

STUDY ON SOFTENING CONSTITUTIVE MODEL OF SOFT ROCK USING STRAIN SPACE BASED UNIFIED STRENGTH THEORY

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Received 15 June 2008

Revised 23 June 2008

This study attempts to modify the unified strength theory by considering compression as a positive load in geotechnical engineering. It also aims to establish a unified elastoplastic strain softening constitutive model which can accurately describe the strain softening behavior of one kind of soft rocks distributed in Japan. The hardening function parameters of the unified elastoplastic strain softening constitutive model are determined from experiments. In addition, numerical simulations of this model are performed to compare the pre-peak, post-peak and the residual strengths of soft rock predicted by this study and experimental results. Simulation results demonstrated that the proposed constitutive equations in strain space can well describe the softening behavior and accurately predict the peak and residual strengths of soft rock. While the proposed equation is applicative for normally consolidated state and overconsolidated state according to the simulation results.

Keywords: Soft rock; strain softening; strain space; unified elasto-plastic; constitutive model.

1. Introduction

Soft rock¹ is often found in the construction of dams², tunnels³, mines⁴, nuclear power stations, and bridges. It is a geotechnical material featuring notable plastic deformation. As a weather sensitive nonlinear material in geotechnical engineering⁵, soft rock has attracted many researches, with emphasis placed on investigation of constitutive models⁶ that can accurately predict the soft rock deformation and failure^{7,8}, softening, and peak and residual strengths⁹.

Although most studies on deformation⁴, failure⁵ and constitutive models⁶ of soft rock are performed in stress space, they are inadequate in terms of fully describing the behavior of unstable materials such as geotechnical materials in stress space¹⁰, because the rupture and failure of geotechnical materials are functions of strain, which can be

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determined experimentally. Therefore, a constitutive model involving strain space criteria can be established with respect to the three principal strain axes. While some models⁸ consider strain space, they do not consider the difference in tensile and compressive strengths, and the effect of intermediate principal stress of soft rock. In practice, however, it is important to establish a constitutive model of soft rock in strain space, which can describe both the difference of tensile and compressive strengths and the effect of intermediate principal stress.

This study proposes a softening model of a diatom soft rock in strain space, where compression is taken as a positive load by modifying the unified strength theory for accommodating practical convenience^{11,12}. A modified strain space yield function of unified strength theory is derived, and a triaxial elasto-plastic constitutive relationship of soft rock is established. Finally, numerical simulations are conducted to verify the validity of this relationship.

2. Theoretical Background

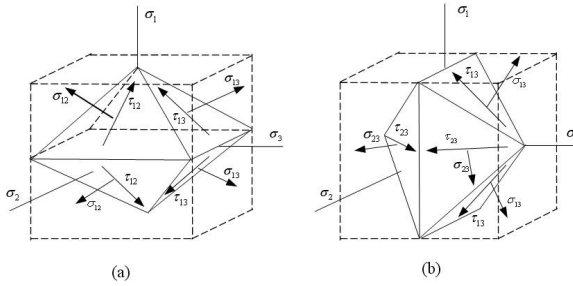


Fig. 1. Stress components on twin-shear stress element model.

An orthogonal octahedral element was introduced in the unified strength theory¹² shown in Fig.1. For accommodating practical convenience in geotechnical engineering, this study modifies all equations of the unified strength theory by taking compression as a positive load. The modified parameters β and C of the unified strength theory can be determined as $\beta = (\alpha - 1) / (\alpha + 1)$ and $C = \sigma_c (1 + b) \alpha / (\alpha + 1)$ by using test results, taking compression as a positive load as $\sigma_1 = \sigma_c$, $\sigma_2 = \sigma_3 = 0$ and $\sigma_1 = \sigma_2 = 0$, $\sigma_3 = -\sigma_c$. Substituting the modified β and C back into the unified strength theory, the modified equations are then expressed as follows respectively for different stress Lode angles:

$$F = \alpha \sigma_1 - \frac{1}{1+b} (b \sigma_2 + \sigma_3) = \alpha \sigma_c, \quad \text{when } \sigma_2 \leq \frac{\alpha \sigma_1 + \sigma_3}{\alpha + 1}, \quad (1a)$$

$$F' = \frac{\alpha}{1+b} (\sigma_1 + b \sigma_2) - \sigma_3 = \alpha \sigma_c, \quad \text{when } \sigma_2 \geq \frac{\alpha \sigma_1 + \sigma_3}{\alpha + 1}. \quad (1b)$$

where α is the ratio of tension and compression strength of the material, b is the parameter of intermediate principal stress effect.

3. The Unified Elasto-Plastic Softening Constitutive Model

The modified unified strength theory in the form of stress invariants can be obtained by substituting the principal stresses in the form of the stress invariants into the modified unified strength theory in Eqs. (1a) and (1b) as:

$$F = (\alpha - 1) \frac{I_1}{3} + \frac{1-b}{1+b} \sqrt{J_2} \sin \theta + \sqrt{\frac{J_2}{3}} \cos \theta = \alpha \sigma_c, \quad \text{when } \theta \leq \tan^{-1} \frac{\sqrt{3}\alpha}{\alpha+2}, \quad (2a)$$

$$F' = (\alpha - 1) \frac{I_1}{3} + \sqrt{\frac{J_2}{3}} \left(\frac{2-b}{1+b} \alpha + 1 \right) \cos \theta + \sqrt{J_2} \left(\frac{b}{1+b} \alpha + 1 \right) \sin \theta = \alpha \sigma_c, \quad \text{when } \theta \geq \tan^{-1} \frac{\sqrt{3}\alpha}{\alpha+2}, \quad (2b)$$

where I_1 and J_2 are the first invariant of the stress tensor and the second invariant of deviatoric stress tensor, respectively^{10,12} and θ is Lode angle.

The yield function of the unified strength theory in stress space for a softening geotechnical material is expressed as:

$$\Phi = F(\sigma_{ij}) - \kappa = 0. \quad (3)$$

As for triaxial consolidated undrained condition¹⁰, $\varepsilon_v = 0$, $I_1 = 3KI_1' = 3K \cdot \varepsilon_v = 0$, and $\theta = 0 \leq \tan^{-1} \sqrt{3}\alpha/(\alpha+2)$, where ε_v is the volume strain.

In addition, $\sigma_1 > \sigma_2 = \sigma_3$; then $J_2 = \frac{1}{3}(\sigma_1 - \sigma_3)^2 = \frac{1}{3}q^2$.

Eq. (2a) can be reduced to $F = \sqrt{\frac{J_2}{3}} = \sigma_i$,

and Eq. (3) to $\psi = \sqrt{\frac{J_2}{3}} - \sigma_i - \kappa = 0$, i.e., $\psi = \frac{1}{3}q - \sigma_i - \kappa = 0$, or

$$\kappa = \frac{1}{3}q - \sigma_i. \quad (4)$$

If the hardening function is chosen as the function of the plastic shear strain, $\kappa = \kappa(\varepsilon_s^p)$, then

$$q = 3G\varepsilon_s^e = 3G(\varepsilon_s - \varepsilon_s^p). \quad (5)$$

The consistency condition is introduced as:

$$d\psi = \frac{\partial \psi}{\partial \varepsilon_s} d\varepsilon_s + \frac{\partial \psi}{\partial \varepsilon_s^p} d\varepsilon_s^p + \frac{\partial \psi}{\partial \kappa} d\kappa = 0, \quad (6)$$

where ε_s is the shear strain and ε_s^p is the plastic shear strain.

Since $d\kappa = \frac{\partial \kappa}{\partial \varepsilon_s^p} d\varepsilon_s^p$, then Eq. (6) can be written as:

$$d\psi = \frac{\partial \psi}{\partial \varepsilon_s} d\varepsilon_s + \left(\frac{\partial \psi}{\partial \varepsilon_s^p} + \frac{\partial \psi}{\partial \kappa} \cdot \frac{\partial \kappa}{\partial \varepsilon_s^p} \right) d\varepsilon_s^p = 0, \quad (7)$$

and

$$d\varepsilon_s^p = d\lambda \left[\left(\frac{\partial Q}{\partial q} \right)^2 + \left(\frac{1}{q} \frac{\partial Q}{\partial \theta_\sigma} \right)^2 \right]^{\frac{1}{2}}, \quad (8)$$

where θ_σ is the stress Lode angle and Q is the potential function.

Substituting Eq. (8) into Eq. (7), $d\lambda$ can be obtained and incremental form of Eq. (5) can be written as

$$dq = 3G \cdot d\varepsilon_s^e = 3G(d\varepsilon_s - d\varepsilon_s^p) = 3G \cdot \left(1 - \frac{\frac{\partial \psi}{\partial \varepsilon_s}}{\frac{\partial \psi}{\partial \varepsilon_s} + \frac{\partial \psi}{\partial \kappa} \cdot \frac{\partial \kappa}{\partial \varepsilon_s^p}} \right) d\varepsilon_s = 3G \cdot \left(1 - \frac{\frac{\partial \psi}{\partial \varepsilon_s}}{\frac{\partial \psi}{\partial \varepsilon_s} + \frac{\partial \kappa}{\partial \varepsilon_s^p}} \right) d\varepsilon_s$$

According to the conventional triaxial compression tests¹⁰, $\partial \psi / \partial \varepsilon_s = G$, hence,

$$dq = 3G \left(1 - \frac{G}{G + \frac{\partial \kappa}{\partial \varepsilon_s^p}} \right) d\varepsilon_s \quad (9)$$

Thus, the elasto-plastic constitutive relationship under triaxial stress is presented by using the elastic plastic theory in strain space and the unified strength theory, taking compression as a positive load, as established in this paper.

4. Numerical Simulation and Discussion

Experimental data⁸ are compared with the results derived in this paper. The specimen is a diatom soft rock in Japan. Specimens were made with a diameter of $0.05m$ and a height of $0.1m$. Consolidated undrained triaxial experiments controlled by stress and/or strain were conducted under different confining pressures and different loading rates. The specimens with high void ratios were fully saturated. The preconsolidation pressure was $1,500 kPa$.

The hardening function in Eq. (9) is chosen as given in reference 13, shown in Fig. 2.

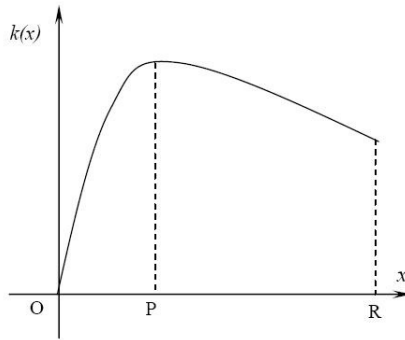


Fig. 2. Hardening function.

$$\kappa(x) = \frac{H}{A} e^{Ax} \left(1 + \frac{1}{PA} - \frac{x}{P} \right) - \frac{H}{A} \left(1 + \frac{1}{PA} \right) \quad , \quad (10)$$

where P , H , and A are hardening function parameters determined from experiments. P is the corresponding value of x when the stress reaches its peak value. Here x is plastic shear strain. H and A can be determined by using Eq. (10) with the peak and residual strengths of the soft rock, respectively, listed in Table 1.

The stress-strain curves of the proposed model can be drawn using the numerical integration results obtained with Eqs. (9) and (10) shown in Figs.3 and 4 (preconsolidation pressure 1500kPa and strain rate $\dot{\epsilon}_a = 0.175\%/min$). The strain softening behavior as well as the peak and residual strengths are described fairly well which are significant for practical engineering applications. The error between simulation and experimental results lies within 10% for normally consolidated state and 5% for overconsolidated state, respectively. So the established model in this paper is found to be effective and applicable.

Table 1. Hardening function parameter values of the unified elasto-plastic softening constitutive model.

Parameter	100 kPa	500 kPa	1000 kPa	1500 kPa	2000 kPa	2500 kPa	3000 kPa	3500 kPa
P	0.0127	0.0161	0.0225	0.0318	0.0285	0.0302	0.0283	0.0312
A	-114.597	-92.898	-65.503	-48.663	-58.876	-56.356	-61.222	-62.251
H (e4)	9.8887	8.7834	7.1651	5.6624	6.6302	7.0142	8.3980	9.8378

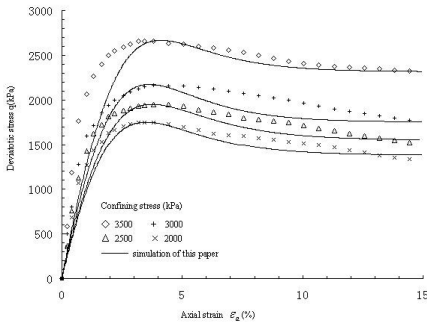


Fig. 3. Normally consolidated state results.

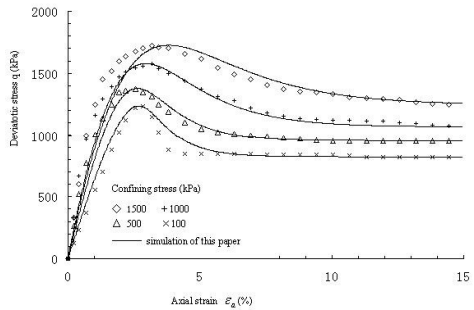


Fig. 4. Overconsolidated state results.

5. Conclusion

From the study in this paper, by taking compression as a positive load, the unified strength theory equation is modified to accommodate practical convenience in geotechnical engineering, where compression is customarily taken as positive. Also, elasto-plastic constitutive models in strain space are proven to be applicable to the softening behavior of soft rock. Finally, a triaxial unified elasto-plastic softening constitutive equation of soft rock is derived in strain space, the associated flow rule, and the unified strength theory. The equation is used to simulate the strain softening behavior of a type of soft rock with a proper hardening function. The simulation results show that this model can well describe softening behavior and can predict the peak and residual strengths of soft rock. The proposed softening constitutive model can describe the residual and peak strengths of overconsolidated soft rock better than the ones of normally consolidated rocks.

Acknowledgment

This work was supported by Inha University.

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