Editorial Preface

"Let there be light!"
– *Genesis* 1:3

The book you are reading is commonly referred to as the *Wild Book*. The proof of the Krohn-Rhodes Prime Decomposition Theorem in the 1960s revealed a deep but completely unexpected connection between algebra (semigroups) and computation of all sorts (especially computation requiring only bounded memory). The book in draft manuscript form was first completed by John Rhodes around 1969 and quickly became an underground classic, potentially destined to become the source for fundamental advances in many branches of science. It contains radical ideas on the application of automata and semigroup theory via a rigorously developed theory of mathematical complexity to traditional and non-traditional areas for applications of mathematics. The book introduces a highly insightful approach to create a general, new applied mathematics of finite systems. It is "wild" in that it invades areas as diverse as philosophy (epistemology and the purpose of life), psychology (mathematical theory of psychoanalysis), physics (finite phase-space systems), biology (including metabolism, development and evolution), and games (complexity of games), with a completely original unifying viewpoint and rigorous mathematical methods.

The draft manuscript was used in courses on algebra and automata at the University of California at Berkeley, and was known to students of Rhodes and those lucky enough to find it in the Berkeley Mathematics Library in a simple spiral binding (dated 1971). Many physicists and mathematicians around the world, from Japan to Santa Fe, who became aware of it through academic rumor and word of mouth made photocopies for themselves and disseminated them to colleagues. By the 1990s, many scientists including Professor Morris Hirsch at Berkeley, who has kindly written the foreword, were encouraging Rhodes to publish the *Wild Book*. They realized that the approach of introducing sequential coordinates into the understanding of any phenomenon that can be described by a finite-state (or more general) automaton is a fundamental and general contribution to science whose time is coming, even if its full exploitation may require generations of scientists. As with many crucial ideas in mathematics and science that require time to mature, given the depth of these methods and the publication of this book only now even after 47 years since the Krohn-Rhodes Theorem was discovered, it is still only shortly after the first illuminating light-rays of dawn. Let there be light.

Much more has been done and published by John Rhodes and followers of his school, in particular in developing the abstract mathematical theory of finite semigroups and automata, since the first draft of this book was written. Yet due to the limited availability of the material to other scientists and interdisciplinary researchers, most of the suggested applications still need to be developed much further. Applications are also notoriously unfashionable (sometimes even considered scandalous) in the prevalent culture of many pure mathematicians, therefore many of those exposed to and inspired by the material were not being motivated to pursue these aspects.* If nothing else, even casual readers with a traditional view toward applications will likely find their prejudices about the confines of what comprises applied mathematics vigorously shaken by this book, and with just a little further effort may even awaken to substantially broadened horizons of what applied mathematics and applied algebra could look like in the future.

Happily, algorithmic and computational tools implementing Rhodes' ideas in order to automatically generate sequential coordinate systems are finally becoming available to mathematicians and interdisciplinary scientists thanks to the special abilities of a new generation versed not only in the mathematics and its potential application areas, but capable also of efficiently harnessing the computer algebra and high-performance parallel

*Indeed, a negative attitude toward applications is common in the orientation of many narrowly enculturated or specialized mathematicians (as the present editor knows only too well from his own intellectual development), but such a viewpoint is either an error of mere prejudice in the prevailing mathematical culture or is based on profound ignorance of the relationship of mathematics to its applications areas. Pure and applied mathematics can drive each other's creative development, as examples from the work of Archimedes, Isaac Newton, John von Neumann, or John Rhodes (e.g. the present volume) make plain.

computation necessary to achieve the full potential of these ideas.† These recent developments are also now approaching the point where some of the techniques described here can be fruitfully applied to natural systems in biology and to artificial intelligence to reveal insights into the hidden algebraic structures and symmetries of these systems in a manner that goes well beyond what is computable by an unaided human being.‡

For this first published edition, John Rhodes has written a new philosophical section included here as a prologue, outlining background of the overall viewpoint. Many of the references have been updated in this new edited version of the book while retaining its essential character, errors and typos have been corrected, and an index has been introduced. An almost completely successful effort has been made to retain the numbering of mathematical equations and assertions to agree with the original version. Editorial footnotes are indicated by non-numerical symbols, while the author's own footnotes are indicated with numerals.

Readers of the book, whether from mathematics or other fields, including physicists, biologists of various stripes, computer scientists, psychologists, philosophers, and game theorists, will find much original thought to stimulate the development of their own ideas here. The non-mathematical reader is advised to skip most of the sections developing the mathemati-

†The first realizations of software implementations of the Krohn-Rhodes Theorem were achieved only at the dawn of the 21st century with the remarkable PhD work of Attila Egri-Nagy at the University of Hertfordshire (see A. Egri-Nagy & C. L. Nehaniv, "Algebraic Hierarchical Decomposition of Finite State Automata: Comparison of Implementations for Krohn-Rhodes Theory", *Springer Lecture Notes in Computer Science* 3317:315-316, 2005). Ongoing work is associated with our newer software implementations for mathematical synthesis of hierarchical coordinate systems (i.e. wreath product decompositions usable for prediction, manipulation, understanding and automated solution of problems in finitary discrete dynamical systems). These include Krohn-Rhodes coordinate systems on transformation semigroups (via the holonomy method) and, as a special case, Frobenius-Lagrange coordinate systems on permutation groups. The open-source software (implemented as a package for the GAP computer algebra system) is freely available at `http://sourceforge.net/projects/sgpdec/` with some at present minimal documentation.

‡For some current and recent work in this direction, the reader is referred to A. Egri-Nagy & C. L. Nehaniv, "Hierarchical Coordinate Systems for Understanding Complexity and Its Evolution, with Applications to Genetic Regulatory Networks", *Artificial Life* (Special Issue on Evolution of Complexity), 14(3):299-312, 2008; A. Egri-Nagy, C. L. Nehaniv, J. L. Rhodes, & M. J. Schilstra, "Automatic Analysis of Computation in BioChemical Reactions", *BioSystems*, 94(1-2):126-134, 2008; A. Egri-Nagy & C. L. Nehaniv, "Algebraic Properties of Automata Associated to Petri Nets and Applications to Computation in Biological Systems", *BioSystems*, 94(1-2):135-144, 2008; as well as to subsequent and forthcoming papers by these authors and collaborators.

cal theory (Chapters 2, 3 and 5) on first reading and concentrate on the applications sections (the prologue introducing the philosophical view of semigroups as algebraic models of time, the brief overview of the book in Chapter 1, and the very important and deeply insightful Chapter 4 on coordinate systems for understanding phenomena in science, physics, and biochemistry, and then the four substantial Parts of Chapter 6 on metabolism, biology, psychology, and games, with their wide ranging vision and more detailed developments of the viewpoint for various fields). Most of these sections can be understood prior to developing a detailed understanding of the underlying mathematics. Given these applications as motivations, these readers can later pursue the more mathematical chapters. Chapters 2 and 3 explain and justify the research program for finite semigroups from the mathematical viewpoint of finite algebra (especially group theory) and complexity. The motivating material in these chapters introduces a guiding viewpoint on finite semigroup theory as a generalization of finite group theory, with the natural development of the complexity theory framing the study of finite semigroups (and therefore, as the reader shall see in later Chapters, of finite state automata and, therefore, myriad other topics). These parts will be most accessible to those with some background in abstract algebra. Chapter 5 explains the mathematical core of the Krohn-Rhodes prime decomposition theory for finite state machines and then develops the associated complexity theory. The appendix to Chapter 5 describes the connection between the cascade product and to series-parallel product of circuits and provides mathematical proofs of many of the assertions of Chapter 5; this is the most technical section of the book, providing valuable insights for more mathematically sophisticated readers.

The ideas presented in Chapter 6 on applications to biology and psychology (together with their philosophical interpretations), to games, and in Chapter 4 to physics are, for the most part, a still untapped intellectual gold mine for applications of (group-)complexity.

The editor is grateful to the University of Hertfordshire for support and encouragement during the preparation of this volume (especially to Professors Jill Hewitt, Bruce Christianson, John Senior, Kerstin Dautenhahn, and Martin Loomes). In typesetting, copyediting, correcting, putting together this material, and/or indexing, the editor depended on the various skills and invaluable work of Deborah Craig, Attila Egri-Nagy, Laura Morland Rhodes, and the always helpful staff at World Scientific Publishing Co., although they cannot be blamed for any remaining errors, gaffes or inconsistency in style, nor the decision to retain the author's unique

citation style (against advice to the contrary). Artist Anita Chowdry provided wonderfully appropriate mathematically wild artwork for the front and back covers and page ii. Many thanks to her, and also to Lionel de Rothschild and Najma Kazi for permission to reproduce these exquisite creations! The editor also thanks Dr. Maria Schilstra and again especially Dr. Attila Egri-Nagy, both of the Royal Society / Wolfson Foundation Bio-Computation Research Laboratory at the University of Hertfordshire, for their advice leading to some technical corrections and improvements. As could have been expected, although it is still far from perfect, the work of editing has taken many years longer than planned. But it would be unfair to delay any further and not make this book available to those with the gifts to pursue its richness, rigor, depth, and to explore and develop the new vistas it opens for science and mathematics.

The material is intellectually demanding, but the rewards will more than repay the effort. Depending on the reader, various bits of the contents and style of this book may come as inspiration, revelation, or shock, as those who have studied with Rhodes will know from his inimitable lectures which hold a special intellectual excitement. In lectures and half-day long café meetings with John Rhodes, many of us as young mathematicians (including 26 PhD students he has seen through) learned to really think creatively for ourselves (wordlessly being expected to fill in gaps as if being instructed by a gentle mathematical Zen master), to drop naive and narrow minded notions in mathematics, to be optimistically curious and adventurous, to lash out with creativity and fearless metaphor in mathematical exploration, to create our own mathematical language and concepts as needed, to come back to discipline and reflective judgment backed up by rigorous proofs, and to begin to see the algebra in everything.[§] A similar spirit pervades this book. May it help the reader find illumination!

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Welwyn Garden City, U.K.
First Day of Spring, 20 March 2009

[§]Attending at least one lecture by John Rhodes has been suggested as an educational requirement for all Berkeley PhD students in mathematics. There are many colorful tales to tell about Professor Rhodes' lectures, such as the invention of new Greek letters as needed, fluid co-opting of pictures to denote algebraic operations, proofs presented (and understood by the students) without language, relentless perseverance through calculations and in pushing viewpoints, enlightening breakthrough insights, repeated re-engagement with things already proved yielding new questions and perspectives, unfinished sentences, blank slides, etc., but these stories must be told another time.