Preface

Generally speaking, a phenomenon of synchronization can be defined as a correlation (adjustment) in time of two or more different processes, but it is obvious that in the scientific bibliography of the problem there exist plenty of various definitions and terms describing this phenomenon. Recently, the idea of synchronization took on a more interdisciplinary character, apart from physical and technological applications only, and it has become an object of great interest in many other areas of science. Moreover, this concept has also been adopted for chaotic systems. Earlier, it was supposed that synchronization phenomenon only concerns periodic systems of regular dynamics, while deterministic chaos and synchronization mutually exclude each other due to the sensitivity to the initial state. However, many researchers have demonstrated that two or more chaotic systems can be synchronized by linking them with a mutual coupling or with a common signal.

The results of the above-mentioned research have been described in numerous papers in scientific journals. The authors of larger publications, i.e., books, monographs or surveys, usually give a general overview of the synchronization phenomenon and its possible applications, while this monograph is mainly focused on the complete synchronization problem, i.e., conditions of its occurrence, speaking more precisely. This problem is demonstrated with respect to a type of coupling, which is applied between dynamical systems. Therefore, a detailed classification of such possible couplings is introduced.

Another aspect distinguishing this book among other publications on synchronization is an application of the synchronization properties for the estimation of Lyapunov exponents, especially for non-differentiable

systems. As is well known, these exponents are one of the most sophisticated tools to identify a character of motion of dynamical systems and their calculation or estimation is one of the fundamental tasks in studies of these systems. For practical applications, it is enough to know the largest Lyapunov exponent. If it is positive, then the system is chaotic. A non-positive maximum number indicates regular system dynamics. On the other hand, in the experimental practice a regular motion is manifested by the synchronization phenomenon. In turn, a lack of synchronization indicates irregular dynamics of chaotic or stochastic nature. Thus, there appears an explicit correlation between synchronous/ desynchronous states and values of Lyapunov exponents. Hence, the synchronization process can be applied to detect chaos and also to determine the largest Lyapunov exponent of an arbitrary dynamical system.

Generally, the proposed monograph is composed of two parts, where:

1. modern techniques for determining the synchronization thresholds,
2. application of the complete synchronization for the estimation of Lyapunov exponents, are analyzed and described.

In the first part (Chapters 1 – 4), after a general introduction to the synchronization phenomenon, a classification of couplings between dynamical systems is presented. Next, a review of analytical and numerical methods (e.g., a graph method, a concept called master stability function) to determine the synchronization thresholds for identical or slightly different dynamical systems is demonstrated and illustrated with several examples of the oscillatory networks with single or disconnected synchronous range(s) of the parameter space. The second part of the book (Chapters 5 – 7) is devoted to a description of the synchronization method for the estimation of the largest Lyapunov exponent with a review of its possible applications. This depiction is preceded with a short survey of classical and modern techniques for the determination of Lyapunov exponents. The properties of the diagonal coupling considered in the first part of the book are the basis for the proposed technique. The main advantage of this approach lies in its usefulness for dynamical systems with discontinuities or time delay,

where classical attempts are not easily applicable. Therefore, the proposed method is mainly exemplified with cases of such systems.

February 2009 *A. Stefanski*
Lodz