

# MODELING BY NONLINEAR DIFFERENTIAL EQUATIONS

Dissipative and Conservative Processes

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# MODELING BY NONLINEAR DIFFERENTIAL EQUATIONS

Dissipative and Conservative Processes

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To our wives, Patricia and Inge

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