

Preface

Since I left the Institute of Mathematics at Academia Sinica in 1991, I have conducted a graduate course on number theory every year at the National Chung Cheng University. The title of the course is usually “Special Topics in Number Theory” or “Theory of Modular Forms of Several Variables”. Unfortunately, there are no textbooks available for such a course and I had to write my own lecture notes, lifting materials from books by T. Shimura [48], E. Freitag [31], W. L. Baily, Jr. [6] and T. Miyake [42]. To enrich the contents of the lecture notes, I have included some of the results from my own research work.

In such a course as mentioned above, we usually begin with the theory of modular forms of one variable, followed by the theory of zeta functions, new approaches to Bernoulli identities and Kummer Congruences, the theory of modular forms of several variables, the theory of Jacobi forms over Cayley numbers and finally the theory of Jacobi forms of several variables over Cayley numbers. The theory of Jacobi forms is based on my research papers from 1991 to 2000. Students are encouraged to conduct research related to the materials mentioned above. Eventually, a total of 6 Ph.D. students and more than 30 M.Sc. students graduated under this programme.

The theory of modular forms of several variables [31] was initiated by C. L. Siegel around 1945 in an attempt to generalize the theory of modular forms of one variable. The theory was employed by M. Eichler and D. Zagier [15] in 1985 to give a definition of Jacobi forms. Indeed the coefficients of the Fourier-Jacobi expansions of modular forms of rank two are the main sources of Jacobi forms. It is still a difficult problem to construct Jacobi forms besides the Eisenstein series. Even the Eisenstein series of Jacobi

forms of several variables are difficult to construct. All we can say is that many results concerning modular forms of several variables remain to be studied in depth.

Georg-August-Universität Göttingen is the center of development of modular forms with masters like Siegel and U. Christian. I spent a year at Georg-August-Universität Göttingen as a Sonderforschungsbereich (special research fellow) in 1985, three years after my graduation from the University of Chicago in 1982.

It is almost 60 years since the birth of modular forms of several variables in Germany. It is an appropriate time to have a textbook on the theory in English rather than in German. This is my main motivation for writing this book.

Minking Eie
April 19, 2008