Chapter 1

Planets and Inspiration

1.1 Venus

Only two people are known to have observed the transit of Venus across the sun on the 24th of November, 1639: Jeremiah Horrocks (or Horrox) and his friend William Crabtree, of Toxtenth, a village near Liverpool, England [5]. Johannes Kepler had predicted the 1631 transit, and thought 1639 would be a near miss; Horrocks found Kepler was mistaken. (Horrocks' friend Christopher Townley gave Horrocks's prediction of a transit of Mercury for the 23rd of October, 1651, to Jeremy Shakerley, who moved to Surat, India, to observe it.)

The transit of June 6, 1761, was different: despite the ongoing Seven Years' War making battlegrounds on nearly every continent the major nations of the world launched scientific expeditions which gave many French and British astronomers and surveyors the chance to experience new intensities of frustration, typically at the weather, interrupted with capture and parole by the opposite nation. The scientific result from this which impressed everyone was the roughly modern estimate for the distance between the Earth and the Sun.

But there was another result, one which brings particular inspiration to us, which came from that transit. It was made by the Russian astronomer Mikhail Lomonosov, at the Petersburg Observatory, who saw light refracted around the disc of Venus, indicating there should be an atmosphere. There was a whole new atmosphere and a new meteorology which could be explored, at least provided reliable observations could be made.

But Venus is a difficult planet about which to observe details: 1761 was also the final year in which a putative moon of Venus was observed; observers had seen it, on occasion, since 1645, and even the renowned Gio-

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vanni Cassini spotted this moon. Observations of the planet found what appeared to be stable enough formations for many estimates of the planet's day, the majority of them quite close to 24 hours, to be repeatedly made and to sometimes agree with one another. Fine points like the refraction of light around the tiny disc of Venus seen in the precious few moments of the start or end of a transit, an observation which could be made at best twice in a century was slender evidence, were difficult to build a compelling observational history on.

Still, transits are not the only observations that might give evidence for an atmosphere. The German astronomer Johann Hieronymous Schröter observed in 1793 that Venus appears slightly concave at a time in its orbit when it should be exactly half-illuminated. This suggests an atmosphere must be present, and the observation could well be repeated and confirmed. But even there difficulties remain: Schröter also in that time observed evidence suggesting a lunar atmosphere. (Schröter was correct in his analysis of Venus's atmosphere producing the concavity; it would not be until 1996 that this was proven compellingly, however.)

Chester Smith Lyman observed a ring around the whole planet implying light refracted through an atmosphere, but beyond that, any conclusive investigation would have to wait until spectroscopy and photography could be brought in and to make analysis less dependent on too-short moments of good seeing by eyes which could be quite trained and sensitive but which could not be independently checked.

Spectroscopic astronomy could give some suggestions about the content of the upper atmosphere, its top few kilometers, and that showed a lack of oxygen and of water. What was principally learned from the ground was that Venus was very hot, that its rotation was extremely slow. Details would need to wait for planetary probes.

On February 5, 1974, the NASA probe Mariner 10[1] began taking photographs for its flyby of Venus, which began with the photographing of the atmosphere of Venus to seek evidence of cloud tops or other structures. By the 13th of February the probe had taken 4,165 images of the planet, and the atmosphere was finally seen in considerable detail. Among the many fascinating results was the discovery that, in ultraviolet light, one can see the lower atmosphere, up to 100 km, rotates around the planet roughly every four (Earth) days, compared to the relatively sluggish 243 days Venus requires to complete a rotation. Verner E Suomi, one of the founders of

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[1] Mariner 2 and Mariner 5 had flown by Venus, and the Venera 3 through 8 probes had impacted or orbited Venus before then, but they did not have cameras.

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satellite meteorology and inventor of the Spin Scan Radiometer, which allowed the Geostationary Operational Environmental Satellite production of time-lapse motions of cloud images, pointed out cellular structures within the clouds. These cells could reach two hundred to three hundred kilometers across.

With the Pioneer Venus orbiter, operating from December 1978 to June 1980 (with new operations begun in 1991) an abundance of photographic evidence of this super-rotation, and finer information on the structures of this layer, were available. It also discovered a vortex in the north pole. In April 2006 the Venus Express mission photographed over the south pole not only a vortex, but a polar dipole. Polar vortices are known on Earth to form in the stratosphere and the higher layers of the troposphere, typically forming for the winter months. Those on Venus are quadruple the size of Earth's, and the dipole is an exciting new challenge.

Here the atmosphere is thinning, roughly one-tenth the atmospheric pressure of Earth at sea level (and near a thousandth that of Venus's ground level), with the surprisingly cool temperature of about -30 degrees Celsius. At this altitude there is virtually no difference between the day and night temperatures, with the atmosphere's day temperature being only about one degree warmer than the night side.

Sutherland's formula allows us to estimate the dynamic viscosity of an ideal gas as a function of temperature, with the knowledge of some physical constants. With η_0 the viscosity of the gas at a reference temperature T_0, and a constant C unique to each gas, we can estimate the dynamic viscosity η for any temperature T:

$$\eta = \eta_0 \left(\frac{T_0 + C}{T + C} \right) \left(\frac{T}{T_0} \right)^{\frac{3}{2}} \tag{1.1}$$

For sulphur dioxide C is 416 at the reference temperature of 293.65 Kelvin, while the reference viscosity is $\eta_0 = 12.54 \times 10^{-6} Pa{\cdot}s$. This provides an estimate for the dynamics viscosity at that layer to be around $10.2 \times 10^{-6} Pa{\cdot}s$. In comparison the viscosity of normal Earth air is about $18.27 \times 10^{-6} Pa{\cdot}s$. Since we can accept the treatment of the atmosphere of Earth as an inviscid fluid for certain analyses, we can extend this same consideration to the super-rotating layer of the atmosphere of Venus.

And now we have the makings for our physical model: a slender fluid layer, in thermal balance, on a rotating planet. These will be assumptions made which make for straightforward analysis and for numerical simulations. We also have a specific target of the fascinating super-rotation mode which we would like to reproduce.

1.2 Titan

The natural next question is are there other atmospheres known to show super-rotation? This takes us to Titan, largest moon of Saturn, second largest moon in the solar system, discovered in 1655 by the Dutch astronomer Christiaan Huygens. Like Venus, Titan has a thick, heavy and opaque atmosphere on a slowly-rotating planet. Unlike Venus which is a fore-runner of the global-warming that could be earth's fate as a result of greenhouse gases such as water-vapor and carbon dioxide, Titan is nearly a billion km away from the sun, with a surface temperature around 94 kelvin — one of the many reasons why its thick and luxurious atmosphere has persisted for billions of years — and an atmosphere dominated by nitrogen and hydrocarbons such as methane. Although Titan's atmosphere did not undergo a run-away greenhouse phenomenon, its methane gas near the triple-point plays a role similar to water-vapor on earth. It will turn out from the recent Huygen's observations that the upper atmosphere has very high super-rotating winds, not unlike those on Venus. Unfortunately, Titan and Venus are the only data points in our solar system for clearly super-rotating or sub-rotating atmospheres. Nonetheless, with the rapid advancement in telescopy of modern types, the discovery of planets in other solar systems with significant atmospheres, will quickly increase in numbers and it is hoped that some of these new data points will further validate the theories discussed in this book.

Past the fact that it existed, however, there was not much to say definitively about its properties for just short of three hundred years. The moon is too small, too far away, and too close to a large bright object, at least for the observations a human eye and optical telescope can easily make.

Gerard Peter Kuiper, working at the Yerkes observatory for the University of Chicago, discovered in 1944 a curious spectrum in the light from Titan. The absorption bands, at 6190 and 7250 Angstroms, proved to be those of methane at low pressures. Titan was a most curious moon enjoying an atmosphere with at least 10 kPa partial pressure of methane. Kuiper would further learn over the course of a decade that Titan was still unique: there were no other Saturnian moons to have the same abundance of methane.

Was there more to the atmosphere? How thick was the atmosphere? By 1975, limb darkening gave evidence for there being a thick atmosphere, and Laurence Trafton found evidence of absorption which showed that either the methane atmosphere was at least ten times as thick as that which

Kuiper observed or else that there was much more to the atmosphere than just methane. Trafton found tentative evidence of molecular hydrogen, and later evidence of various hydrocarbons would be added to the understanding of what makes up this atmosphere.

Pioneer 11 flew by Saturn and Titan September 1, 1979, showing Titan to be too cold for life as we know it, and leaving unanswered the question of how there could be hydrocarbons long after energy from the Sun would suggest they should have been burned off. Voyager 1, flying by in November 1980, was sent to photograph the haze of Titan's atmosphere (incidentally sending the probe out of the plane of the solar system, ending its planetary science mission) which proved to reach as much as 300 to 350 kilometers above the moon's surface, and to be rich in nitrogen. Voyager 2, which would be sent to the outer planets, was unable to closely photograph Titan.

On July 1, 2004, the probe Cassini entered orbit of Saturn, starting a long and detailed observational campaign for the system. It had its first close flyby of Titan on October 27 that year. Models built in the 1990s by Goddard Institute for Space Sciences researchers suggested that it might be a super-rotating atmosphere — with Titan rotating about once every 16 (Earth) days and having a diameter only slightly larger than Earth's moon this does not require an enormous velocity — and Cassini's sensors showed exactly this sort of speed, with hurricane-force winds in the lower atmosphere.

The Huygens probe dropped into Titan winds of around 120 meters per second at an altitude 120 kilometers above the moon's surface, with wind speeds dropping below the altitude of about 60 kilometers. The highest layers of the atmosphere rotate west-to-east, with a reversal of direction about seven kilometers above the surface, and another reversal at about 700 meters above ground.

1.3 The Great Red Spot

Any discussion of vortices in planetary atmospheres comes swiftly to the Great Red Spot of Jupiter. This spot, which might have been observed by Robert Hooke in May 1664 — and might be the spot observed by Giovanni Cassini from 1665 to 1713 (initially, in July 1665, near to the shadow of Ganymede) — has been reliably observed since 1831. These qualifiers are necessary: Hooke's observation seems to have been, from his writings, of something appearing in the North Equatorial Belt. While Cassini's observations are those of a skilled observer over many decades, there is nevertheless a curious gap in recorded observations from 1713 to 1831.

The Spot fluctuates in most every observable quantity: its size has shrunk about a third in length from its size in the 1890s. Its color has varied from the intense red which gave it is name to almost invisibility against the surrounding South Tropical Zone; it has not been noticeably red, in visible light, since the 1970s. It is conceivable that the Spots of Hooke and of Cassini were unrelated to the one now observed; it is conceivable that Cassini's at least was the present one and simply faded past detection for over a century. When the spot was observed again from 1831 it was seen primarily in its "hollow", the distortion of the bands of surrounding clouds. It would be in 1878-1881 that the spot grew dramatically more red and became famous as the Great Red Spot.

As very nearly the only long-lived object that is not simply a zonal band on Jupiter the spot has been a natural reference point to use in trying to define longitudes on the planet. Since 1892 and the establishment of A Marth's Invaluable Ephemerides there have several systems for establishing longitude. System I is used for atmospheric features within ten degrees of the equator, and is based on the rotation of features around the axis of the planet every nine hours, 50 minutes, thirty seconds. System II is used outside that band and its motion is based on the average speed of the Great Red Spot. Its coordinates rotate around the planet every nine hours, 55 minutes, 40.6 seconds. It is a touch unsettling that the Great Red Spot has been drifting eastward even in this reference frame. The rotation of the Spot also appears to be slowing as the spot shrinks. (There is also a longitudinal System III, tied to the motion of radio-detectable objects within Jupiter, with a rotation period of nine hours, 55 minutes, 29.7 seconds.)

Its latitude, however, remains reasonably constant, hovering within about one degree of its average position. This is an interesting physical property which may be subject to modelling in the statistical mechanics treatment.

Explaining such a long-lasting anticyclonic structure is a naturally tempting target for any equilibrium statistical mechanics model and we will try to form some understanding of it. Frustrating many efforts to explain the storm as a hurricane-like structure — Kuiper was inspired to use this to try explaining its structure and survival, and the hurricane is still used as a shorthand for explaining it — is the Spot's apparent lack of much of a hurricane's structure, most noticeably in the absence of an eye or eye-wall.

While the Great Red Spot is the most famous anticyclonic storm it is far from the only one: large, white ovals appear in the South Tropical Bounds and in the South-South Tropical Bands, and a long chain of them formed part of the captivating moving images of Jupiter as reconstructed from Pioneer and Voyager observations. The ovals in the South Tropical Band have been observed to last at least half a century, while those in the South-South Temperate Belt seem to survive — or at least to be detectible from Earth — for only a few years.

In the northern hemisphere are more anticyclonic white ovals appear in the North Equatorial Belt, in the North-North Temperate Belt, and in the North Temperate Belt. Curiously, these are short-lived phenomena compared to their southern counterparts. The North Equatorial Belt storms rarely last more than one or two years, and the others are shorter-lived still. Similarly Little Red Spots forming in the region between the North Equatorial Belt and the North Tropical Zone — the analogue, it would appear, to the Great Red Spot's location — form but typically expire in about a year.

An exciting recent development has been the forming of the Oval BA storm. This is roughly half the size of the Great Red Spot, and it was created from the merger of smaller white ovals in the South Temperate Belt between 1998 and 2000. Its darkening into the same color as the Great Red Spot by March 2006 reinforces that the forces which formed and sustain the Great Red Spot are not unique to that phenomenon, and that new physical models of it can be tested with current data. The idea of anticyclonic storms merging into larger structures more reminiscent of the Great Red Spot has been often explored, as for example in simulations by Andrew Ingersoll and P G Cuong.

1.4 Polar Vortices and Other Curiosities

Another interesting large-scale structure known to almost all the bodies with atmospheres is that of the polar vortex. On Earth there are vortices for both poles, with a growth or decay in strength corresponding to the season. These vortices became noteworthy in the popular imagination in the late 1980s and the 1990s in their correspondence to the ozone holes. The coldest air within the polar vortices allows for the depletion of ozone by sunlight and by chlorine-bearing compounds, and as there appears to be relatively little transport of air between the polar vortices and the atmosphere in general this produces a considerable depletion of ozone over either pole.

But the conditions which produce a polar vortex are not unique to Earth. Mariner 10 and then Pioneer Venus found evidence of a major vortex around the south pole of Venus, and the Venus Expression mission in 2006 reinforced not just the existence of this structure but indicated the existence of a dipole there. This feature is yet under investigation.

Mars, too, features a polar vortex, there complicated by the curious fact that much of the carbon dioxide in its atmosphere will condense out of the atmosphere and fall on the polar ice caps. In what appears to be a part of this cycle, atmospheric argon increases in the southern polar regions in autumn, and dissipate in winter and spring. While these are fascinating phenomena they require a level of detail to model — particularly in requiring a change in density and atmospheric composition, as well as relatively sophisticated ground effects — that put it for now beyond what we wish to study.

On Saturn, recent study by the Cassini probe suggests the existence of a super-hurricane, reaching approximately eight thousand kilometers in diameter and centered on the south pole. This is a storm clearly analogous to hurricanes on earth: it has the well-defined eye, and it is surrounded by a ring of clouds between thirty and eighty kilometers taller than the storm's center.

The north pole of Saturn meanwhile has a curious hexagonal cloud surrounding it. This storm was first seen in the images from Voyager 1 and 2, and its rotation period of 10 hours, 39 minutes, 24 seconds matches that of the variational period for the planet's radio emissions. Polygonal shapes are reproducible in spinning fluids in the laboratory, of course, and it is hard not to think of Thomson rings of vortices when hearing of them. Of course, a ring of six vortices is stable on the sphere only when the six vortices are at a pole; even allowing that Saturn's north pole is a bit flatter than a sphere would be, the fact that the polygon stretches over ten thousand kilometers across makes this yet another challenge.

And finally Saturn does have its share of anticyclonic storms reminiscent of the Great Red Spot, although these appear to be seasonal phenomena, appearing when the northern hemisphere is tilted sunwards. These Great White Spots were first observed in 1876 by Asaph Hall, who is also known to astronomers for his 1877 discovery of the two moons of Mars. He is also obscure to specialists in Monte Carlo for an 1873 paper in which he reported the results of performing Buffon's needle problem to experimentally estimate the value of π. Hall was able to use the Great White Spot to provide the first good estimates for the rotation rate of Saturn.

When Voyager 2 flew past Neptune in 1989 it observed a large anticyclonic storm in the southern hemisphere, featuring winds blowing as fast as 2400 kilometers per hour. However, by 1994, the storm had disappeared, according to observations made by the Hubble Space Telescope. It might be that the storm was a particularly transient phenomenon, a hole in the methane layer or the like; but a very similar spot, known as the Northern Great Dark Spot, has appeared in the northern hemisphere and has remained for years. There is also a Small Dark Spot, a southern and cyclonic spot observed during the Voyager 2 flyby.

One of the astronomical events of the past decades which drew considerable scientific and popular interest, the collision of comet Shoemaker-Levy 9 into Jupiter, produced short-lived "scars" on the outer atmosphere, and by its effects provided information about the structure of the inner atmospheric layers. However, these spots were small, short-lived, and ultimately lacked the structure of the cyclonic or anticyclonic spots in which we are interested. The information about properties such as the location of tropopause contribute to our numerical modeling, data which lets us more rigorously test how our model compares to the actual atmosphere.

1.5 Outline

There are two substantially different sorts of planetary atmosphere which this book attempts to understand: the first is the super-rotating atmosphere like that of Venus and Titan, with slowly rotating planet surfaces and fast atmosphere layers. For this we construct model based on a thin layer of a barotropic (non-divergent) fluid on a rotating planet. This model is informed by earlier work on the non-rotating planet described in some detail in [64]. The addition of planetary rotation to the model introduces energy in a way opening new physical considerations. Most noticeably in the non-rotating planet when superrotation occurs is that it may (as expected) form an "axis of vorticity" in any arbitrary direction, while on the rotating planet there are energy considerations encouraging alignment parallel or antiparallel with the axis of rotation. Furthermore there is evidence of multiple phase transitions as a function of the statistical mechanics temperature.

Most of the material in this part of the book is based on new results obtained by the senior author and Dr. Xueru Ding, in collaboration with Rajinder Singh Mavi (chapter 8), Zhu Da and Nuwan Indurugwege. Included in this part is the exact closed-form solutions for the phase transitions of the Barotropic Vorticity Model by Lim, motivated by the discoveries, uncovered by Ding's simulations, of a significant asymmetry between super-rotating and anti-rotating vertically-averaged barotropic flows. The statement of this discovery, prediction and theorem is: a slowly-rotating planet can support a super-rotating barotropic atmosphere at high enough energies, but not an anti-rotating one; a necessary condition for anti-rotation of a barotropic atmosphere is sufficiently fast planetary spin. Only a few important physical quantities are needed as input in this theory, namely, in addition to planetary parameters such as spin-rate and radius, we need to input the relative enstrophy of the atmosphere, and a narrow range of averaged energy-momentum levels of the associated reservoir.

The multi-faceted discussions, mean-field solutions, exact integration by spherical model method based on the device that the enstrophy constraint is a higher-dimensional sphere and simulated super-rotating and anti-rotating end-states, in the first part of this book, goes beyond providing pedagogical completeness and an easy entry point for the interested reader, especially planetary astronomers, towards a resolution of a continuing enigma in the atmospheric dynamics of terrestrial planets and major moons [87].

The Metropolis-Hastings Monte Carlo algorithm proves useful in producing statistical equilibrium end-states which allow the gathering of numerical evidence for these transitions. That we want to use a Metropolis-Hastings procedure will encourage us to look for several quantities which can be kept microcanonically and a relevant energy function which we can allow to vary canonically in order to find stable equilibriums in positive and in negative temperature regions. This will be the topic of chapter 6, followed by two chapters employing respectively a simple mean-field theory and the Bragg mean-field theory, on variants of the Barotropic Vorticity Model where the enstrophy constraint for example might be relaxed to an inequality. In chapter 9 we return to the Barotropic Vorticity Model with its microcanonical constraints on total circulation and relative enstrophy, treating the latter as a spherical constraint in an application of Kac's spherical model to obtain exact or closed-form solutions for the phase transitions to super- and sub-rotating barotropic flows on a massive rotating sphere. This body of calculations will show that the mean-field approach is justified for the problem of barotropic flows on a rotating sphere although it should be emphasized that the Monte Carlo simulations and spherical model method in respectively chapter 6 and 9 are performed without any assumptions of mean-field. On the other hand, there is as yet no convincing evidence that a mean-field approach will do for the more complex divergent models known as the Shallow Water Models discussed in the second part of chapter 2 and in chapters 10 and 11.

The second type of planetary atmosphere we mean to model is that of the Jovian atmospheres, with rapidly-rotating planets where the surface height of the active fluid layer is dynamic. We have in mind possible simple explanations for an atmospheric layer divided into alternating zones and belts of varying vorticity (or velocity, or atmospheric pressure — these all prove to be relevant quantities) and the appearance of cyclonic or anticyclonic storms. In mind through this model construction will be the Great Red Spot, serving as an example of a large anticyclonic storm in the southern hemisphere which has high rim velocities and which also allows for a collection of other, smaller anticyclonic storms which appear in the same hemisphere. At issue here are the on-going debates about the anticyclonic predominance and the north-south asymmetry that exists on Jupiter [24], [46]. Most of the material for this part of the book is based on the recent work of the senior author and Dr Xueru Ding.

To model the Jovian atmospheres, we will build a Shallow-Water Model, in which the rapid spinning of the planet and the strong horizontal and non-quasi-geostrophic nature of the flows will help explain, in terms of simple physical quantities such as the fluid's angular momentum and its changing moment of inertia due to varying surface heights, the key large-scale features of these atmospheres. In this case we will look for cyclonic and anticyclonic spots, with size, location, and predominance of cyclonic or anticyclonic spots proving to be dependent on enstrophies and on the rotation rate of the planet. And as in the Barotropic Vorticity Model and the super-rotation of Venus and Titan there is evidence for phase transitions in both the positive and negative temperature ranges. Once again Metropolis-Hastings proves to be a good tool for visualizing these states and for finding evidence of phase transitions, as discussed in chapters 10 and 11. At this point, it is not known whether a mean-field approach will work for the shallow water problem. Furthermore, no exact solutions like those for the barotropic problem are known.

These numerical experiments will in turn indicate that the angular momentum of the atmosphere — itself a reflection of the energy derived from the rotation of the planet — is a critical component to the formation of Great Red Spot-like storms, and for that matter in the formation of the prominent features which motivated these models. The models also surprisingly capture details of the planetary atmospheres not explicitly in mind when the physical models or the numerical simulations for them were constructed. This indicates we have constructed robust models with physical significance that call for more study.

Some background material in the fine points of spin-lattice models and their analytic implications, as well as the topics in statistical mechanics most directly relevant to the questions we mean to ask, are included following the introduction of the barotropic and the Shallow-Water Models which grow to dominate the book.

A point of emphasis of the book is the formulation of a simple and powerful unified statistical mechanics theory for the modelling of the emergence of large-scale coherent structures in planetary atmospheres through phase transitions. One significant point is that the general unified theory encompasses both the Barotropic Vorticity Model for super-rotating and sub-rotating vertically averaged barotropic flows on slowly-spinning terrestrial planets and major moons, and the Shallow-Water Model for strongly-divergent, non-quasi-geostrophic shallow flows on rapidly-rotating Gas Giants. The reader should keep in mind that, as he explores the many-faceted aspects of these theories, the powerful methods on which they are based, and the good qualitative agreement of their theoretical predictions with recent observations obtained by space missions of NASA and the ESA, the overall objective of this book's models is not to replicate with high fidelity the results of intensive numerical simulations on dynamical rotating Shallow-Water Equations models. These important works have been organized and discussed by amongst others, Ingersoll, Del Genio, Salmon, Shepherd, Schubert, Young and their collaborators in too many seminal papers to be adequately referenced in this small monograph, and they form a highly informative and valuable background for the discerning reader. The reader should also read several excellent texts on the relevant background on geophysical fluid dynamics, such as Holton [43], Pedlosky, and more recently Vallis.