

Preface

Contemporary quantum field theory is mainly developed as quantization of classical fields. Classical field theory thus is a necessary step towards quantum field theory. This book provides an exhaust mathematical foundation of Lagrangian classical field theory and its BRST extension for the purpose of quantization.

Lagrangian theory of Grassmann-graded (even and odd) fields on fibre bundles and graded manifolds is presented in the book in a very general setting. It is adequately formulated in geometric and algebraic topological terms of the jet manifolds and the variational bicomplex. The main ingredients in this formulation are cohomology of the variational bicomplex, the global first variational formula, variational symmetries and supersymmetries, the first Noether theorem, Noether identities, the direct and inverse second Noether theorems, and gauge symmetries.

Degenerate Lagrangian field theories are comprehensively investigated. The hierarchies of their non-trivial reducible Noether identities and gauge symmetries are described in homology terms. The relevant direct and inverse second Noether theorems are formulated in a very general setting.

The study of degeneracy of Lagrangian field theory straightforwardly leads to its BRST extension by Grassmann-graded antifields and ghosts which constitute the chain and cochain complexes of non-trivial Noether identities and gauge symmetries. In particular, a gauge operator is prolonged to a nilpotent BRST operator, and an original field Lagrangian is extended to a non-trivial solution of the classical master equation of Lagrangian BRST theory. This is a preliminary step towards quantization of classical Lagrangian field theory in terms of functional integrals.

The basic field theories, including gauge theory on principal bundles, gravitation theory on natural bundles, theory of spinor fields and topolog-

ical field theory, are presented in the book in a very complete way.

Our book addresses to a wide audience of theoreticians and mathematical physicists, and aims to be a guide to advanced differential geometric and algebraic topological methods in field theory.

With respect to mathematical prerequisites, the reader is expected to be familiar with the basics of differential geometry of fibre bundles. We have tried to give the necessary mathematical background, thus making the exposition self-contained. For the sake of convenience of the reader, several relevant mathematical topics are compiled in Appendixes.