

Preface

Matrix orders have fascinated mathematicians and applied scientists alike for many years. Several matrix orders have been developed during the past four decades by researchers working in Linear Algebra. Some of them are pre-orders such as the space pre-order and the Drazin pre-order, while others are partial orders like the minus order, the sharp order and the star order. These developments have grown symbiotically with advances in other areas such as Statistics and Matrix Generalized Inverses.

Two closely connected concepts - parallel sums and shorted operators for non-negative definite matrices - play a major role in the study of electrical networks. Both of these share nice relationships with matrix partial orders as also between themselves. Several extensions of these concepts have been developed in the recent past by researchers working in these and related areas.

There are many research articles in the areas of matrix orders and shorted operators that are scattered in various journals. This is the first full length monograph on these topics. The aim of this monograph is to present the developments in the fields of matrix orders and shorted operators for finite matrices in a unified way and illustrate them with suitable applications in Generalized Inverses, Statistics and Electrical Networks. In the process of this compilation, many new results have evolved. Virtually every chapter in the monograph contains results unpublished hitherto. In fact, Chapter 13 on partial orders of modified matrices comprises entirely of new material.

We believe that dissection of matrices through matrix decompositions helps in clear understanding of the anatomy of various matrix orders in a transparent manner. We employ simultaneous decompositions such as the simultaneous normal form, the simultaneous singular value decomposition and the generalized singular value decomposition extensively in developing the properties of matrix orders. Accordingly, the reader will find new and

intuitive proofs to several known results in this monograph. We also pose some open problems which should be of interest to researchers in these topics. There are a number of exercises at the end of virtually every chapter. These are expected to serve the dual purpose of helping in the understanding of the topics covered in the text and of introducing other related results that have not been included in the main text.

This monograph is aimed at (i) graduate students and researchers in Matrix Theory and (ii) researchers in Statistics and Electrical Engineering who may use these concepts and results as tools in their work. The monograph can be used as a text for a one-semester graduate course in advanced topics in Matrix Theory and it can also serve as self-study text for those who have knowledge in basic Linear Algebra, perhaps at the level of [Rao and Bhimasankaram (2000)].

We deliberately avoided the study of majorization as there is an excellent book [Marshall and Olkin (1979)] on this topic. In this monograph, we consider matrix orders and shorted operators for finite matrices over a field. We do not consider matrices over more general algebraic structures or operators over more general spaces. There has been a good deal of work in these directions - see [Drazin (1958)], [Hartwig (1979)], [Hartwig (1980)], [Jain, Srivastava, Blackwood and Prasad (2009)], [Morley and William (1990a,b)] and [Mitch (1986)], to name a few. We have not included them here in order to keep the monograph focused and less bulky. We have also not gone into order preserving and order reversing transformations (for the minus, star, sharp and one-sided orders) and application of the sharp order in the analysis of Markov chains. These are exciting topics of research that may develop over the next few years.

This monograph was conceived by the first author, S. K. Mitra who was a pioneer in the development of theory of matrix partial orders and shorted operators. He introduced the sharp order, one-sided orders and a unified theory of matrix partial orders. He developed several approaches to the shorted operators along with applications in Generalized Inverses and Statistics.

The first author who is no more, has been the mentor and the main source of inspiration for the other two authors of the monograph. However, we, the second and third authors are responsible for any errors in the monograph.

July, 2009

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