

Chapter 1

Introduction

One often reads about the beauty and importance of general relativity (GR). It is viewed almost universally as our premier theory of gravity, superseding Newton's theory of gravity in a profound way. We share this view whole-heartedly. However, while there have been a very large number of papers written in the field, it is disconcerting to note how little there is in the way of a connection of this large accumulated effort with the great body of the world's physicists and astronomers. Many of these papers are of great mathematical complexity, decipherable only by the experts in the field. Even when decoded, many, if not most, have little relevance to the important issues of physics and astronomy. This is unfortunate as gravity is assuming a greater role of significance in science and the best theory of gravity should be readily accessible to the non-experts.

Our goal in this book is multi-fold: first, we wish to present the essentials of general relativity in a simple way so that any physicist who might have missed training in the field, once having digested the primary concepts and equations, will not have his or her eyes glaze over when confronted by some reference to or equation for general relativity. To do so, we will also cover the essentials of special relativity to provide a smooth transition to the general theory. For a detailed development of special relativity, the reader is directed to the truly classic work of L. D. Landau and E. M. Lifshitz [3] and the very intuitive approach of H. Bondi [4]. The special relativistic treatment in this book has been particularly influenced by these excellent thinkers.

The reader who is comfortable with the standard development of special relativity but who is unfamiliar with Bondi's intuitively appealing approach might wish to start with Chapter 3. In the standard approach, we discuss how the physical equivalence of all inertial reference frames plus the experimental result that the velocity of light is an invariant form the foundation of special relativity, Einstein's theory of space and time in the absence of gravity. These two cornerstones show us that the old transformations of the space-time coordinates with the Cartesian coordinates with which we are familiar, no longer hold and that time is no longer an absolute. The correct transformations using Cartesian coordinates preserve both the value and the form of the spacetime interval, the important measure linking time and space that will lead us into general relativity. Most importantly, we will show that the lengths of bodies and the intervals of time are seen to vary in remarkable ways when viewed in different frames of reference. We will get our first taste of the importance of extrema in physics, how the minimum possible time interval between the ticks of a clock is read in the rest frame of that clock (the "proper time interval") whereas the maximum possible length of a body is the length that is measured in the rest frame of that body (the "proper length"). Being extrema of opposite sense underlines the reciprocal nature of time and space.

In the development of special relativity, we have our first contact with the vectors and tensors of four-dimensional spacetime. These mathematical structures are used with the important Principle of Least Action, which forms the basis of the fundamental equations of physics and relativistic dynamics.

In Chapter 3, Bondi's approach to special relativity builds upon the Doppler factor between two observers in relative motion. Some of the basic results of special relativity are re-derived using Bondi's simple and appealing framework. It enables us to resolve the so-called twin or clock paradox, the asymmetric aging of twins who reunite after a voyage of separation. We will see that there is nothing mysterious that occurs at the turnaround point in the voyage of the accelerating twin. Different paths in spacetime track different spans of time in analogy with the different paths in space which track different spans of distance.

Having covered the essentials of special relativity, we proceed into

the next major step in Chapter 4, the development of general relativity, Einstein's theory of gravity. (The reader who is comfortable with the basics of general relativity might wish to proceed to Chapter 5 or 6, depending upon his/her familiarity with the subject.) We first discuss the Principle of Equivalence, the local equivalence between accelerated reference frames and gravitational fields that was the guiding light for Einstein in his quest for a relativistic theory of gravity. We also focus on the importance of the approximate aspect of the Equivalence Principle, how it is spacetime curvature rather than accelerated reference frames that constitutes true gravity, a point well-articulated by J. L. Synge. That being said, the usefulness of the Equivalence Principle remains, as illustrated in the lead-in to the spacetime metric tensor as describing a gravitational field.

We then proceed with the basic mathematics, tensor calculus, that is required for technical work in general relativity. (Depending upon the degree to which the reader may wish to follow technical aspects, he or she may wish to skim over the sections that follow in Chapter 4 and then concentrate more carefully beginning with Chapter 5.) With the basic aspects of general coordinate transformations covered, we proceed to explore the nature of curved spaces, introducing the important Riemann tensor, which characterizes gravity in an invariant manner. We then focus upon a key GR departure from Newtonian gravity, the removal of gravity from the category of forces. A freely gravitating body is seen to move with zero intrinsic acceleration, following the extremal paths, the geodesics of the spacetime that the body occupies. Motion under gravity as a force is replaced with free motion following the special paths in curved spacetime in analogy with airline pilots who follow the geodesics, the great circles on the globe, to minimize distance between two points.

The energy and momentum conservation laws of special relativity are first generalized to arbitrary coordinate systems using the new generalized derivative, the "covariant derivative". Guided by the Principle of Equivalence, we are led to the conservation laws for general relativity. Consistency with these laws and the demand for melding with Newtonian gravity under appropriate conditions brings us to the Einstein tensor. This incorporates gravity on the left hand side of the Einstein field equations to equal the source, the energy-

momentum tensor, on the right hand side. Having established the Einstein field equations with the basic background structures and concepts, we are prepared to study the relativistic world of gravity.

We first study the simple important and very interesting Schwarzschild solution of the Einstein equations in Chapter 5, the gravitational field in vacuum under the condition of spherical symmetry. To do so, we require the concepts of proper distance and proper time in general metrics, including gravity. The study of the Schwarzschild spacetime introduces interesting issues, event horizons, black holes and singularities, issues that have ignited the imagination of the general public for decades. As compared to special relativity, there is a paucity of experimental corroboration for the theory of general relativity. We outline the various issues regarding these tests.

As there are waves of electromagnetic nature emanating from the acceleration of charges, Einstein's general relativity predicts that there must be waves of gravitational nature arising from the acceleration of masses. In Chapter 6, we discuss these waves briefly. Gravity waves have never yet been observed directly but their existence is inferred from the motion of certain sources such as the binary pulsar, PSR1913+16. However, issues concerning energy and momentum for such waves are not clear-cut. Energy localization has been an enduring controversy in general relativity. We will bring forward our hypothesis that energy, including the contribution from gravity, is most logically localized in the regions of the energy-momentum tensor. This has the unsettling implication that gravitational waves, assuming the reality of their existence to which we certainly subscribe, are not carriers of energy in vacuum.

Part of the natural appeal of physics is that it encompasses all scales of dimension, from the very tiniest size, a size so small as to challenge the imagination, to the universe itself which may in fact be infinite. In Chapter 7, we wind our way through the hierarchy of scales in nature, connecting them to the fundamental forces. We know that the macroscopic physics of our everyday experience breaks down completely when we reach the quantum scale of atomic physics at dimension 10^{-10} m. At this level and smaller, the world as we know it is gone. New forces come into play, the "strong force" binding the nucleus and the "weak force" responsible for the decay of particles such as the neutron. Even the familiar electromagnetic force from

macroscopic physics must be “quantized”, taking on a new character.

It has been almost universally assumed that gravity must also be quantized at a certain stage. We have argued that this is not necessarily the case since gravity is fundamentally different: all particles and fields other than gravity exist *within* spacetime whereas gravity *is* spacetime, i.e. its curvature. In our view, this aspect sets gravity apart as the enveloper of all the rest of physics and removes the necessity for its quantization. However, it is well to ask whether gravity might play a non-classical role at a scale that arises from equating the Compton wavelength of a particle where a quantum duality sets in, to the dimension of contraction at which a body exhibits an event horizon, bringing into play the full nonlinearity of general relativity. This occurs at the Planck scale, of dimension 10^{-35} m. While we can look at this number with its long chain of zeros, we cannot begin to visualize it as an actual length in any normal sense.

In his earliest work in atomic physics, Bohr had set into motion the quantization of the hydrogen atom with *ad hoc* rules that were remarkably successful for their time. We were inspired by the work of Bohr to attempt an *ad hoc* quantization at the Planck scale, adding spin and charge to the Planck mass.

Interestingly, at the extreme of Planck quantum states, a new level of the dimensionless fine structure constant α arises, namely $1/128$ as opposed to the approximate value of $1/137$ of atomic physics. The $1/128$ value has a serendipitous aspect as this is almost precisely the α value governing high energy radiation in Z-boson production and decay. We know of no particular reason for these numbers to match. Perhaps it is a sheer coincidence. However, R. P. Feynman used to impress upon us that it is the confluence of numbers that can foreshadow important truths in physics.

The preceding focused upon the smallest of scales. Proceeding to the very largest series of scales beginning in Chapter 8, we first take a brief excursion into cosmology, the very largest scale in nature. We provide some perspective by building the image of the vast dimensions that we will encounter by the use of scaling, reducing the size of the Sun to that of the head of a pin. From there, we can better picture the vastness of empty space between the stars and then the immensity of a galaxy. We will discuss some of the early ideas about the cosmos and how the modern picture developed.

There has been much recent interest in the idea that the universe is currently in a state of *accelerated* expansion. We will discuss this aspect briefly in conjunction with the cosmological term in the Einstein equations. We will argue that this term should be viewed most logically as another form of matter, albeit exotic, and not a “geometrical” adjunct to the theory.

As a primary goal, in Chapters 9 and 10, we will show why general relativity must be brought into the greater sweep of dynamical problems in the universe. These entail the motions of stars in the galaxies and the motions of galaxies within clusters. Until now, it was believed that Newtonian gravity was the appropriate theory for these scales and that general relativity came into significant play only in situations of ultra-strong (or at least very strong) gravity or for the dynamics of the universe as a whole. This is a bizarre view: overall, the gravity of the universe is weak. Since general relativity is seen as necessary to describe this largest scale, why would one expect GR to be unnecessary for the second and third largest scales in nature, those of the clusters of galaxies and of the galaxies themselves? To this point, when one encountered the expression “general relativistic dynamics”, the reference was to those very special situations in nature such as the case of a closely orbiting pair of neutron stars where very strong gravitational fields and very high velocities prevailed. These cases are certainly very interesting but they are of very limited range. It is worth repeating: the new reality to be faced is that general relativity reaches into the dynamics of essentially all of the key basic building blocks in nature, the arrays of the billions of stars in the galaxies and the clusters of the galaxies themselves. In the vast majority of these cases, the gravity is not very strong and the velocities are not very high by standard relativity measure.

The importance in following this new path is immediate: without general relativity, one is left with serious issues first brought to bear from the work of F. Zwicky and V. Rubin in having to account for the higher-than-expected velocities of stars within galaxies and of galaxies within clusters, velocities having to be rationalized on the basis of Newtonian gravity. This had led to the belief that the normal baryonic matter that we see is but a small fraction of the total matter in the universe, that there is an immense quantity of so-called “dark matter” that is required to drive these anomalous velocities. This

matter was seen to reveal its existence solely by gravitation. Clearly if the gravitational laws that underpin this belief are removed, the paradigm shifts. Many would argue that the galactic motions are no longer the main reasons for belief in dark matter, that the primary reason is now the need for the extra matter to quickly form the galaxies and their clusters shortly after matter decoupled from radiation in the early universe. While early universe studies are the glamorous focus of current interest, some perspective is useful here: progress in early universe study has been impressive, but firm pronouncements as to what is required to explain COBE and WMAPs strike us as unjustified. Alternative scenarios should be, and surely will be explored in the years to come.

In Chapter 9, we will describe the paradigm shift due to general relativity. We will describe how S. Tieu and the author accounted for the high velocities of stars in spiral galaxies by the application of general relativity and without the requirement for the vast stores of exotic dark matter as is the case when Newtonian gravity is taken as the underlying theory of gravity. This raises the phenomenon of *general* relativistic velocity to a level of fundamental importance as an observational tool for situations of weak gravity and velocities far below that of light, domains previously reserved for Newtonian gravity. An essential point is this: where general relativity gives a result for a given physical system different from that provided by Newtonian gravity, we choose the general relativistic result. There is nothing controversial about this choice—the great majority of the world’s physicists would do likewise as general relativity is regarded as our premier theory of gravity. The issue is the generally prevailing belief that the galactic systems that we studied should have given the same descriptions with the applications of both Newtonian gravity and general relativity. While our approach and departure from expectations have received a great deal of attention world-wide in the media and by many physicists and astronomers in hundreds of communications, it has also been opposed by an interesting variety of critics. We have answered their critiques in detailed papers and in Appendix A, we present a simpler account of both the nature of the criticisms and our replies to them.

In Chapter 10, we will then proceed to the next size scale to present an account of how we can rationalize the relatively large ve-

locities of galaxies within clusters without the aid of dark matter, again using general relativity. We will focus on our study of the spherical collapse under gravity of a pressureless ball of fluid as an idealized model of a freely gravitating cluster of galaxies. We will present the contrast between the local and asymptotic measures of the velocities of the particles, the former that would be perceived by observers in the vicinity of the galaxies and the latter that astronomers would actually perceive over the vast distances separating us from the populations of galaxies under study. It is well understood that even in classical physics, velocity is an observer-dependent quantity. In special relativity, that dependence assumes a more complicated form and it is even more complicated in general relativity. While these aspects of velocity have been understood, they have been under-appreciated in the case of general relativity. Our approach in taking these aspects into careful account leads to an alternative to dark matter as an explanation for the high galactic velocities seen in clusters of galaxies.

A key theme of this book is one of repositioning general relativistic as opposed to Newtonian dynamics as an essential tool for the complete description of the motions of bodies under gravity. Our work with Tieu on galactic dynamics focuses on the hitherto unjustified neglect of the application of general relativity and the interesting effects its incorporation produces. By contrast, in Chapter 11 we will also discuss our work on the *misuse* of general relativistic dynamics in the notion of “closed timelike curves”, often interpreted as time machines, and how they can be seen most logically as mere mathematical rather than physical constructs.

Finally, in Chapter 12 we will discuss the overall direction of current theoretical physics research, its fixation on the need for unification of the fundamental forces with gravity. We will argue that there are reasons for regarding gravity as fundamentally different from the other forces in nature, in fact that it is not a force at all. Rather than the standard forces that serve as mechanisms coupling objects to each other as ingredients *within* spacetime, gravity is a property of spacetime itself, its curvature. This is the essence of gravity. While most relativists, if pressed, would acknowledge this to be the case, the focus has been lost (or never fully appreciated) by many. One aim of this book is to restore the realization of gravity’s essence.

This book will have accomplished its mission if readers will appreciate the wonders that Einstein has brought to our recognition of relativity in a now much broader framework as one of the cornerstones of modern physics.