Preface

The present book is intended mainly to be a textbook for physics students at the advanced undergraduate and beginning graduate levels, especially those with a theoretical inclination. Its chief purpose is to give a systematic introduction to the main ingredients of the fundamentals of quantum theory, with special emphasis on those aspects of *quantum dynamics*, *group theory* and *differential geometry* that are relevant in modern developments of the subject. Of these three areas, the first is without doubt of primary importance from the viewpoint of physics, and is usually given prominent place in most texts. Its treatment from first principles is also accorded the same status in the present text, and indeed occupies a good forty per cent of the material. But it is really the second and the third areas that the author wishes to highlight, in relation to the first. While the importance of group theory in quantum mechanics is almost universally recognized, and there indeed exist numerous excellent monographs and texts on this specialized topic at the graduate level, its inclusion at the undergraduate level is still a matter of controversy. The recognition of the importance of differential geometry and topology in quantum theory, for example, in the topics of *geometric phases* and *topological quantum numbers*, is of even more recent vintage. Although they are beginning to be mentioned in quantum mechanics texts, their mathematical underpinnings (in terms of the geometric notions of *fiber bundles*) are usually well hidden. The present book is an attempt to inject *both* group theory and differential geometry into the traditional framework of non-relativistic quantum dynamics in an integral fashion, beginning at the undergraduate level. In doing so it also seeks to present the fundamental physical ideas and mathematical framework in a unified manner, with the underlying purpose of providing the student with an overview of the key elements of the theory, as well as a solid preparation in calculational techniques which are universally applicable to diverse disciplines of physics where quantum mechanics plays a role. The focus of the book is on fundamental theory as well as practical calculations. Experimental aspects of the subject, though indispensable epistemologically in an overall appreciation, will not be emphasized for the sake of stylistic and thematic unity.

Perhaps a brief explanation of the title of the book is in order here. It is meant to convey the thematic unity of what the author sees as the three interlocking elements of modern quantum theory. While *dynamics* – the understanding of the space-time evolution of a physical system based on the relevant equations of motion – is the immediate objective of the theory, its effective use, and deep understanding, cannot be achieved without an appreciation of the constraints at a deep level due to the presence of various kinds of symmetries (both space-time and internal, the latter also referred to as *gauge symmetries*) in a physical system. These constraints manifest themselves as *conservation principles*, independent of many details of the system. Group theory *is* the mathematical language of symmetries in general; and *group representation theory*, in particular, provides the most natural framework for the description and indeed, the classification, of the quantum states of a system possessing various symmetries. Finally, the geometric notions of *complex vector and tensor bundles*, with identified *gauge groups*, provide the exact mathematical framework for the description of the dynamics of quantum states (represented by vectors in complex vector spaces) "living" in space-time and constrained by *gauge forces*. In many interesting cases, as in that of "conserved" *topological quantum numbers*, the physical constraints are manifested as a result of topological ones, at an even deeper level than geometrical ones. All of what is said in this paragraph so far is also relevant in relativistic quantum theory, namely, *quantum field theory*, although this subject, one that is much more involved technically than its non-relativistic counterpart, is not covered in the present text.

Some of the distinguishing features of the book are as follows. The topics are mostly introductory; yet the presentation is more advanced and in-depth than customary. Unlike many traditional texts, more care is devoted to the motivation and explanation of the relevant mathematical ideas than usual, and an attempt has been made to demonstrate their thematic unity beyond seemingly unrelated applications. On occasion, proofs of certain key mathematical theorems are presented in some detail; but only when they are relatively simple and of importance in applications. Calculational techniques are also stressed throughout. For this purpose, a mode of exposition with a degree of mathematical rigor somewhere between a physics text and mathematics text is adopted.

Another quite unique feature of the book is the inclusion in the first few chapters of the historical genesis of quantum theory derived from the canonical works of Schrödinger, Heisenberg, and Dirac. Again, this fascinating material is not usually included in modern texts at either the undergraduate or graduate levels. We hope that it will allow students to appreciate better the significant continuity of theoretical themes (if not the interpretational ones) in the passage from classical to quantum mechanics. On the other hand, there are numerous topics in non-relativistic quantum theory that are of vital importance but not treated in this text. These include, among others, many-particle systems, quantum statistics and non-relativistic quantum field theory, and topics of more current interests such as quantum decoherence, quantum entanglement, and quantum computation. It is hoped, however, that the present text will lay the requisite groundwork for these more specialized topics.

I have deliberately tried to arrange the material of this book into a large number of topically well-defined chapters, with each more or less focused on a single topic that is clearly discernable from the title. The chapters, however, are far from logically independent. Almost without exception, relatively unfamiliar concepts and results found anywhere in the text can be traced to a point of origin in an earlier part, and it is hoped that the organic connections between many topics will become apparent as they are discussed within different contexts. To facilitate this, some effort has been made to provide for clear cross references within the text by specific chapter numbers, equation numbers, or other means, as well as a comprehensive index. To promote further study, I have also not shied away from including more advanced literature as well as references to original work in the Bibliography. Problems of various degrees of difficulty, sometimes accompanied by generous hints, are interspersed within the text at strategic points. These are meant to complement the text in crucial ways; and the reader is encouraged to attempt as many as possible in order to derive maximal benefit from the book.

Organizationally, the flow of the book is as follows. The first three chapters give a portrait of the "pre-history" of quantum mechanics. Chapters 4 to 16 treat the fundamental principles of quantum dynamics, embodied in the Schrödinger equation, the Heisenberg equation, and the uncertainty relation. In these chapters, the basic mathematical framework of quantum mechanics employing the theory of self-adjoint operators in Hilbert spaces is also introduced, and discussed by means of several specific applications. Beginning with Chapter 17, and running through Chapter 30, the relevance of the notion of symmetries in quantum theory and its mathematical description by means of group theory are discussed and illustrated by various examples. The main groups discussed are the Lie groups $SU(2)$ and $SO(3)$ (for their relevance in rotational invariance), and the discrete symmetric groups (for their relevance in systems of identical particles). Chapters 31 through 37 resume with more conventional topics within the purview of dynamics, where the focus is on specific perturbative and non-perturbative techniques of the solution of the Schrödinger equation. The final part of the book, from Chapters 38 to 44, deals with the applications of topology and geometry in quantum mechanics. The main point of entry into this fascinating area is the physics of geometric phases. In this part, we also provide a rather unconventional introduction of sorts to the mathematical machinery of homotopy, homology and de Rham cohomology, differential forms, connections on vector bundles, Chern classes, and Chern-Simons classes. There is probably more than sufficient material in this book for an entire academic year's worth of instruction. It can be used in a sequential fashion; but the arrangement of topics as described above may also allow for the selection of groups of chapters to focus on any of the three main themes of the book.

In terms of the physics background required of the reader, she/he is expected to be familiar with the fundamentals of classical mechanics and electromagnetism at the advanced undergraduate level, including an appreciation of Lagrangian and Hamiltonian mechanics, and the concept of gauge transformations of the electromagnetic vector potential. A course in introductory modern physics at the sophomore level, including some exposure to the beginnings of quantum theory, is not absolutely required but will be useful. The requisite mathematical background is perhaps not so easily spelled out. In general, a working knowledge of those topics usually covered in a mathematical physics sequence at the junior to senior level is also assumed. This include, roughly speaking, vector calculus, some ordinary and partial differential equations, some complex analysis and special functions, and especially, a good dose of linear algebra. Within the text, however, a body of mathematical knowledge much beyond these items will be presented. The rudiments of functional analysis (Hilbert space theory) are developed, as far as needed, in the text itself, as are those of the more non-traditional topics of group theory and topology/differential geometry. The presentation of these latter mathematical topics in a physics text presents special challenges. As far as possible, and in general, I aimed at providing some physical motivation before delving into the exposition of the mathematics; and abstract mathematical concepts and facts (of which there are unavoidably many in this book) are introduced in light of hopefully more intuitive and concrete physical examples. To this end, I have, somewhat ambivalently, refrained from the more usual practice of relegating the collection of useful mathematical definitions and theorems to appendices, and have, instead, tried to "weave in" the mathematics with the physics. I realize that this is done at the price of mathematical rigor, completeness, and at times even accuracy, and only hope that the physics reader will be more engaged, by being assured at each point that the mathematics will be put to good use, and by not having to skip back and forth between the main text and the appendices. My modest wish is that the physics student, after a serious perusal of the text, will be persuaded of the essential importance of group theory and differential geometry in physics, and motivated to engage in a more systematic study of these subjects. On the other hand, I will also be thrilled if an occasional mathematics reader will be enticed to look within these pages for some non-trivial use of abstract mathematical formalisms in fundamental physics problems.

The first half or so of the book had been developed from an evolving set of class notes for an upper-division quantum mechanics sequence that I have taught at Cal Poly Pomona, on and off, since the early 1990's. These notes were made available to students, and many of them have provided valuable feedback and encouragement over the years. In a very real sense, they were the motivating force for me to write the book. Motivation and even a strong desire to write would not have been sufficient for progress, had not another key requirement – time – been generously fulfilled by the Faculty Sabbatical Program of the California State University. The late, preeminent geometer, Shiing-Shen Chern, through his generous and patient guidance on a collaborative textbook on differential geometry (Chern, Chen and Lam 1999), taught me most of what is found in these pages on that subject, and imparted to me a heightened appreciation of the often tortured and nuanced relationship between physics and mathematics. Dr. Soumya Chakravarti, my colleague in the Physics Department at Cal Poly Pomona, has kindly and with good humor acted as a sounding board for many of my half-baked ideas on how to present mathematical concepts convincingly in a physics text, and provided many valuable suggestions. Dr. Ertan Salik, another Physics colleague of mine, and Mr. Hector Maciel, also from our Physics Department, have both provided useful advice on the preparation of the figures. Ms. E. H. Chionh, my able editor at World Scientific,

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