

Preface

Even if more than 60 years have passed since their first appearance in Feynman's PhD thesis, Feynman path integrals have not lost their fascination yet.

They give a suggestive description of quantum evolution, reintroducing in quantum mechanics the classical concept of trajectory, which had been banned from the traditional formulation of the theory. In fact, they can be recognized as a bridge between the classical description of the physical world and the quantum one. Not only do they provide a quantization method, allowing to associate, at least heuristically a quantum evolution to any classical Lagrangian, but also they make very intuitive the study of the semiclassical limit of quantum mechanics, i.e. the study of the detailed behavior of the wave function when the Planck constant is regarded as a small parameter converging to zero.

Nowadays, the physical applications of Feynman's ideas go beyond non relativistic quantum mechanics and include quantum fields, statistical mechanics, quantum gravity, polymer physics, geometry. Nevertheless, in most cases, Feynman path integrals remain a mathematical challenge as they are not well defined from a mathematical point of view.

Since 1960, a large amount of work has been devoted to the mathematical realization of Feynman path integrals in terms of a well defined functional integral. Despite the several interesting results that have been obtained in the last decades, the feeling that Feynman integrals are only an heuristic tool is still a widespread belief among mathematicians and physicists.

The present book provides a detailed and self-contained description of the rigorous mathematical realization of Feynman path integrals in terms of infinite dimensional oscillatory integrals, a particular kind of functional

integrals that can be recognized as the direct generalization of classical oscillatory integrals to the case where the integration is performed on an infinite dimensional space, in particular on a space of paths.

The book describes the mathematical difficulties, the first results obtained in the 70's and the 80's, as well as the more recent development and applications. Special attention has been paid to enlightening the mathematical techniques, including infinite dimensional integration theory, asymptotic expansions and resummation techniques, without losing the connection with the physical interpretation of the theory.

A large amount of references allows the reader to get a deeper knowledge of the most interesting mathematical results as well as of the modern physical applications.

I am grateful to many coworkers, friends and colleagues for fruitful discussions. Special thanks are due to S. Albeverio for his help and support, as well as for reading the manuscript and making lots of useful comments. I am also particularly grateful to G. Greco, V. Moretti, E. Pagani, M. Toller and L. Tubaro.

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