

## Foreword

This volume is very appropriately titled ‘Exploration of a Nonlinear World’, because this is just the path which Howell Tong has followed these thirty-odd years: the study of the time series analysis of nonlinear models. I have admired his work and am grateful for this chance to offer my own tribute, even if to do so also means admitting that I have seen the subject vanish over my horizon during those years.

My own work in the nineteen-fifties was firmly rooted in the linear world, which seemed challenging enough at the time. My principal contribution (Whittle, 1951) was to obtain the asymptotic (large  $n$ ) expression

$$\frac{n}{2\pi} \int_{-\pi}^{\pi} \left[ \frac{\hat{f}(\omega)}{f(\omega)} + \log f(\omega) \right] d\omega$$

(modulo constants) for the log-likelihood of a sample of  $n$  jointly-normal observations. Here  $f(\omega)$  is the spectral density of the process and  $\hat{f}(\omega)$  is its raw estimate, the periodogram. A good deal follows from this and its spatial analogue. I obtained the result by the unrigorous approximation of the Laurent covariance matrix occurring in the likelihood by a circulant matrix. The natural statistical path is of course to appeal to an autoregressive representation of the process, and it is just when such a representation exists that the above expression is valid.

I realised very well the importance and ubiquity of nonlinear phenomena, ecology providing one fertile source, but gave up hope of any substantial analysis. Nonlinearity was forced on me observationally, however, when a seiche study (Whittle 1954) revealed the existence of subharmonics. Two principal ‘periodic’ components appeared (characterised by peaks of a smoothed periodogram) which could be physically identified as distinct seiches. However, components also appeared whose *period* was a sum of integral multiples of these two basic periods. Some thought suggested a physical mechanism, which was in fact a threshold model.

Howell began his career in control theory, but moved within a few years to a mature and individual study of time series and their statistical analysis. By 1978 he had seized on the threshold theme, which he developed progressively to a study of nonlinear models generally, developing a coherent theory and practical methods. These are set forth particularly in his seminal paper with Lim of 1980, and his book of 1983.

Howell’s instinct to stretch an envelope so as to bring in subtler features manifests itself repeatedly. In both the linear and the nonlinear case he saw the estimation of order (dimension) as being a crucial problem. A much more radical step was to extend the study of nonlinear models to that of chaotic models (see his paper of 1995 and his books of 1990 and 2001). An essential feature of chaotic models is the discontinuous (but highly structured) dependence of the path upon the initial state value, and it is just this feature which one would expect to be lost in a stochastic version.

The review papers in the present volume bring out Howell’s qualities with discernment. Some do so also with charm (Cutler), some with idiosyncrasy (Fomby) and some with substantial technical muscle. As an example of the latter, Cline gives a very thorough account

of the way the properties (stability etc.) of a dynamical system transform under stochasticisation of that system. An, Brockwell and Rosenblatt all see nonlinear time series models as stochastic versions of dynamical systems, and concur with Howell on the importance in this context of Richard Tweedie's 1975 paper. Yao and Lawrance attack the difficult chaos/randomness question by asking how one would determine from observations whether the 'deterministic' part of the generator of a random process is or is not a chaotic generator. Leng *et al*, Gao and Tjøstheim all consider the determination of dimension, the latter in particular contrasting the cross-validation method with the methods associated with Akaike, Mallows and Rissanen. Ling addresses the central consistency question.

Howell was always interested in particular applications, and we see Geweke and Stenseth finding continuing interest in the hare/lynx data, and Li and Tsay treating the essential nonlinearities of the financial models which are now so important.

The striking feature of Howell Tong's 150 papers, three books and book contributions and also his personal exposition and engagement is the continuing freshness, boldness and spirit of enquiry which inform them – indeed, proper qualities for an explorer. He stands as the recognised innovator and authority in his subject, while remaining disarmingly direct and enthusiastic. This collection stands as a tribute to his achievements, although there is no expectation that these will cease.

## References

1. P. Whittle (1951) *Hypothesis testing in time series analysis*. Almqvist and Wicksell, Uppsala.
2. P. Whittle (1954) The statistical analysis of a seiche record. *J. Marine. Res.* **13**, 78-100.

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