

Preface

Topology, created by H. Poincaré in the late 19th and early 20th century as a new branch of mathematics under the name “Analysis Situs” differed in its style and character from other parts of mathematics: it was less rigorous, more intuitive and visible than the other branches. It was not by chance that topological ideas attracted physicists and chemists of the 19th century, for instance, Maxwell, Kelvin and Betti, as well as other scientists residing at the junction of mathematics and physics, such as Gauss, Euler and Poincaré. Hilbert thought it necessary to make this beautiful part of mathematics more rigorous; as it was, it seemed to Hilbert alien.

As a result of the rapid development of 1930s–1960s, it was possible to make all achievements of previously known topology more rigorous and to solve many new deep problems, which seemed to be inaccessible before. This leads to the creation of new branches, which changed not only the face of topology itself, but also of algebra, analysis, geometry — Riemannian and algebraic, — dynamical systems, partial differential equations and even number theory. Later on, topological methods influenced the development of modern theoretical physics. A number of physicists have taken a great interest in pure topology, as in 19th century.

How to learn classical topology, created in 1930s–1960s? Unfortunately, the final transformation of topology into a rigorous and exact section of pure mathematics had also negative consequences: the language became more abstract, its formalization — I would say, excessive, took topology away from classical mathematics. In the 30s and 40s of the 20th century, some textbooks without artificial formalization were created: “Topology” by Seifert and Threlfall, “Algebraic Topology” by Lefschetz, “The topology of fiber bundles” by Steenrod. The monograph “Smooth manifolds and their applications in homotopy theory” by Pontrjagin written in early 50s and, “Morse Theory” by Milnor, written later, are also among the best examples. One should also recommend Atiyah’s “Lectures on K-Theory” and Hirzebruch’s “New Topological Methods in Algebraic Geometry”, and also “Modern geometric structures and fields” by Novikov and Taimanov and Springer Encyclopedia Math Sciences, Vol. 12, Topology-1 (Novikov)

and Vol. 24, *Topology-2* (Viro and Fuchs), and *Algebraic Topology* by A. Hatcher (Cambridge Univ. Press).

However, no collection of existing textbooks covers the beautiful ensemble of methods created in topology starting from approximately 1950, that is, from Serre's celebrated "Singular homologies of fiber spaces". The description of this and following ideas and results of classical topology (that finished around 1970) in the textbook literature is reduced to impossible abstractly and to formally stated slices, and in the rest simply is absent. Luckily, the best achievements of this period are quite well described in the original papers — quite clearly and with useful proofs (after the mentioned period of time even that disappears — a number of fundamental "Theorems" is not proved in the literature up to now).

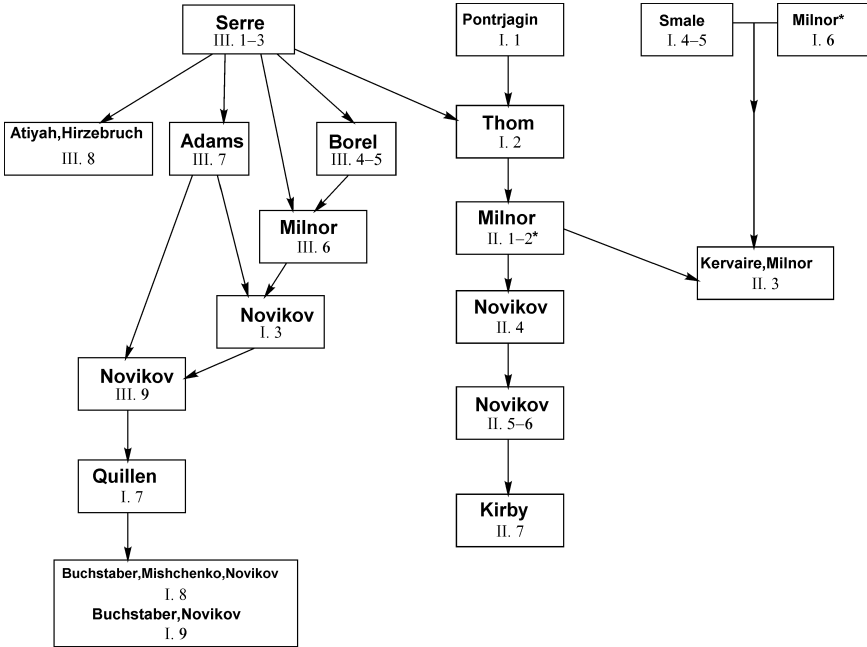
We have decided to publish this collection of works of 1950s–1960s, that allow one to learn the main achievements of the above-mentioned period. Something similar was done in late 1950s in the USSR, when the celebrated collection "Fiber spaces" was published, which allowed one to teach topology to the whole new generation of young mathematicians. The present collection is its ideological continuation. We should remark that the English translations of the celebrated papers by Serre, Thom, and Borel which are well-known for the excellent exposition and which were included in the book of "Fiber spaces" were never published before as well as the English translation of my paper "Homotopical properties of Thom complexes".

Its partition into three volumes is quite relative: it was impossible to collect all papers in one volume. The algebraic methods created in papers published in the third volume are widely used even in many articles of the first volume, however, we ensured that several of the initial articles of the first volume employ more elementary methods. We supply this collection by the graph which demonstrates the interrelation of the papers: if one of them has to be studied after another this relation is shown by an arrow. We also present the list of additional references to books which will be helpful for studying topology and its applications.

We hope that this collection would be useful.

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The interrelation between articles listed in the Russian edition of the Topological Library looks as follows:



Milnor’s books “Lectures on the h-cobordism Theorem” and “Lectures of Characteristic Classes” (Milnor I.6 and Milnor II.2) are not included into the present edition of the series.¹

¹Due to the omission of the two articles, the numerical order of the present edition has been shifted.

Complementary References:

Springer “Encyclopedia of Math. Sciences” books.

Topology I General Survey, Novikov, S. P., Vol. 12, 1996.

Topology II, Homotopy and Homology: Fuchs, D. B., Viro, O. Y. Rokhlin, V. A., Novikov, S. P. (Eds.), Vol. 24, 2004.

Novikov, S. P., Taimanov, I. A., Modern Geometric Structures and Fields, AMS, 2006.

Milnor, J. W. Morse Theory. Princeton, NJ: Princeton University Press, 1963.

Atiyah, M. F. K-theory, W. A. Benjamin, New York, 1967.

S. Lefschetz, Algebraic Topology, AMS, 1942.

Algebraic Topology, to Appear, available from <http://www.math.cornell.edu/~hatcher/#ATI>

Hirzebruch, F. Topological Methods in Algebraic Geometry, Springer, NY, 1966.