Chapter 1

The Theory of Interest

One of the first types of investments that people learn about is some variation on the savings account. In exchange for the temporary use of an investor's money, a bank or other financial institution agrees to pay **interest**, a percentage of the amount invested, to the investor. There are many different schemes for paying interest. In this chapter we will describe some of the most common types of interest and contrast their differences. Along the way the reader will have the opportunity to renew their acquaintanceship with exponential functions and the geometric series. Since an amount of capital can be invested and earn interest and thus numerically increase in value in the future, the concept of **present value** will be introduced. Present value provides a way of comparing values of investments made at different times in the past, present, and future. As an application of present value, several examples of saving for retirement and calculation of mortgages will be presented. Sometimes investments pay the investor varying amounts of money which change over time. The concept of **rate of return** can be used to convert these payments in effective interest rates, making comparison of investments easier.

1.1 Simple Interest

In exchange for the use of a depositor's money, banks pay a fraction of the account balance back to the depositor. This fractional payment is known as **interest**. The money a bank uses to pay interest is generated by investments and loans that the bank makes with the depositor's money. Interest is paid in many cases at specified times of the year, but nearly always the fraction of the deposited amount used to calculate the interest is called the **interest rate** and is expressed as a percentage paid per year.

For example a credit union may pay 6% annually on savings accounts. This means that if a savings account contains \$100 now, then exactly one year from now the bank will pay the depositor \$6 (which is 6% of \$100) provided the depositor maintains an account balance of \$100 for the entire year.

In this chapter and those that follow, interest rates will be denoted symbolically by r. To simplify the formulas and mathematical calculations, when r is used it will be converted to decimal form even though it may still be referred to as a percentage. The 6% annual interest rate mentioned above would be treated mathematically as $r = 0.06$ per year. The initially deposited amount which earns the interest will be called the **principal amount** and will be denoted P. The sum of the principal amount and any earned interest will be called the **capital** or the **amount due**. The symbol A will be used to represent the amount due. The reader may even see the amount due referred to as the **compound amount**, though this use of the adjective "compound" is independent of its use in the term "compound interest" to be explored in Section 1.2. The relationship between P, r, and A for a single year period is

$$A = P + Pr = P(1 + r).$$

In general if the time period of the deposit is t years then the amount due is expressed in the formula

$$A = P(1 + rt). \tag{1.1}$$

This implies that the average account balance for the period of the deposit is P and when the balance is withdrawn (or the account is closed), the principal amount P plus the interest earned Prt is returned to the investor. No interest is credited to the account until the instant it is closed. This is known as the **simple interest** formula.

Some financial institutions credit interest earned by the account balance at fixed points in time. Banks and other financial institutions "pay" the depositor by adding the interest to the depositor's account. The interest, once paid to the depositor, is the depositor's to keep. Unless the depositor withdraws the interest or some part of the principal, the process begins again for another interest earning period. If P is initially deposited, then after one year, the amount due according the Eq. (1.1) with $t = 1$ would be $P(1 + r)$. This amount can be thought of as the principal amount for the account at the beginning of the second year. Thus two years after the initial deposit the amount due would be

$$A = P(1+r) + P(1+r)r = P(1+r)^2. \tag{1.2}$$

Continuing in this way we can see that t years after the initial deposit of an amount P, the capital A will grow to

$$A = P(1+r)^t \tag{1.3}$$

A mathematical "purist" may wish to establish Eq. (1.2) using the principle of induction.

Banks and other interest-paying financial institutions often pay interest more than a single time per year. The yearly interest formula given in Eq. (1.2) must be modified to track the compound amount for interest periods of other than one year.

1.2 Compound Interest

The typical interest bearing savings or checking account will be described by an investor as earning a nominal annual interest rate compounded some number of times per year. Investors will often find interest compounded semi-annually, quarterly, monthly, weekly, or daily. In this section we will compare and contrast compound interest to the simple interest case of the previous section. Whenever interest is allowed to earn interest itself, an investment is said to earn **compound interest**. In this situation, part of the interest is paid to the depositor once or more frequently per year. Once paid, the interest begins earning interest. We will let n denote the number of compounding periods per year. For example for interest "compounded monthly" $n = 12$. Only two small modifications to the simple interest formula (1.2) are needed to calculate the compound interest. First, it is now necessary to think of the interest rate per compounding period. If the annual interest rate is r, then the interest rate per compounding period is r/n. Second, the elapsed time should be thought of as some number of compounding periods rather than years. Thus with n compounding periods per year, the number of compounding periods in t years is nt. Therefore the formula for compound interest is

$$A = P\left(1+\frac{r}{n}\right)^{nt}. \tag{1.4}$$

Eq. (1.4) simplifies to the formula for the amount due given in Eq. (1.2) when $n = 1$.

Example 1.1 Suppose an account earns 5.75% annually compounded monthly. If the principal amount is \$3104 then after three and one-half years the amount due will be

$$A = 3104\left(1 + \frac{0.0575}{12}\right)^{(12)(3.5)} = 3794.15.$$

The reader should verify using Eq. (1.1) that if the principal in the previous example earned only *simple* interest at an annual rate of 5.75% then the amount due after 3.5 years would be only \$3728.68. Thus happily for the depositor, compound interest builds capital faster than simple interest. Frequently it is useful to compare an annual interest rate with compounding to an equivalent simple interest, *i.e.* to the simple annual interest rate which would generate the same amount of interest as the annual compound rate. This equivalent interest rate is called the **effective interest rate**. For the rate mentioned in the previous example we can find the effective interest rate by solving the equation

$$\begin{aligned}\left(1 + \frac{0.0575}{12}\right)^{12} &= 1 + r_e \\ 0.05904 &= r_e\end{aligned}$$

Thus the nominal annual interest rate of 5.75% compounded monthly is equivalent to an effective annual rate of 5.90%.

In general if the nominal annual rate r is compounded n times per year the equivalent effective annual rate r_e is given by the formula:

$$r_e = \left(1 + \frac{r}{n}\right)^n - 1. \tag{1.5}$$

Intuitively it seems that more compounding periods per year implies a higher effective annual interest rate. In the next section we will explore the limiting case of frequent compounding going beyond semiannually, quarterly, monthly, weekly, daily, hourly, *etc.* to continuously.

1.3 Continuously Compounded Interest

Mathematically when considering the effect on the compound amount of more frequent compounding, we are contemplating a limiting process. In symbolic form we would like to find the compound amount A which satisfies the equation

$$A = \lim_{n\to\infty} P\left(1+\frac{r}{n}\right)^{nt}. \tag{1.6}$$

Fortunately there is a simple expression for the value of the limit on the right-hand side of Eq. (1.6). We will find it by working on the limit

$$\lim_{n\to\infty}\left(1+\frac{r}{n}\right)^{n}.$$

This limit is indeterminate of the form 1^∞. We will evaluate it through a standard approach using the natural logarithm and l'Hôpital's Rule. The reader should consult an elementary calculus book such as [Smith and Minton (2002)] for more details. We see that if $y = (1+r/n)^n$, then

$$\begin{aligned}\ln y &= \ln\left(1+\frac{r}{n}\right)^{n}\\ &= n\ln(1+r/n)\\ &= \frac{\ln(1+r/n)}{1/n}\end{aligned}$$

which is indeterminate of the form $0/0$ as $n\to\infty$. To apply l'Hôpital's Rule we take the limit of the derivative of the numerator over the derivative of the denominator. Thus

$$\begin{aligned}\lim_{n\to\infty}\ln y &= \lim_{n\to\infty}\frac{\frac{d}{dn}\left(\ln(1+r/n)\right)}{\frac{d}{dn}\left(1/n\right)}\\ &= \lim_{n\to\infty}\frac{r}{1+r/n}\\ &= r\end{aligned}$$

Thus $\lim_{n\to\infty} y = e^r$. Finally we arrive at the formula for **continuously compounded interest**,

$$A = Pe^{rt}. \tag{1.7}$$

This formula may seem familiar since it is often presented as the exponential growth formula in elementary algebra, precalculus, or calculus. The quantity A has the property that A changes with time t at a rate proportional to A itself.

Example 1.2 Suppose \$3585 is deposited in an account which pays interest at an annual rate of 6.15% compounded continuously. After two and one half years the principal plus earned interest will have grown to

$$A = 3585e^{(0.0615)(2.5)} = 4180.82.$$

The effective simple interest rate is the solution to the equation

$$e^{0.0615} = 1 + r_e$$

which implies $r_e \approx 6.34\%$.

1.4 Present Value

One of the themes we will see many times in the study of financial mathematics is the comparison of the value of a particular investment at the present time with the value of the investment at some point in the future. This is the comparison between the **present value** of an investment versus its **future value**. We will see in this section that present and future value play central roles in planning for retirement and determining loan payments. Later in this book present and future values will help us determine a fair price for stock market derivatives.

The future value t years from now of an invested amount P subject to an annual interest rate r compounded continuously is

$$A = Pe^{rt}.$$

Thus by comparison with Eq. (1.7), the future value of P is just the compound amount of P monetary units invested in a savings account earning interest r compounded continuously for t years. By contrast the present value of A in an environment of interest rate r compounded continuously for t years is

$$P = Ae^{-rt}.$$

In other words if an investor wishes to have A monetary units in savings t years from now and they can place money in a savings account earning interest at an annual rate r compounded continuously, the investor should deposit P monetary units now. There are also formulas for future and present value when interest is compounded at discrete intervals, not continuously. If the interest rate is r annually with n compounding periods per year then the future value of P is

$$A = P\left(1 + \frac{r}{n}\right)^{nt}.$$

Compare this equation with Eq. (1.4). Simple algebra shows then the present value of P earning interest at rate r compounded n times per year for t years is

$$P = A\left(1 + \frac{r}{n}\right)^{-nt}.$$

Example 1.3 Suppose an investor will receive payments at the end of the next six years in the amounts shown in the table.

Year	1	2	3	4	5	6
Payment	465	233	632	365	334	248

If the interest rate is 3.99% compounded monthly, what is the present value of the investments? Assuming the first payment will arrive one year from now, the present value is the sum

$$\begin{aligned}&465\left(1+\frac{0.0399}{12}\right)^{-12} + 233\left(1+\frac{0.0399}{12}\right)^{-24} + 632\left(1+\frac{0.0399}{12}\right)^{-36}\\&\quad + 365\left(1+\frac{0.0399}{12}\right)^{-48} + 334\left(1+\frac{0.0399}{12}\right)^{-60} + 248\left(1+\frac{0.0399}{12}\right)^{-72}\\&= 2003.01\end{aligned}$$

Notice that the present value of the payments from the investment is different from the sum of the payments themselves (which is 2277).

Unless the reader is among the very fortunate few who can always pay cash for all purchases, you may some day apply for a loan from a bank or other financial institution. Loans are always made under the assumptions of a prevailing interest rate (with compounding), an amount to be borrowed, and the lifespan of the loan, *i.e.* the time the borrower has to repay the loan. Usually portions of the loan must be repaid at regular intervals (for example, monthly). Now we turn our attention to the question of using the amount borrowed, the length of the loan, and the interest rate to calculate the loan payment.

A very helpful mathematical tool for answering questions regarding present and future values is the **geometric series**. Suppose we wish to find the sum

$$S = 1 + a + a^2 + \cdots + a^n \tag{1.8}$$

where n is a positive whole number. If both sides of Eq. (1.8) are multiplied by a and then subtracted from Eq. (1.8) we have

$$\begin{aligned} S - aS &= 1 + a + a^2 + \cdots + a^n - (a + a^2 + a^3 + \cdots + a^{n+1}) \\ S(1-a) &= 1 - a^{n+1} \\ S &= \frac{1 - a^{n+1}}{1-a} \end{aligned}$$

provided $a \neq 1$.

Now we will apply this tool to the task of finding out the monthly amount of a loan payment. Suppose someone borrows P to purchase a new car. The bank issuing the automobile loan charges interest at the annual rate of r compounded n times per year. The length of the loan will be t years. The monthly installment can be calculated if we apply the principle that the present value of all the payments made must equal the amount borrowed. Suppose the payment amount is the constant x. If the first payment must be made at the end of the first compounding period, then the present value of all the payments is

$$\begin{aligned} & x(1+\frac{r}{n})^{-1} + x(1+\frac{r}{n})^{-2} + \cdots + x(1+\frac{r}{n})^{-nt} \\ &= x(1+\frac{r}{n})^{-1} \frac{1-(1+\frac{r}{n})^{-nt}}{1-(1+\frac{r}{n})^{-1}} \\ &= x\frac{1-(1+\frac{r}{n})^{-nt}}{\frac{r}{n}} \end{aligned}$$

Therefore the relationship between the interest rate, the compounding frequency, the period of the loan, the principal amount borrowed, and the payment amount is expressed in the following equation.

$$P = x\,\frac{n}{r}\left(1 - \left[1 + \frac{r}{n}\right]^{-nt}\right) \tag{1.9}$$

Example 1.4 If a person borrows \$25000 for five years at an interest rate of 4.99% compounded monthly and makes equal monthly payments, the payment amount will be

$$x = 25000(0.0499/12)\left(1 - [1 + (0.0499/12)]^{-(12)(5)}\right)^{-1} = 471.67.$$

Similar reasoning can be used when determining how much to save for retirement. Suppose a person is 25 years of age now and plans to retire at age 65. For the next 40 years they plan to invest a portion of their monthly income in securities which earn interest at the rate of 10% compounded monthly. After retirement the person plans on receiving a monthly payment (an annuity) in the absolute amount of \$1500 for 30 years. The amount of money the person should invest monthly while working can be determined by equating the present value of all their deposits with the present value of all their withdrawals. The first deposit will be made one month from now and the first withdrawal will be made 481 months from now. The last withdrawal will be made 840 months from now. The monthly deposit amount will be be denoted by the symbol x. The present value of all the deposits made into the retirement fund is

$$x \sum_{i=1}^{480} \left(1 + \frac{0.10}{12}\right)^{-i} = x \left(1 + \frac{0.10}{12}\right)^{-1} \frac{1 - \left(1 + \frac{0.10}{12}\right)^{-480}}{1 - \left(1 + \frac{0.10}{12}\right)^{-1}} \approx 117.765x.$$

Meanwhile the present value of all the annuity payments is

$$1500 \sum_{i=481}^{840} \left(1 + \frac{0.10}{12}\right)^{-i} = 1500 \left(1 + \frac{0.10}{12}\right)^{-481} \frac{1 - \left(1 + \frac{0.10}{12}\right)^{-360}}{1 - \left(1 + \frac{0.10}{12}\right)^{-1}} \approx 3182.94.$$

Thus $x \approx 27.03$ dollars per month. This seems like a small amount to invest, but such is the power of compound interest and starting a savings plan for retirement early. If the person waits ten years (*i.e.*, until age 35) to begin saving for retirement, but all other factors remain the same, then

$$x \sum_{i=1}^{360} \left(1 + \frac{0.10}{12}\right)^{-i} \approx 113.951x$$

$$1500 \sum_{i=361}^{720} \left(1 + \frac{0.10}{12}\right)^{-i} \approx 8616.36$$

which implies the person must invest $x \approx 75.61$ monthly. Waiting ten years to begin saving for retirement nearly triples the amount which the future retiree must set aside for retirement.

The initial amounts invested are of course invested for a longer period of time and thus contribute a proportionately greater amount to the future value of the retirement account.

Example 1.5 Suppose two persons will retire in twenty years. One begins saving immediately for retirement but due to unforeseen circumstances must abandon their savings plan after four years. The amount they put aside during those first four years remains invested, but no additional amounts are invested during the last sixteen years of their working life. The other person waits four years before putting any money into a retirement savings account. They save for retirement only during the last sixteen years of their working life. Let us explore the difference in the final amount of retirement savings that each person will possess. For the purpose of this example we will assume that the interest rate is $r = 0.05$ compounded monthly and that both workers will invest the same amount x, monthly. The first worker has upon retirement an account whose present value is

$$x \sum_{i=1}^{48} \left(1 + \frac{0.05}{12}\right)^{-i} \approx 43.423x.$$

The present value of the second worker's total investment is

$$x \sum_{i=49}^{240} \left(1 + \frac{0.05}{12}\right)^{-i} \approx 108.102x.$$

Thus the second worker retires with a larger amount of retirement savings; however, the ratio of their retirement balances is only $43.423/108.102 \approx 0.40$. The first worker saves, in only one fifth of the time, approximately 40% of what the second worker saves.

The discussion of retirement savings makes no provision for rising prices. The economic concept of **inflation** is the phenomenon of the decrease in the purchasing power of a unit of money relative to a unit amount of goods or services. The rate of inflation (usually expressed as an annual percentage rate, similar to an interest rate) varies with time and is a function of many factors including political, economic, and international factors. While the causes of inflation can be many and complex, inflation is generally described as a condition which results from an increase in the amount of money in circulation without a commensurate increase in the amount of available goods. Thus relative to the supply of goods, the value of the currency is decreased. This can happen when wages are arbitrarily increased without an equal increase in worker productivity.

We now focus on the effect that inflation may have on the worker planning to save for retirement. If the interest rate on savings is r and the inflation rate is i we can calculate the **inflation-adjusted rate** or as it is sometimes called, the **real rate of interest**. This derivation will test your understanding of the concepts of present and future value discussed earlier in this chapter. We will let the symbol r_i denote the inflation-adjusted interest rate [Broverman (2004)]. Suppose at the current time one unit of currency will purchase one unit of goods. Invested in savings, that one unit of currency has a future value (in one year) of $1+r$. In one year the unit of goods will require $1+i$ units of currency for purchase. The difference

$$(1+r)-(1+i)=r-i$$

will be the real rate of growth in the unit of currency invested now. However, this return on saving will not be earned until one year from now. Thus we must adjust this rate of growth by finding its present value under the inflation rate. This leads us to the following formula for the inflation-adjusted interest rate.

$$r_i=\frac{r-i}{1+i} \tag{1.10}$$

Note that when inflation is low (i is small), $r_i \approx r-i$ and this latter approximation is sometimes used in place of the more accurate value expressed in Eq. (1.10).

Returning to the earlier example of the worker saving for retirement, consider the case in which $r=0.10$, the worker will save for 40 years and live on a monthly annuity whose inflation adjusted value will be \$1500 for 30 years, and the rate of inflation will be $i=0.03$ for the entire lifespan of the worker/retiree. Thus $r_i \approx 0.0680$. Assuming the worker will make the first deposit in one month the present value of all deposits to be made is

$$\begin{aligned} x\sum_{i=1}^{480}\left(1+\frac{0.068}{12}\right)^{-i} &= x\left(1+\frac{0.068}{12}\right)^{-1}\frac{1-\left(1+\frac{0.068}{12}\right)^{-480}}{1-\left(1+\frac{0.068}{12}\right)^{-1}} \\ &\approx 164.756x. \end{aligned}$$

The present value of all the annuity payments is given by

$$\begin{aligned} 1500\sum_{i=481}^{840}\left(1+\frac{0.068}{12}\right)^{-i} &= 1500\left(1+\frac{0.068}{12}\right)^{-481}\frac{1-\left(1+\frac{0.068}{12}\right)^{-360}}{1-\left(1+\frac{0.068}{12}\right)^{-1}} \\ &\approx 15273.80. \end{aligned}$$

Thus the monthly deposit amount is approximately \$92.71. This is roughly four times the monthly investment amount when inflation is ignored. However, since inflation does tend to take place over the long run, ignoring a 3% inflation rate over the lifetime of the individual would mean that the present purchasing power of the last annuity payment would be

$$1500\left(1+\frac{0.03}{12}\right)^{-840} \approx 184.17.$$

This is not much money to live on for an entire month. Retirement planning should include provisions for inflation, varying interest rates, the period of retirement, the period of savings, and desired monthly annuity during retirement.

1.5 Rate of Return

The present value of an item is one way to determine the absolute worth of the item and to compare its worth to that of other items. Another way to judge the value of an item which an investor may own or consider purchasing is known as the **rate of return**. If a person invests an amount P now and receives an amount A one time unit from now, the rate of return can be thought of as the interest rate per time unit that the invested amount would have to earn so that the present value of the payoff amount is equal to the invested amount. Since the rate of return is going to be thought of as an equivalent interest rate, it will be denoted by the symbol r. Then by definition

$$P = A(1+r)^{-1} \quad \text{or equivalently} \quad r = \frac{A}{P} - 1.$$

Example 1.6 If you loan a friend \$100 today with the understanding that they will pay you back \$110 in one year's time, then the rate of return is $r = 0.10$ or 10%.

In a more general setting a person may invest an amount P now and receive a sequence of positive payoffs $\{A_1, A_2, \ldots, A_n\}$ at regular intervals. In this case the rate of return per period is the interest rate such that the present value of the sequence of payoffs is equal to the amount invested. In this case

$$P = \sum_{i=1}^{n} A_i(1+r)^{-i}.$$

It is not clear from this definition that r has a unique value for all choices of P and payoff sequences. Defining the function $f(r)$ to be

$$f(r) = -P + \sum_{i=1}^{n} A_i (1+r)^{-i} \tag{1.11}$$

we can see that $f(r)$ is continuous on the open interval $(-1, \infty)$. In the limit as r approaches -1 from the right, the function values approach positive infinity. On the other hand as r approaches positive infinity, the function values approach $-P < 0$ asymptotically. Thus by the Intermediate Value Theorem (p. 108 of [Smith and Minton (2002)]) there exists r^* with $-1 < r^* < \infty$ such that $f(r^*) = 0$. The reader is encouraged to show that r^* is unique in the exercises.

Rates of return can be either positive or negative. If $f(0) > 0$, *i.e.*, the sum of the payoffs is greater than the amount invested then $r^* > 0$ since $f(r)$ changes sign on the interval $[0, \infty)$. If the sum of the payoffs is less than the amount invested then $f(0) < 0$ and the rate of return is negative. In this case the function $f(r)$ changes sign on the interval $(-1, 0]$.

Example 1.7 Suppose you loan a friend \$100 with the agreement that they will pay you at the end of each year for the next five years amounts $\{21, 22, 23, 24, 25\}$. The rate of return per year is the solution to the equation,

$$-100 + \frac{21}{1+r} + \frac{22}{(1+r)^2} + \frac{23}{(1+r)^3} + \frac{24}{(1+r)^4} + \frac{25}{(1+r)^5} = 0.$$

Newton's Method (Sec. 3.2 of [Smith and Minton (2002)]) can be used to approximate the solution $r^* \approx 0.047$.

1.6 Exercises

(1) Suppose that \$3659 is deposited in a savings account which earns 6.5% simple interest. What is the amount due after five years?

(2) Suppose that \$3993 is deposited in an account which earns 4.3% interest. What is the compound amount after two years if the interest is compounded

   (a) monthly?
   (b) weekly?
   (c) daily?

(d) continuously?

(3) Find the effective annual interest rate which is equivalent to 8% interest compounded quarterly.
(4) You are preparing to open a bank which will accept deposits into savings accounts and which will pay interest compounded monthly. In order to be competitive you must meet or exceed the interest paid by another bank which pays 5.25% compounded daily. What is the minimum interest rate you can pay and remain competitive?
(5) Suppose you have \$1000 to deposit in one of two types of savings accounts. One account pays interest at an annual rate of 4.75% compounded daily, while the other pays interest at an annual rate of 4.75% compounded continuously. How long would it take for the compound amounts to differ by \$1?
(6) Many textbooks determine the formula for continuously compounded interest through an argument which avoids the use of l'Hôpital's Rule (for example [Goldstein *et al.* (1999)]). Beginning with Eq. (1.6) let $h = r/n$. Then

$$P\left(1 + \frac{r}{n}\right)^{nt} = P(1 + h)^{(1/h)rt}$$

and we can focus on finding the $\lim_{h\to 0}(1 + h)^{1/h}$. Show that

$$(1 + h)^{1/h} = e^{(1/h)\ln(1+h)}$$

and take the limit of both sides as $h \to 0$. Hint: you can use the definition of the derivative in the exponent on the right-hand side.
(7) Which of the two investments described below is preferable? Assume the first payment will take place exactly one year from now and further payments are spaced one year apart. Assume the continually compounded annual interest rate is 2.75%.

Year	1	2	3	4
Investment A	200	211	198	205
Investment B	198	205	211	200

(8) Suppose you wish to buy a house costing \$200000. You will put a down payment of 20% of the purchase price and borrow the rest from a bank for 30 years at a fixed interest rate r compounded monthly. If you wish your monthly mortgage payment to be \$1500 or less, what is the maximum annual interest rate for the mortgage loan?

(9) If the effective annual interest rate is 5.05% and the rate of inflation is 2.02%, find the nominal annual real rate of interest compounded quarterly.

(10) Use the Mean Value Theorem (p. 235 of [Stewart (1999)]) to show the rate of return defined by the root of the function in Eq. (1.11) is unique.

(11) Suppose for an investment of \$10000 you will receive payments at the end of each of the next four years in the amounts $\{2000, 3000, 4000, 3000\}$. What is the rate of return per year?

(12) Suppose you have the choice of investing \$1000 in just one of two ways. Each investment will pay you an amount listed in the table below at the end of each year for the next five years.

Year	1	2	3	4	5
Investment A	225	215	250	225	205
Investment B	220	225	250	250	210

(a) Using the present value of the investment to make the decision, which investment would you choose? Assume the annual interest rate is 4.33%.

(b) Using the rate of return per year of the investment to make the decision, which investment would you choose?