Preface

Many problems in science and engineering have their mathematical formulation as an operator equation

$$Tx = y,$$

where T is a linear or nonlinear operator between certain function spaces. In practice, such equations are solved approximately using numerical methods, as their exact solution may not be often possible or may not be worth looking for due to physical constraints. In such situation, it is desirable to know how the so-called *approximate solution* approximates the *exact solution*, and what would be the error involved in such procedures.

This book is concerned with the investigation of the above theoretical issues related to approximately solving linear operator equations. The main tools used for this purpose are the basic results from functional analysis and some rudimentary ideas from numerical analysis. However, no in-depth knowledge on these disciplines is assumed for reading this book.

Although there are many monographs and survey articles on particular topics under discussion, there exists hardly any book which can be used for a mathematical curriculum to deal with operator theoretic aspects of both *well-posed* and *ill-posed* equations. This book is an attempt to fill this gap so as to be used for a second level course for an M.Sc. programme or for a pre-Ph.D. programme. Such a course will enable students to know how the *important theorems* of functional analysis are in use for solving practical problems that occur in other disciplines.

In the first chapter the concepts of well-posedness and ill-posedness are formally introduced and a few examples of such problems are listed. The second chapter equips the reader with the basics of functional analytic results which have been used throughout the text. The problem of approximately solving *well-posed equations*, in particular, the *second-kind*

equations, is treated in the third chapter. The fourth chapter is concerned with the problems associated with *ill-posed equations* and their regularization. In the fifth chapter, ill-posed equations with approximately specified operators have been treated.

Although the book discusses some of the results published in a very recent past, especially, on topics in ill-posed problems, this book is meant only as an introductory text dealing with the issues concerning well-posedness and ill-posedness of linear operator equations. Hence, the topics covered and the discussions carried out in this text are far from exhaustive. Readers interested in the topics under discussion are advised to look into some of the excellent books on the subjects. For instance, the book [5] by Atkinson is a very good reference for second kind operator equations, whereas the books by Baumeister [8], Louis [39], Engl, Hanke and Neubauer [17] and Kirsch [34] give fairly good account on ill-posed operator equations. In fact, there are many journals exclusively devoted to the subject on operator equations, for example, *Integral Equations and Operator Theory, Journal Integral of Equations, Numerical Functional Analysis and Optimization, Inverse Problems*, *Journal of Inverse and Ill-Posed Problems*, etc., and the readers are encouraged to refer to these journals to have up-to-date status on the topics.

Examples, illustrations, remarks and exercises are interspersed throughout the text. They, along with the problems at the end of each chapter, are intended to help the reader in understanding the concepts under discussion.

Now, some words about numberings and certain notations. Lemmas, propositions, theorems, corollaries, examples and remarks are numbered consecutively within each category using two digits. To mark the end of a proof of a Lemma, Proposition, Theorem, or Corollary, we use the symbol $\square$ while for marking the end of a Remark and Example, the symbol $\Diamond$ is used. **Bold face** is used when a new terminology is defined, and *italics* are used to emphasize a terminology or a statement.

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