\omega \\ & = |(\mu + \omega)A^{12}y_2(t_0)|^\mu \cdot |(\mu + \omega)A^{11}y_1(t_0)|^\omega \end{aligned} \quad (A.19)$$

which was given in Eq. (1.28) in the text. The motion of these trajectories is depicted in the phase plane of Fig. 1.1. This equation can be rewritten into the expression of the original variables $A(t)$ and $k(t)$ with the use of the relations (A.3) and (A.4) as

$$\begin{aligned} & \left| \left(\frac{\mu}{\alpha\rho} \right) \ln \left(\frac{A(t)}{A_*} \right) + \ln \left(\frac{k(t)}{k_*} \right) \right|^\mu \cdot \left| \left(\frac{\omega}{\alpha\rho} \right) \ln \left(\frac{A(t)}{A_*} \right) - \ln \left(\frac{k(t)}{k_*} \right) \right|^\omega \\ & = |(\mu + \omega)A^{12}y_2(t_0)|^\mu \cdot |(\mu + \omega)A^{11}y_1(t_0)|^\omega, \end{aligned} \quad (A.20)$$

which was Eq. (1.24) in the text. The behavior of growth paths are easily depicted in $(A(t), k(t))$ -plane.

Individual growth equations of $A(t)$ and $k(t)$ are obtained from Eqs. (A.15) and (A.16) by changing variables, using Eqs. (A.3) and (A.4). That is,

$$A(t) = A(t_0) \exp[(A^{11}y_1(t_0)e^{-\mu(t-t_0)} + A^{12}y_2(t_0)e^{\omega(t-t_0)})\alpha\rho] \quad (A.21)$$

$$k(t) = k(t_0) \exp(-\mu A^{11}y_1(t_0)e^{-\mu(t-t_0)} + \omega A^{12}y_2(t_0)e^{\omega(t-t_0)}). \quad (A.22)$$

These equations give us the unknown constants, $A^{11}y_1(t_0)$ and $A^{12}y_2(t_0)$ in Eqs. (A.19) and (A.20) in consideration of the initial conditions, $A(t_0)$ and $k(t_0)$, as Eqs. (1.25) and (1.26) in the text.

In the phase plane of $(x_1(t), x_2(t))$, the asymptotic lines for the trajectories of Eq. (A.19) are found to be

$$\mu x_1(t) + \lambda x_2(t) = 0 \quad \text{and} \quad \omega x_1(t) - \lambda x_2(t) = 0, \quad (A.23)$$

which leads to the asymptotic curves for growth paths of Eq. (A.20) in the form

$$\left(\frac{\mu}{\alpha\rho} \right) \ln \left(\frac{A(t)}{A_*} \right) + \ln \left(\frac{k(t)}{k_*} \right) = 0$$

and

$$\left(\frac{\omega}{\alpha\rho} \right) \ln \left(\frac{A(t)}{A_*} \right) - \ln \left(\frac{k(t)}{k_*} \right) = 0. \quad (A.24)$$

These play a crucial role in characterizing the behavior of trajectories in both planes of $(x_1(t), x_2(t))$ and $(A(t), k(t))$, as explained in the text.

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