

Chapter 1

Noncovariant Gauges at Zero and Nonzero Temperature

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I review the formalism for gauge-field perturbation theory in noncovariant gauges, particularly the temporal axial gauge. I show that, even at zero temperature, there are complications and it is not known whether a formalism exists for handling these that is correct for all calculations. For thermal field theory in the imaginary time formalism, there are different difficulties, whose solution so far is known only up to lowest order in the coupling g .

1.1. Introduction

Noncovariant gauges in which the gauge field A_μ is defined to satisfy

$$n \cdot A = n^\mu A_\mu = 0 \quad (1.1)$$

where n^μ is some fixed 4-vector of unit length, have been pioneered by Wolfgang Kummer¹ and later widely used in nonabelian gauge theories.² This is partly because Faddeev-Popov ghosts are believed to decouple in such gauges,³ and partly because a suitable choice of the direction of n^μ often seems to simplify calculations. In thermal field theory, in particular, it is natural to choose, in the rest frame of the ensemble under study,

$$n^\mu = (1, 0, 0, 0) \quad (1.2)$$

The corresponding gauge is called the temporal gauge.

However, there are severe complications with calculating in noncovariant gauges. Indeed, it is not even clear that a consistent universal calculation scheme exists, even for ordinary perturbation theory at zero temperature. The basic problem is that a naive derivation of the gauge-field propagator

gives

$$D_{\mu\nu}(k) = \left[-g_{\mu\nu} + \frac{k_\mu n_\nu + n_\mu k_\nu}{n \cdot k} - n^2 \frac{k_\mu k_\nu}{(n \cdot k)^2} \right] \frac{1}{k^2 + i\epsilon} \quad (1.3)$$

and we do not know how to handle the double pole at $n \cdot k = 0$. Traditionally, it was assumed⁴ that it is correct to apply a principal-value prescription. This gives the right answer for simple calculations, but not when there are Feynman graphs where two gauge-field lines carry the same momentum k , so that $D_{\mu\nu}(k)$ has to be squared. This was shown^{5,6} by comparing the calculation in Feynman gauge and in temporal gauge of a Wilson loop in next-to-leading order, where the sensitive graph is that shown in figure 1.

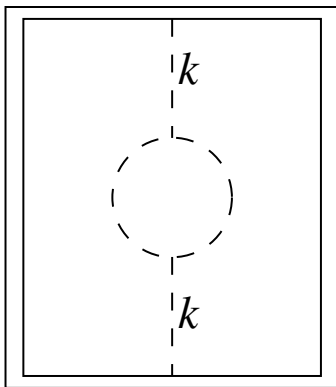


Figure 1.1 Graph in the calculation of a Wilson loop in which a propagator of momentum k is squared

Here I will first review progress, or the lack of it, with this difficulty. I then go on to discuss how things are even more complicated with thermal field theory in the temporal gauge.

1.2. Temporal gauge at finite temperature

The propagator (1.3) satisfies

$$n^\mu D_{\mu\nu}(k) = 0 = D_{\mu\nu}(k) n^\nu \quad (1.4)$$

In particular, in the temporal gauge only the spacelike components are nonzero:

$$D_{ij}(k) = \frac{1}{k^2 + i\epsilon} \left(\delta_{ij} - \frac{k_i k_j}{k_0^2} \right) \quad (1.5)$$

The apparent property that

$$D_{00} = D_{0i} = D_{i0} = 0 \tag{1.6}$$

together with the absence of ghosts, is why the gauge is supposed to be useful: it makes calculations much simpler. However, it is not clear that (1.6) can be assumed to be true.

Certainly, non-leading-order calculations are usually delicate. In my calculation of the Wilson loop,⁶ having established that the principal-value prescription

$$\frac{1}{k_0^2} \longrightarrow \frac{1}{2} \left(\frac{1}{(k_0 + i\eta)^2} + \frac{1}{(k_0 - i\eta)^2} \right) \tag{1.7}$$

gave the wrong answer, I assumed a prescription that was fairly similar:

$$\frac{1}{k_0^2} \longrightarrow \frac{1}{k_0^2 + \eta^2} \tag{1.8}$$

I found that individual graphs diverged as powers of $1/\eta^2$ when $\eta \rightarrow 0$. They must be calculated carefully. The limit must be taken only right at the end of the calculation, after Feynman's $\epsilon \rightarrow 0$. In particular, powers of η from the loop integration for the graph in figure 1.1 must be retained, as they multiply powers of $1/\eta$ coming from the k integration. In the end, all the divergences cancel and the same answer is obtained as in Feynman gauge.

The prescription (1.8) was chosen so as to give the correct answer for this calculation, but there is no derivation of it from first principles and therefore no guarantee that the same will be true for all other calculations.

Attempts to derive a prescription typically start with a gauge close to the one that is wanted. For example^{7,8} one might start by imposing

$$A_0 = \eta \frac{1}{\partial_3} \frac{1}{\partial^2} \partial \cdot \mathbf{A} = \eta \frac{1}{\partial_3} \frac{1}{\partial^2} \partial \cdot \mathbf{A}^L \tag{1.9}$$

with \mathbf{A}^L the longitudinal field,

$$A_i^L = \partial_i \frac{1}{\partial^2} \partial \cdot \mathbf{A}$$

and let $\eta \rightarrow 0$ at the end of any calculation. One can then eliminate A_0 from the Lagrangian, calculate the Hamiltonian and write down the corresponding equations of motion. Eliminating A_0 results in the loss of an equation of motion, which turns out to be Gauss's law. When the gauge-field coupling g is switched off, this is just $\partial \cdot \dot{\mathbf{A}} = 0$. This cannot

be imposed as an operator condition. Instead one requires that its matrix element vanishes:

$$\langle P' | \partial \cdot \dot{\mathbf{A}} | P \rangle = 0 \quad (1.10)$$

Here P and P' denote any pair of physical states, and for setting up perturbation theory it is sufficient to consider asymptotic states. The equations of motion ensure that, if this constraint is imposed at any time, say $t = 0$, it remains satisfied at other times. The constraint (1.10) enables one to pick a complete set of physical states, not uniquely, but most simply one might choose those states that contain no longitudinal gauge particles.

Canonical quantisation then leads to the prescription

$$\frac{1}{k_0^2} \longrightarrow \left(\frac{1}{k_0 + i\eta/k_3} \right)^2 \quad (1.11)$$

This is the so-called Vienna prescription.⁹ It again gives the correct Wilson loop to next-to-leading order,¹⁰ but again one cannot be sure how generally it may be applied. The problem is that, until $\eta \rightarrow 0$, one cannot be sure that it is valid to neglect the elements D_{00}, D_{0i}, D_{i0} of the gauge-field propagator, nor indeed the ghosts.

The conclusion then is that, attractive as noncovariant gauges may seem to be, using them for calculations is, to say the least, problematical even at zero temperature.

1.3. Thermal field theory in temporal gauge

Thermal field theory is formulated by starting with the grand partition function

$$Z = \sum_i \langle i | e^{-(H-\mu N)/T} | i \rangle \quad (1.12)$$

from which nearly all (though not all¹¹) the interesting properties of the system under study may be calculated. Here T is the temperature, and we use units in which Boltzmann's constant $k_B = 1$. The system's Hamiltonian is H and N is some conserved quantum number, such as baryon number, with μ the corresponding chemical potential. The states $|i\rangle$ are a complete orthonormal set of physical states of the system. In scalar field theory all states are physical and so

$$Z = \text{tr} e^{-(H-\mu N)/T} \quad (1.13)$$

which is invariant under changes in the choice of orthonormal basis of states. In the case of gauge theories there are unphysical states, for example longitudinally-polarised photons or gluons, which must be excluded from the summation in (1.12). So then

$$Z = \text{tr } \mathbb{P} e^{-(H-\mu N)/T} \tag{1.14}$$

where \mathbb{P} is a projection operator onto physical states. The presence of \mathbb{P} can make things more complicated.

For scalar field theory in the so-called imaginary-time formalism of thermal field theory, the Feynman rules are just as at zero temperature, except that round each loop of a graph the usual loop-momentum integration undergoes the replacement

$$\int \frac{d^4k}{(2\pi)^4} \rightarrow iT \sum_n \int \frac{d^3k}{(2\pi)^3} \tag{1.15}$$

Here the summation is over discrete values

$$k_0 = n\pi T \quad n = 0, \pm 2, \pm 4, \dots \tag{1.16}$$

which has the consequence that the propagator is periodic in the time t :

$$D(t, \mathbf{k}) = D(t - i/T, \mathbf{k}) \tag{1.17}$$

Were this rule to apply also to a gauge-field theory in the temporal gauge, there would obviously be a difficulty, since the summation would include a contribution from $n = 0$, that is $k_0 = 0$, where the zero-temperature propagator has a double pole.

The transverse gauge-field propagator does behave similarly to a scalar field, with

$$[D_n^T(\mathbf{k})]_{ij} = - \left(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2} \right) \frac{1}{\pi^2 n^2 T^2 + \mathbf{k}^2} \quad n = 0, \pm 2, \pm 4, \dots \tag{1.18}$$

and it has the periodicity (1.17). However,¹² this is not the case for the the longitudinal field: because of the presence of the projection operator in (1.14), canonical quantisation results in a longitudinal propagator $[D^L(t, \mathbf{k})]_{ij}$ that is not periodic in t . But there is a simplification to compensate for this complication: $[D_n^L(\mathbf{k})]_{ij}$ does not have a double pole at $n = 0$. Indeed, it is regular there, but n is not restricted to even values: $[D_n^L(\mathbf{k})]_{ij} = (k_i k_j / \mathbf{k}^2) D_n^L(k_3)$, with

$$D_n^L(k_3) = \begin{cases} 1/(4T^2) & n = 0 \\ -i\epsilon(k_3)/(2\pi n T^2) & n \text{ even} \\ -1/(\pi^2 n^2 T^2) + i\epsilon(k_3)/(2\pi n T^2) & n \text{ odd} \end{cases} \tag{1.19}$$

The non-vanishing of $[D_n^L(\mathbf{k})]_{ij}$ for odd n means that it is easier not to work with $[D_n(\mathbf{k})]_{ij}$ and perform summations over the various n associated with the different lines in a Feynman graph, but instead to work with propagators $[D^L(t, \mathbf{k})]_{ij}$ and integrate over the times associated with the various vertices.

Another complication that has to be taken into account is the constraint (1.10) on the physical states. At zero temperature, or when one is working in the real-time thermal formalism, one uses asymptotic states and this constraint is sufficient. But the imaginary-time formalism rather uses interaction-picture states and so the Gauss operator is now

$$G^a(t, \mathbf{x}) = \partial \cdot \dot{\mathbf{A}}^a(t, \mathbf{x}) - g f^{abc} \mathbf{A}^b(t, \mathbf{x}) \cdot \dot{\mathbf{A}}^c(t, \mathbf{x}) \quad (1.20)$$

and it turns out¹² that one needs to impose a set of constraints

$$\langle P' | \prod_{i=1}^N G^{a_i}(0, \mathbf{x}) | P \rangle = 0 \quad N = 1, 2, \dots \quad (1.21)$$

As one increases the accuracy of one's calculation to higher powers of g , one needs to go up to higher and higher values of N . The physical states can no longer be taken as those with purely-transverse gauge particles. The solution of (1.21) is known only for low-order calculations for which it is sufficient to go up to $N = 2$.

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