

DEPENDENCE OF SCISSION-NEUTRON YIELD ON LIGHT-FRAGMENT MASS FOR $\Omega = 1/2$

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The dependence of the scission-neutron multiplicity on the mass ratio of the fragments in asymmetric fission of ^{236}U was investigated in the frame of the sudden approximation. Only emission from neutron states characterized by the projection of the total angular momentum on the symmetry axis $\Omega = 1/2$ was considered. This dependence was found to be different and less pronounced than for prompt neutrons.

Keywords: Scission-neutrons; sudden-approximation; asymmetric fission; bound and unbound states; neutron multiplicity; fragment mass.

1. Introduction

In a previous study¹ the multiplicity of scission-neutrons emitted during the low-energy symmetric fission of ^{236}U was estimated. For this it was assumed a sudden transition between two nucleon configurations: one just before scission (two fragments connected by a neck characterized by r_{min}) and one immediately after scission (two newly separated fragments characterized by the distance between the inner tips d_{min}). Each initially occupied single-neutron state is thus transformed into a wave packet that is a linear combination of single-neutron states in the final potential well. Some of these states are unbound and the probability to populate such states gives the emission probability.

Since fission into equal fragments represents only 0.01% of the total yield in the thermal-neutron induced fission of ^{236}U , this previous calculation concerns a rare process. Moreover the most striking aspect of neutron emission during fission is the variation of the average neutron multiplicity with the fragment mass. It is therefore necessary to extend our approach to asymmetric fission.

Numerical calculations for each fragment mass ratio and all bound states requires however a considerable amount of CPU time. As a first step towards our goal we report here the results obtained only for a subset of neutron states, defined by a given value of the projection of the total angular momentum along the symmetry axis, namely for $\Omega = 1/2$. It was shown¹ that, during symmetric fission, more than 55% of the scission neutrons are emitted from $1/2$ states. Since this percentage is expected to approximately hold for any mass asymmetry, the present results will give a good idea of the variation of the total number of scission neutrons with the fission-fragments mass ratio.

2. Sudden-approximation formula for the multiplicity of scission neutrons

The probability for a neutron, that just-before-scission had occupied a given state $|\Psi^i\rangle$, to be emitted is

$$P_{em}^i = \sum_f |a_{if}|^2 \quad (1)$$

where $a_{if} = \langle \Psi^f | \Psi^i \rangle$ and $|\Psi^f\rangle$ are the eigenstates in the continuum of the immediately-after-scission single-particle hamiltonian.

To gain precision we replace Eq. (1) by

$$P_{em}^i = 1 - \sum_f |a_{if}|^2 \quad (2)$$

where the sum is now over all final bound states.

Summing these partial emission probabilities P_{em}^i for all initially occupied states one obtains the total number of scission neutrons per fission event:

$$\nu_{sc} = \sum_i v_i^2 P_{em}^i \quad (3)$$

where v_i^2 is the ground state occupation probability of $|\Psi^i\rangle$. For independent neutrons it is a step function: it is 1 for all states below the Fermi level and 0 above.

3. The eigenvalue problem of the single-particle hamiltonian for arbitrary-shape nuclei solved on a grid of cylindrical coordinates

In the previous section we have seen that the main ingredients in our formalism are the single-particle wave functions $|\Psi^i(\epsilon_i)\rangle$ and $|\Psi^f(\epsilon_f)\rangle$ with negative energies $e^i(\epsilon_i)$ and $e^f(\epsilon_f)$ corresponding to the two nuclear configurations, ϵ_i and ϵ_f , between which the sudden transition is supposed to occur.

To describe the nuclear shapes just-before and immediately-after scission we have used as zeroth-order approximations Cassini ovals² with only one deformation parameter: $\epsilon_i = 0.985$ (i.e., $r_{min} = 1.5$ fm) and $\epsilon_f = 1.001$ (i.e. $d_{min} = 0.3$ fm) respectively. Note that $\epsilon = 1.0$ describes a zero neck scission shape. It is known that these ovals are very close to the conditional equilibrium shapes, obtained by minimization of the deformation energy at fixed value of the distance between the centers of mass of the future fragments.^{3,4} To include asymmetric fission it is necessary to introduce a deviation from these ovals defined by a second parameter α_1 - see.⁵

We have recently developed a new numerical method to find the eigenstates (wave functions and energies) of the single-particle hamiltonian for an axially symmetric (otherwise arbitrary shape) nucleus. Rather than diagonalizing in a deformed oscillator basis (as in Nilsson model or in the deformed Woods-Saxon generalizations that followed) we solved the two-dimensional stationary Schrödinger equation on a grid in cylindrical coordinates (ρ, z) . The numerical method consists in calculating the eigensolutions of the matrix resulting from the discretization by central finite differences of our two-dimensional hamiltonian with zero Dirichlet boundary conditions.

The wave functions have two components, corresponding to spin "up" and spin "down" as follows

$$\Psi = f(\rho, z)e^{i\Lambda_1\phi}|\uparrow\rangle + g(\rho, z)e^{i\Lambda_2\phi}|\downarrow\rangle. \quad (4)$$

The values Λ_1, Λ_2 are defined by:

$$\Lambda_1 = \Omega - \frac{1}{2}, \quad \Lambda_2 = \Omega + \frac{1}{2}.$$

Ω is the projection of the total angular momentum along the symmetry axis and it is a good quantum number.

Taking into account the spin-orbit coupling, the hamiltonian has also two components: H_1 and H_2 (see⁶). Considering in addition the axial symmetry, we have

$$H_1\Psi = O_1f - 2K(S_ag + S_cf), \quad (5)$$

$$H_2\Psi = O_2g - 2K(S_b f + S_d g) \quad (6)$$

where K is a constant and

$$O_1 = -\frac{\hbar^2}{2\mu}\left(\Delta - \frac{\Lambda_1^2}{\rho^2}\right) + V, \quad O_2 = -\frac{\hbar^2}{2\mu}\left(\Delta - \frac{\Lambda_2^2}{\rho^2}\right) + V$$

with

$$\Delta = \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{\partial^2}{\partial \rho^2} + \frac{\partial^2}{\partial z^2}$$

$$S_a = \frac{\partial V}{\partial \rho} \frac{\partial}{\partial z} - \frac{\partial V}{\partial z} \left(\frac{\partial}{\partial \rho} + \frac{\Lambda_2}{\rho} \right), \quad S_b = -\frac{\partial V}{\partial \rho} \frac{\partial}{\partial z} + \frac{\partial V}{\partial z} \left(\frac{\partial}{\partial \rho} - \frac{\Lambda_1}{\rho} \right),$$

$$S_c = \frac{\partial V}{\partial \rho} \frac{\Lambda_1}{\rho}, \quad S_d = -\frac{\partial V}{\partial \rho} \frac{\Lambda_2}{\rho}.$$

The approximation by finite differences leads to

$$H_1\psi_{i,j} = -\frac{\hbar^2}{2\mu} \left(\frac{1}{\rho_i} \frac{f_{i+1,j} - f_{i-1,j}}{2\Delta\rho} + \frac{f_{i+1,j} - 2f_{i,j} + f_{i-1,j}}{\Delta\rho^2} + \frac{f_{i,j+1} - 2f_{i,j} + f_{i,j-1}}{\Delta z^2} - \frac{\Lambda_1^2}{\rho_i^2} f_{i,j} \right) + V_{i,j} f_{i,j} - 2K \quad (7)$$

$$\left[\frac{\partial V_{i,j}}{\partial \rho} \frac{g_{i,j+1} - g_{i,j-1}}{2\Delta z} - \frac{\partial V_{i,j}}{\partial z} \left(\frac{g_{i+1,j} - g_{i-1,j}}{2\Delta\rho} + \frac{\Lambda_2}{\rho_i} g_{i,j} \right) + \frac{\partial V_{i,j}}{\partial \rho} \frac{\Lambda_1}{\rho_i} f_{i,j} \right]$$

$$H_2\psi_{i,j} = -\frac{\hbar^2}{2\mu} \left(\frac{1}{\rho_i} \frac{g_{i+1,j} - g_{i-1,j}}{2\Delta\rho} + \frac{g_{i+1,j} - 2g_{i,j} + g_{i-1,j}}{\Delta\rho^2} + \frac{g_{i,j+1} - 2g_{i,j} + g_{i,j-1}}{\Delta z^2} - \frac{\Lambda_2^2}{\rho_i^2} g_{i,j} \right) + V_{i,j} g_{i,j} - 2K \quad (8)$$

$$\left[-\frac{\partial V_{i,j}}{\partial \rho} \frac{f_{i,j+1} - f_{i,j-1}}{2\Delta z} + \frac{\partial V_{i,j}}{\partial z} \left(\frac{f_{i+1,j} - f_{i-1,j}}{2\Delta\rho} - \frac{\Lambda_1}{\rho_i} f_{i,j} \right) - \frac{\partial V_{i,j}}{\partial \rho} \frac{\Lambda_2}{\rho_i} g_{i,j} \right]$$

where the subscript (i, j) corresponds to the grid point (ρ_i, z_j) .

The deformed Coulomb plus nuclear potential $V(\rho, z)$ is defined in terms of the above mentioned Cassini ovals.

To obtain the eigenstates we are using the software package **ARPACK**, which solves large algebraic eigenvalue problems based on the implicitly restarted Arnoldi method.⁷

Since the above hamiltonian depends on Ω , the computation has, in principle, to be repeated for all possible values: $1/2, 3/2, 5/2, \dots$. However bound states in ^{236}U have $\Omega < 11/2$.

The main advantages of our new approach are:

1. Reflexion asymmetric nuclear shapes are calculated with the same program as reflection symmetric ones, without additional numerical effort. The Nilsson-type models require another basis that doesn't conserve parity, i.e. another program.

2. Generalization to non-axiality can be simply done by keeping the 3rd cylindrical coordinate ϕ in the Schrödinger equation.

3. The tails of the wave functions are properly described and not inevitably cut by the finite dimension of the basis.

This last advantage is important at least in three situations:

a) When calculating properties of single-neutron or single-proton states near the drip-line or in halo nuclei.

b) When preparing initial quasi-stationary states for the time-dependent approach to deep quantum tunnelling. Due to the extremely high precision necessary to calculate extremely small tunnelling probabilities, only high purity (essentially one component) initial wave packets can be numerically handled.

c) When calculating stripping or pick-up reaction cross sections that are extremely sensitive to the tail of the nucleonic wave functions.

4. Results and conclusions

For each light-fragment mass A_L we have first calculated the value of the parameter α_1 that defines a perturbed Cassini ovaloid that is asymmetric under reflection at a plane perpendicular to the axis of symmetry and has the required ratio A_L/A_H . For a given A_L , α_1 depends on the deformation parameter ϵ and we have thus obtained two different values α_1^i and α_1^f . Then we have calculated the two sets of bound states $|\Psi_i(\epsilon_i, \alpha_1^i)\rangle$ and $|\Psi_f(\epsilon_f, \alpha_1^f)\rangle$ as described in the previous section. We have finally used them in Eqs.(2) and (3) to estimate $\nu_{sc}(A_L)$. So far we have done this only for neutron states characterized by $\Omega = 1/2$.

The results are presented in Fig.1 where the approximate variation of all neutrons is also sketched. We notice the large difference between the two behaviours. This is due to the fact that the scission neutrons reflect the properties of the extremely elongated fissioning nucleus while the prompt neutrons reflect the properties of the primary fragments.⁸ We have considered a step-function for v_i^2 in Eq. (3) (i.e., independent neutrons) that

makes the results sensitive to the quantum numbers of the last occupied state. For pairing correlated neutrons the solid curve in Fig.1 is expected to be smoother.

In conclusion, the variation of the scission-neutron yield with the fragment mass ratio is predicted to be less pronounced but more complicated than for the rest of the prompt fission neutrons.

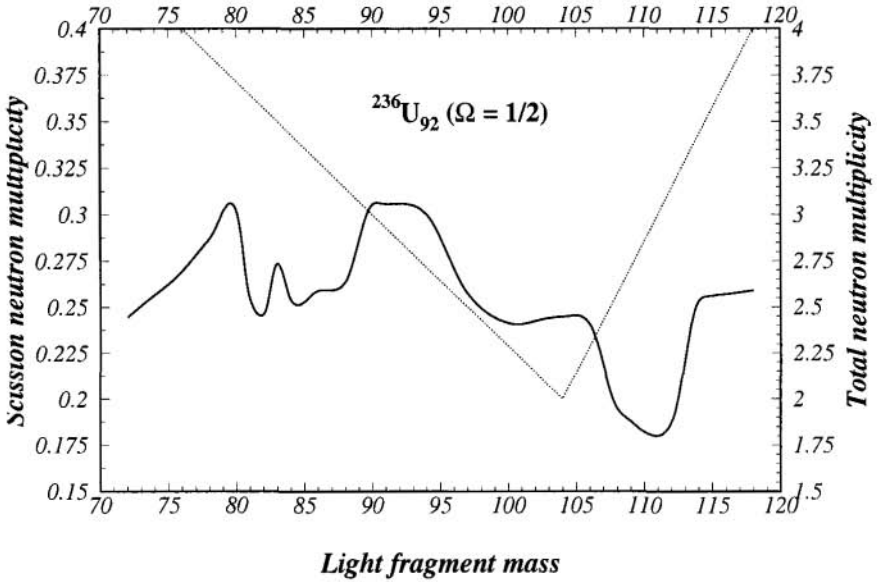


Fig. 1. Estimated scission-neutron yields (solid curve and left-hand axis) compared with experimental total-neutron yields (shown schematically by the dotted lines and right-hand axis).

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