

Chapter 1

Historical Introduction to Electric and Magnetic Moments

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The historical development of the discovery of spin and magnetic moments is reviewed, along with the development of searches for electric dipole moments.

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1.1. The Discovery of Spin

As physics developed at the beginning of the 20th century, a number of intriguing puzzles existed that could only be explained by radically new ideas. In 1911 Rutherford proposed the nuclear atom [1]. This hypothesis, combined with Thompson's discovery of the electron [2] and Millikan's discovery that the electron charge is quantized [3], implied that electrons were somehow in orbit around the positive nucleus, leading to a neutral atom. Classically such a system is unstable, and in 1913 Bohr proposed his quantum theory [4]. Of course, many conceptual problems remained, which began to be understood once Schrödinger's wave equation [5] was published in 1926.

In 1921, two interesting proposals were published: Compton proposed [6] a spinning electron to explain ferromagnetism, which he realized

was difficult to explain by any other means.^a Stern proposed an experiment to study space quantization [7] to test the Sommerfeld quantum theory, where he presented the details of what we now call the Stern–Gerlach experiment. An atomic beam of silver atoms was to be projected through a gradient magnetic field where the net force on the magnetic dipole would separate the different magnetic quantum states. For a classical dipole the deflection would be continuous, since the direction of the dipole moment could have any value.^b

Over the next two years the famous experiments were carried out [8], and the two-band structure observed. By 1924, Stern and Gerlach concluded that to within 10%, the magnetic moment of the silver atom in its ground state was one Bohr magneton [9]. Their papers made no reference to the developments in spectroscopy, and in their 1924 review article, no conclusions beyond the magnetic moment were drawn from the two-band structure.

Independently, in 1925 Uhlenbeck and Goudsmit [10] proposed the “spinning electron” to explain the fine-structure observed in the anomalous Zeeman effect in atomic spectra.^c The fine-structure splitting can be understood as the interaction of the magnetic dipole moment of the electron with the magnetic field produced by the nuclear motion, which in the electron’s rest frame appears to be orbiting about the electron. The electron’s magnetic dipole moment is along its spin and is given by

$$\vec{\mu} = g \left(\frac{q}{2m} \right) \vec{s}, \quad (1.1)$$

where $q = \pm e$ is the charge of the particle in terms of the magnitude of the electron charge e , and the proportionality constant g is the g -factor for spin (which is sometimes written as g_s). In their second paper [11], Uhlenbeck and Goudsmit conclude that the g -factor for spin is twice that for orbital angular momentum, however the calculated fine-structure splitting was then twice as large as the observed splitting. Only later in 1926, when Thomas showed that the factor of 2 discrepancy between experiment and calculation was a kinematic effect [12], did spin start to become an accepted

^aIn his paper Compton acknowledges A.L. Parson (Smithsonian Misc. Collections, 1915) as first proposing that the electron was a spinning ring of charge. Compton modified this proposal to be a much smaller distribution “concentrated principally near its center.” Compton’s paper is almost unknown.

^bSee Allan Franklin, <http://plato.stanford.edu/entries/physics-experiment/app5.html> *Stanford Encyclopedia of Philosophy*, Appendix 5, for a nice discussion putting the Stern–Gerlach experiment into historical context.

^cIn their Nature paper [11] of 1926, they acknowledge Compton’s independent suggestion of spin.

theory. Thomas later wrote to Goudsmit, indicating that Kronig had also suggested spin [13]:

I think you and Uhlenbeck have been very lucky to get your spinning electron published and talked about before Pauli heard of it. It appears that more than a year ago, Kronig believed in the spinning electron and worked out something; the first person he showed it to was Pauli. Pauli ridiculed the whole thing so much that the first person became also the last and no one else heard anything of it. Which all goes to show that the infallibility of the Deity does not extend to his self-styled vicar on earth.

Incidentally, no mention is made of the Stern–Gerlach measurements in the Uhlenbeck and Goudsmit papers. However, the Stern–Gerlach result was noticed by Phipps and Taylor at the University of Illinois at Urbana, and they did draw the connection between the Stern–Gerlach experiment and the electron spin proposed by Uhlenbeck and Goudsmit. They repeated the Stern–Gerlach experiment with an atomic beam of hydrogen in 1926. While technically more challenging than the silver experiment, they reached a similar conclusion, viz. that the magnetic moment of the hydrogen atom was also one Bohr magneton [14].

Today, we understand that the magnetic moment measured in both of these atomic-beam experiments was that of the un-paired atomic electron. We can conclude that a magnetic moment of one Bohr magneton implies that the g -factor for spin is 2. Although in our undergraduate modern physics courses we emphasize that the Stern–Gerlach experiment showed clearly the existence of half-integer spin, historically it seems to have played a much less important role than spectroscopy did.^d In his book, *The Story of Spin*, Tomonaga does not mention the Stern–Gerlach result [16].

1.2. Dirac’s Theory and Beyond

It was not until Dirac’s famous 1928 paper [17], where he introduced his relativistic wave equation for the electron, that the picture became clear. Dirac pointed out that an electron in external electric and magnetic fields has “the two extra terms^e

$$\frac{e\hbar}{c} (\boldsymbol{\sigma}, \mathbf{H}) + i \frac{e\hbar}{c} \rho_1 (\boldsymbol{\sigma}, \mathbf{E}), \quad (1.2)$$

^dThe recollections of Goudsmit agree with this assessment, see Ref. [15].

^eHere we use Dirac’s original notation.

... when divided by the factor $2m$ can be regarded as the additional potential energy of the electron due to its new degree of freedom.” These terms represent the magnetic dipole (Dirac) moment and electric dipole moment interactions with the external magnetic and electric fields.^f Dirac theory predicts that the electron magnetic moment is one Bohr-magneton (viz. $g = 2$), consistent with the value measured by the experiments.^g Dirac later commented: “It gave just the properties that one needed for an electron. That was an unexpected bonus for me, completely unexpected [18].”

As an aside, Dirac had little use for the electric dipole moment (EDM), and stated “The electric moment, being a pure imaginary, we should not expect to appear in the model. It is doubtful whether the electric moment has any physical meaning, since the Hamiltonian ... that we started from is real, and the imaginary part only appeared when we multiplied it up in an artificial way in order to make it resemble the Hamiltonian of previous theories.” We now understand that the presence of an electric dipole moment violates both parity (P) and time reversal (T) symmetries, and CP as well if CPT holds.

1.2.1. *The discovery of anomalous magnetic moments*

For some years, the experimental situation remained the same. The electron had $g = 2$, and the Dirac equation seemed to describe nature. Then a surprising and completely unexpected result was obtained. In 1933, against the advice of Pauli who believed that the proton was a pure Dirac particle [16], Stern and his collaborators [19] showed that the g -factor of the proton was ~ 5.5 , a long way from the expected value of 2. Even more surprising was the discovery in 1940 by Alvarez and Bloch [20] that the neutron had a large magnetic moment (see Eq. (1.1)). These two results remained quite mysterious for many years, and are still not perfectly understood. With the advent of the quark model, one does get a 10 to 20% description of baryon magnetic moments, but given that experiments show that very little of the proton spin is carried by the quarks, the whole spin structure of baryons remains a topic of investigation.^h It became convenient

^fHowever, it appears that the Dirac complex phase is an artifact of his second-order formalism analysis rather than a real EDM.

^gThe Dirac equation also predicts that the g -factor associated with orbital angular momentum $g_\ell = 1$.

^hA.W. Thomas claims that this crisis is resolved [21], but according to R.L. Jaffe [22] this is a minority view.

to break the magnetic moment into two pieces:

$$\mu = (1 + a) \frac{q\hbar}{2m} \quad \text{where} \quad a = \frac{g - 2}{2}. \quad (1.3)$$

The first piece, predicted by the Dirac equation and called the Dirac moment, is 1 in units of the appropriate magneton, $q\hbar/2m$. The second piece is the anomalous (Pauli) moment [23], where the dimensionless quantity a is sometimes referred to as the *anomaly*.

The development of radio frequency engineering and microwave technology during the Second World War was quickly put to use afterward in the laboratory. In 1947, motivated by measurements of the hyperfine structure in hydrogen that obtained splittings larger than expected from the Dirac theory [24–26], Schwinger [27] showed that from a theoretical viewpoint these “discrepancies can be accounted for by a small additional electron spin magnetic moment” that arises from the lowest-order radiative correction to the Dirac moment.ⁱ In his paper, Schwinger points out three important features of his new theory.

The new Hamiltonian is superior to the original one in essentially three ways: it involves the experimental electron mass, rather than the unobservable mechanical mass; an electron now interacts with the radiation field only in the presence of an external field . . . the interaction of an electron with an external field is now subject to a finite radiative correction.

In today’s language, Schwinger pointed out that one replaces the bare mass and charge with the physical (dressed) mass and charge (see Chapter 3 for additional details).

The one-loop contribution to a is shown diagrammatically in Fig. 1.1(b) and has the value $a_e = \alpha/(2\pi) \simeq 0.00116\dots$, which is independent of mass and is the same for a_μ and a_τ .

In the same year, Kusch and Foley [29] measured a_e with 4% precision, and found that the measured electron anomaly agreed well with Schwinger’s prediction. They state that: “. . . the results can be described by $g_\ell = 1$ and $g_s = 2(1.00119 \pm 0.00005)$.”^j

ⁱIn response to Nafe, *et al.* [24], Breit [28] conjectured that this discrepancy could be explained by the presence of a small Pauli moment. It’s not clear whether this paper influenced Schwinger’s work, but in a footnote Schwinger states: “However, Breit has not correctly drawn the consequences of his empirical hypothesis.”

^jThe choice that $g_\ell = 1$ and $g_s > 2$ was guided by theoretical prejudice. The modern experiments, which confine a single electron in a Penning trap, measure g_s directly and fully justify this assumption.

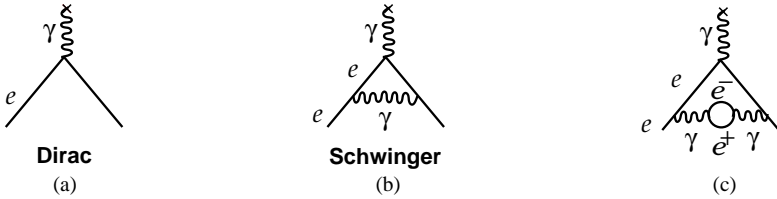


Fig. 1.1. The Feynman graphs for: (a) $g = 2$; (b) the lowest-order radiative correction first calculated by Schwinger; and (c) the vacuum polarization contribution, which is one of five fourth-order, $(\alpha/\pi)^2$, terms.

In the intervening time since the Kusch and Foley paper, many improvements have been made in the precision of the electron anomaly [30–32], as well as in the theory (see Chapters 3 and 4). Most recently, a_e has been measured to a relative precision of 0.24 ppb (parts per billion) [32], and the comparison with theory is limited by the knowledge of the fine-structure constant, α . See Chapters 3 and 6 for the most recent theory and experimental values of a_e .

The ability to calculate the higher-order QED contributions to the anomaly has gone well beyond what could have been imagined by the inventors. In response to a question about how the QED pioneers viewed the theory Freeman Dyson said [33]:

The main point was that all of us who put QED together, including especially Feynman, considered it a jerry-built and provisional structure which would either collapse or be replaced by something more permanent within a few years. So I find it amazing that it has lasted for fifty years and still agrees with experiments to twelve significant figures. It seems that Nature is telling us something. Perhaps she is telling us that she loves sloppiness.

The muon anomaly has been measured to a precision of 0.54 ppm [34]. Naively, this level of precision would seem to limit the physics reach of the muon anomaly when compared to that of the electron. However, since the relative sensitivity of the anomaly to higher mass scales goes as $(m_\mu/m_e)^2 \simeq 43,000$, the muon anomaly has measurable sensitivity up to the several hundred GeV scale, as discussed in the Chapter 2.

1.3. The Search for Electric Dipole Moments

Dirac [17] discovered an electric dipole moment (EDM) term in his relativistic electron theory. Like the magnetic dipole moment, the electric dipole

moment must be along the spin. We can write an expression similar to Eq. (1.1),

$$\vec{d} = \eta \left(\frac{q}{2mc} \right) \vec{s}, \quad (1.4)$$

where η is a dimensionless constant that is analogous to g in Eq. (1.1). While magnetic dipole moments (MDMs) are a natural property of charged particles with spin, electric dipole moments (EDMs) are forbidden both by parity and by time reversal symmetry.

The search for an EDM dates back to the suggestion of Purcell and Ramsey [35] in 1950, well in advance of the paper by Lee and Yang [36], that a measurement of the neutron EDM would be a good way to search for parity violation in the nuclear force. An experiment was mounted at Oak Ridge [37] soon thereafter which placed a limit on the neutron EDM of $d_n < 5 \times 10^{-20}$ e-cm, although the result was not published until after the discovery of parity violation.

Once parity violation was established, Landau [38] and Ramsey [39] pointed out that an EDM would violate both P and T symmetries. This can be seen by examining the Hamiltonian for a spin one-half particle in the presence of both an electric and magnetic field,

$$\mathcal{H} = -\vec{\mu} \cdot \vec{B} - \vec{d} \cdot \vec{E}. \quad (1.5)$$

The transformation properties of \vec{E} , \vec{B} , $\vec{\mu}$ and \vec{d} are given in Table 1.1, and we see that while $\vec{\mu} \cdot \vec{B}$ is even under all three symmetries, $\vec{d} \cdot \vec{E}$ is odd under both P and T . Thus the existence of an EDM implies that both P and T are not good symmetries of the interaction Hamiltonian, Eq. (1.5). In the context of CPT symmetry, an EDM implies CP violation.

Table 1.1. Transformation properties of the magnetic and electric fields and dipole moments.

	\vec{E}	\vec{B}	$\vec{\mu}$ or \vec{d}
P	-	+	+
C	-	-	-
T	+	-	-

The Standard Model value for the electron (muon) EDM is $\leq 10^{-38}$ e-cm ($\leq 2 \times 10^{-36}$ e-cm), well beyond the reach of experiments (which are at the 1.6×10^{-27} (1.8×10^{-19}) e-cm level). Likewise, the Standard-Model

value for the neutron is 10^{-32} e-cm, with the present experimental limit of 2.9×10^{-26} e-cm.

Concerning these symmetries, Ramsey states [39]:

However, it should be emphasized that while such arguments are appealing from the point of view of symmetry, they are not necessarily valid. Ultimately the validity of all such symmetry arguments must rest on experiment.

Fortunately this advice has been followed by many experimental investigators during the intervening 50 years. Today the searches for a (*CP* violating) permanent electric dipole moment of the electron, neutron, and of an atomic nucleus have become an important part of the search for physics beyond the Standard Model. Since the Standard Model *CP* violation observed in the neutral kaon and B-meson systems is inadequate to explain the predominance of matter over antimatter in the universe, the search for new sources of *CP* violation beyond that embodied in the CKM formalism takes on a certain urgency. These searches, along with the relevant theoretical framework, form a major portion of this volume.

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