Preface

Sound propagation in the ocean over hundreds and thousands kilometers is possible due to the existence of a natural refractive waveguide called the underwater sound channel (USC) [Flatté *et al.* (1979); Brekhovskikh and Lysanov (1991); Jensen *et al.* (1994)]. The vertical sound speed profile in a deep ocean usually has a minimum at a depth of about 1 km. Therefore, part of the sound energy is captured within the water bulk which prevents it from the interaction with the lossy bottom. Experiments on the long range sound propagation are usually performed at carrier frequencies of order 100 Hz. The dissipation of sound waves in sea water at these frequencies is comparatively small, only a few dB per 1000 km, and signals can be registered with sufficiently high signal-to-noise ratios even at ranges exceeding 10 000 km [Munk *et al.* (1994)].

The theoretical description of sound fields in the USC is based on the standard wave equation [Ahluwalia and Keller (1977); Brekhovskikh and Lysanov (1991); Jensen *et al.* (1994); Brekhovskikh and Godin (1999)]. The problem is linear, and usually it is treated using the same methods which are traditionally employed for analyzing wave fields in other waveguide media. Among them, the method of geometrical optics, or ray approach, is of particular importance [Flatté *et al.* (1979); Kravtsov and Orlov (1990); Brekhovskikh and Lysanov (1991); Brekhovskikh and Godin (1999)]. Since the time when the underwater acoustics became a quantitative science (it happened during World War II), this method has been one of the most efficient and frequently applied tools for a study of sound fields in the ocean. Significant advantages of the ray approach are that it provides a deep insight into the basic mechanisms of signal propagation in a complicated environment and is well suited for deriving analytical estimates.

Extensive theoretical and experimental studies of long range sound propagation in the ocean have been carried out for sixty years. Results obtained in this field are presented in many publications (see, e.g., monographs [De Santo (1979); Brekhovskikh (1980); Brekhovskikh and Lysanov (1991); Jensen *et al.* (1994)] and references therein). Already in the middle 1980s this topic was considered well understood. However, in the last two decades it turned out that there exists a factor, earlier not taken into consideration, which to a significant extent determines the structure of the wave field at long ranges.

This is the phenomenon of ray chaos whose significance was realized from the beginning of the 1980s [Abdullaev and Zaslavsky (1981, 1983, 1984); Palmer *et al.* (1988); Abdullaev and Zaslavsky (1991); Smith *et al.* (1992)]. In the ray approximation, the underwater sound propagation can be modelled by a Hamiltonian system representing a nonlinear oscillator driven by a weak nonstationary external perturbation. A range-independent background sound speed profile plays the role of an unperturbed potential on which a range-dependent perturbation of the sound speed along the waveguide, that can be caused by internal waves, mesoscale eddies, ocean fronts or something else, is superimposed. In the first papers on ray motion in range-dependent waveguides [Abdullaev and Zaslavsky (1981, 1983, 1984)], chaotic instability of rays has been found in idealized models. The point is that the ray path in a deterministic inhomogeneous medium behaves like a random process. Chaotic rays are very unstable. The initially close paths diverge exponentially with range r so that the difference between their vertical coordinates grows as $\exp(\nu r)$, where ν is the Lyapunov exponent. A typical value of the Lyapunov exponent in a deep sea varies from $1/300\ \text{km}^{-1}$ to $1/100\ \text{km}^{-1}$ [Beron-Vera *et al.* (2003)]. At ranges of order 1000 km, or at megameter ranges in terms of ocean acoustics, chaos is well developed and should be taken into account when describing the sound fields. Now the study of ray chaos and its manifestations at a finite wavelength, the so-called wave chaos, is a branch of the theory of long range sound propagation in the ocean [Worcester and Spindel (2005)]. However, the results obtained in this field are scattered over many journal publications and may be unknown even to specialists working in related fields. The objective of this book is to discuss the methods and concepts used in the theory of ray and wave chaos and present a comprehensive survey of the state of the art in this branch of ocean acoustics. Applications of the general approaches are illustrated by considering a number of particular problems which were solved with the participation of the authors.

The phenomena of ray and wave chaos have the well-known prototypes in mechanics called the dynamical and quantum chaos, respectively [Zaslavsky (1981); Gutzwiller (1990); Reichl (1992)]. The point is that the ray trajectory in an inhomogeneous waveguide is governed by the same Hamilton equations as a nonlinear oscillator driven by a nonstationary external force. Typically, the oscillator exhibits a quasi-random behavior [Sagdeev *et al.* (1988); Lichtenberg and Lieberman (1992)]. Analysis of the phase space of this oscillator is a classical problem in the theory of dynamical chaos. The theory of quantum chaos deals with mechanical systems whose classical analogues demonstrate chaotic motion. The nonlinear oscillator subjected to a nonstationary external force falls in this category. Its wave function obeys the Schrödinger equation that has the same form as the standard parabolic equation describing the sound field in the USC.

The above analogy with the nonlinear oscillator allows one to use the methods of dynamical and quantum chaos for a study of acoustic fields. However, despite the coincidence of the basic equations, formulations of the problems in mechanics and acoustics can be very different. For example, the parameter of a sound ray most extensively studied in underwater acoustics is a ray travel time, that is, the arrival time of a sound pulse coming to the receiver through a particular ray path [Simmen *et al.* (1997); Brown and Viechnicki (1998); Smirnov *et al.* (2001); Virovlyansky (2003); Beron-Vera *et al.* (2003); Makarov *et al.* (2004); Smirnov *et al.* (2005b)]. Ray travel times are used as the main observables in different schemes of acoustic monitoring of the ocean temperature fields [Munk and Wunsch (1979); Spiesberger and Metzger (1991); Worcester *et al.* (1999); Dushaw (1999)]. An analogue of this characteristic in mechanics is Hamilton's principal function, or mechanical action, i.e., a characteristic of dynamical systems that is typically not measured experimentally and therefore has not received much attention in the studies of dynamical chaos.

In a deep ocean, it is generally believed that the sound speed inhomogeneities induced by internal waves are the dominant cause of the acoustic fluctuations at long ranges [Flatté *et al.* (1979)]. An environmental model widely used in the studies of chaotic ray motion is a superposition of a smooth range-independent, or slowly varying with range, sound speed field and a weak range-dependent perturbation induced by internal waves. First works on ray chaos in ocean acoustics have considered perturbations with periodic range dependencies. Although such waveguide models look rather artificial, they demonstrate a number of general properties of the wave field observed in more realistic environments. Interest in range-periodic waveguides is caused by the fact that they can be examined by direct application of the methods borrowed from the theory of dynamical and quantum chaos, such, for example, as the method of Poincaré map. Later on, more realistic waveguide models, in which the sound speed perturbation represented the realization of a random field, were explored. In this book models of both types will be considered.

We are interested in long-range propagation of sound in the ocean over distances of the order of hundreds or even thousands of kilometers. We begin Chapter 1 with a brief description of a guided sound propagation in the ocean in Sec. 1.1.

In Sec. 1.2, we briefly describe the main features of the environmental model used throughout the book. The basic equations determining the sound field are also presented. The linear wave equation is the main one of them. In the case of a monochromatic wave field, it reduces to the Helmholtz equation. Our analysis of the ray and wave chaos in this book is based on the standard parabolic equation which follows from the Helmholtz equation in small-angle approximation.

The geometrical optics approach is introduced in Sec. 1.3. The ray dynamics is described in the scope of Hamiltonian formalism expressed in terms of position-momentum and action-angle variables. The main attention is paid to the action-angle variables which are rarely used in ocean acoustics and therefore may be unknown to the readers.

Section 1.4 is devoted to a discussion of the ray travel times in a range-independent waveguide. Although in this case all the rays are regular, their travel times exhibit some general properties which will be observed in the presence of a perturbation giving rise to ray chaos, as well. We consider a distribution of ray arrivals in the time–depth plane. This is the so-called timefront, a characteristic of the ray travel time distribution traditionally used in underwater acoustics. It is shown that there exist surprisingly simple analytical estimates relating the difference between the ray travel times and the action variables of ray trajectories. These estimates give a quantitative description of the timefront structure and its evolution with range.

The ocean is a complicated acoustic medium with a number of inhomogeneities of different scales. The main factors of the ocean variability influencing sound propagation, namely, internal and Rossby waves, fronts, currents, streamers, and eddies, are briefly discussed in Sec. 1.5. The acoustic tomography of the ocean means extracting information about the state of the ocean from measured acoustic data. The Munk–Wunsch scheme of a reconstruction of the ocean state is described in Sec. 1.6.

Section 1.7 is devoted to a review of important experiments on long-range sound propagation. The first one was conducted in 1960 with explosive charges when the sounds were detected by hydrophones at the distance of about 20 000 km. Modern experiments use sources generating coherent acoustic signals. Some results of the Heard Island Feasibility Test (1991) on long-range sound propagation with a base of 18 000 km in deep waters of the Atlantic, Pacific, and Indian Oceans are discussed. We review as well experiments with acoustic sources mounted into the downsloped bottom in shallow waters in Kaneohe Bay (Oahu island) from 1983 to 1989 with a base of 3709 km. Other experiments with a smaller base were conducted in 1999 and 2006 with bottom-mounted sources in shallow waters of the Sea of Japan nearby cape Shultz. A significant role in studies of ray and wave chaos was played by the Acoustic Engineering Test conducted in 1994. The Alternate Source Test in 1996 provided an opportunity to compare wave-field properties at different signal frequencies: 28, 75, and 84 Hz. We end the chapter with a brief review of aims and some results of the international project, known as Acoustic Thermometry of Ocean Climate, that combined remote acoustic and altimetric measurements along with direct temperature measurements to estimate changes in oceanic heat storage.

Chapter 2 is devoted entirely to the analysis of ray chaos in the USC with periodic range dependence. This highly idealized environmental model allows one to realize the basic mechanismes responsible for the emergence of ray chaos. We begin this chapter with a brief review of dynamics and chaos in Hamiltonian systems and introduce the key concepts of invariant tori, stochastic layers, chaotic mixing, local instability, stickiness, normal and anomalous diffusion and transport in the phase space (Sec. 2.1). In Sec. 2.2 the characteristic of instability of rays in an USC, the finite-range Lyapunov exponent, is introduced. The essential feature of range-periodic waveguides is the so-called ray-medium resonance analyzed in Sec. 2.3. Ray-medium resonance is the analogue of nonlinear resonance in Hamiltonian mechanics. The phase portrait of the ray system in such a waveguide, constructed using the Poincare map, has the same form as that of the nonlinear oscillator driven by a periodic external force. There can be two scenarios of the onset of chaos. If a sound-speed inhomogeneity varies smoothly with depth, then remarkable ray chaos emerges due to overlapping of neighbouring ray-medium resonances. This scenario is studied in Sec. 2.4. Overlapping results in the emergence of a chaotic sea in the phase space, with islands of stability within. In this case, ray instability is governed by the so-called stability parameter which is determined in the form of a background sound-speed profile.

In Sec. 2.5 we consider ray motion in a more complicated case where a weak range-periodic perturbation of sound speed is a rapidly oscillating function of depth. Such a perturbation mimics a fine structure produced by high modes of internal waves. Under these conditions, ray chaos arises due to scattering on vertical resonances whose properties are studied in Sec. 2.5. Instability is governed by the ratio of vertical and horizontal length scales of a sound-speed perturbation. We study interrelation between two scenarios of the onset of chaos and show that they are closely related to each other, except for the case when vertical resonance causes bifurcations of periodic ray orbits.

In Sec. 2.6 we consider the ray travel times. The emphasis is on the analysis of the relationship between the ray travel time and the topology (shape) of the ray trajectory characterized by the ray identifier $\pm M$, where $\pm$ determines the sign of the launch angle and M is the number of ray turning points. It appears that the rays with equal identifiers have close travel times. Therefore, the chaotic rays, arriving at a fixed observation point with equal identifiers, form compact clusters. Arrivals of all the rays with a given identifier are located within a compact area of the timefront. Some of these areas are isolated in the sense that they do not overlap with the areas corresponding to different identifiers. A specific property of range-periodic waveguide models is the coexistence of regular and chaotic rays. Ray tracing demonstrates that this property may manifest itself in the appearance of a gap in the timefront. Besides the gap, a focusing of ray travel times was found to exist within a comparatively small temporal interval preceding the gap. In the numerical simulation of sound pulse propagation, this phenomenon manifests itself in the appearance of a bright spot in the distribution of acoustic energy in the time–depth plane. It was shown that the focusing effect is a signature of the stickiness, i.e., the presence of such parts of a chaotic trajectory where the latter exhibits an almost regular behavior.

In Chap. 3 we discuss the wave chaos phenomenon, that is, the manifestation of chaotic ray motion at a finite wavelength, in a waveguide with a periodic range dependence. We begin this chapter by discussing a close connection between the wave chaos and quantum chaos. Many insights and methods of quantum chaos can be and have been applied in underwater acoustics. In Sec. 3.1, a few key concepts of the quantum chaos theory, that seem to be important in the underwater sound propagation, are discussed: the Ehrenfest break distance, fidelity or overlap of wave fields, dynamical localization, and the semiclassical propagator.

In Sec. 3.2 we introduce the modal representation of the wave field and examine the ray-mode relations. A ray-based approach for the description of normal mode amplitudes in a range-dependent waveguide is derived. In the scope of this approach, the mode amplitude is expressed through parameters of the ray paths. It is shown that the normal mode is formed by contributions from rays whose action variables at a given observation range satisfy the quantization rule. This approach is convenient for analysis of manifestations of the ray dynamics (both, regular and chaotic) in range variations of the mode amplitudes. In this section, it is also shown that the description of the wave field in a range periodic waveguide can be greatly simplified using the Floquet theory. Then the field is decomposed into a sum of terms with regular and relatively simple range dependence which we call the Floquet modes. They are complete analogues of the Floquet states in quantum mechanics (quantum states with fixed quasi-energies).

Section 3.3 deals with manifestations of the ray chaos in the modal structure of the wave field. At first, we consider the decomposition of the wave field into the normal modes of an unperturbed (range-independent) waveguide. It is shown that the normal mode dynamics is to a significant extent determined by the mode-medium resonance. This phenomenon — its prototype in quantum mechanics is called the nonlinear quantum resonance — is interpreted as a manifestation of the ray-medium resonance at a finite wavelength. We also consider an alternative representation of the wave field as a decomposition into the sum of the Floquet modes. We use the Wigner representation of the wave field to link the Floquet modes and the ray distribution in the phase space. It is shown that the characteristic features of the phase portrait of the ray system, namely stable islands and a chaotic sea, manifest themselves in the Wigner functions of the Floquet modes. A similar effect was observed in the theory of quantum chaos. We also consider an analogue of another effect from the theory of quantum chaos — the scarring, or an increase of the wave intensity in the vicinity of an unstable periodic ray path.

Typically, horizontal gradients of the variability in the deep ocean are $10^2 \div 10^3$ times weaker than the vertical ones which are caused mainly by high modes of internal waves. The applicability condition of the ray approximation is not met for the low-frequency sound signals propagating over long distances in the environment with a vertical variability even if the latter is small. So we arrive at a conflict between the ray and wave description: the vertical resonance produces strong ray chaos as it is shown in Sec. 2.5, whereas a small-scale vertical perturbation should be ineffective in the wave-field propagation. In Sec. 3.4, we analyze this conflict from the viewpoint of the modal representation of a wavefield. Another way we follow is the analysis of the Floquet modes. We track how discrepancies between ray and wave descriptions increase with decreasing of the vertical scale of a sound-speed perturbation.

Chapter 4 is devoted to studying the wave fields in a realistic environmental model with sound speed fluctuations induced by random internal waves. The wave fields in such a medium are studied from two different and complementary viewpoints by the theory of wave propagation in random media and the theory of ray and wave chaos. The principal difference between these two approaches is explained in the introductory part.

In Sec. 4.1, we construct a statistical description of the chaotic ray dynamics in the environmental model under consideration. The use of the action-angle variables greatly simplifies the analysis. It turns out that the range dependence of the action variable can be modeled by a random Wiener process representing the simplest mathematical model of diffusion. Then the angle variable is modeled by an integral of this process. In this approximation (we call it the Wiener process approximation), surprisingly simple analytical estimates for statistical characteristics of the ray parameters are derived. It is shown how this result can be applied for estimating the sound intensity smoothed over the depth with a sufficiently large smoothing scale. We also describe a rather general method to find regions of stability in the phase space of Hamiltonian systems driven by a weak noise with an arbitrary spectrum. This method is based on a specific Poincaré map. Physical manifestations of these regions of stability are coherent clusters — bunches of sound rays propagating coherently over long distances in an underwater waveguide through a randomly fluctuating ocean.

Section 4.2 is devoted to studying the ray travel times. It is demonstrated that some general properties observed in the range-independent and range-periodic waveguides remain valid in the presence of random internal waves. The most important of them is the effect of clusterization discussed in Sec. 2.6. This phenomenon sheds new light on the surprising stability of the early portion of the arrival pattern observed in both numerical simulations and field experiments. The quantitative explanation of this effect is obtained by combining an approximate analytical relation for the difference between travel times of chaotic and regular rays and stochastic ray theory derived in Sec. 4.1.

In Sec. 4.3, we explore the influence of the random perturbation on the modal structure of the wave field. This problem is solved by combining two results: (i) relations expressing mode amplitudes through the parameters of ray paths from Sec. 3.2 and (ii) stochastic ray theory from Sec. 4.1. For a monochromatic wave field, a simple analytical estimate is obtained for a coarse-grained distribution of acoustic energy between normal modes. Significant attention is paid to a study of the mode pulses, that is, sound pulses carried by individual modes. Analytical estimates for the spread of a mode pulse and the bias of its mean travel time in the presence of internal waves are derived.

In Sec. 4.4, we consider a wave beam radiated by a vertical antenna placed in an USC. We begin with deriving a ray-based formalism for the description of the beam propagation. First, this approach is used to find a beam whose mean width in a range-independent waveguide is less than that of any other beam with the same initial width. Then the ray-based approach, combined with the ray-mode relations, is applied for studying the modal structure of the beam in the fluctuating environment. The spread of the modal spectrum due to wave scattering at the sound speed fluctuations is estimated.

In Sec. 4.5, we study the delays of the sound pulses caused by a synoptic eddy (an example of the mesoscale inhomogeneity) in the presence of random internal waves. These delays are used as input parameters in the method of ocean acoustic tomography for reconstructing the temperature field variation caused by the eddy. The solution of the inverse problem is considerably hindered by the fact that in the case of signal registration by a point receiver, only the sound pulses propagating along steep rays can be resolved. We show that the use of a vertical antenna allows one to loosen this restriction. An appropriate space–time processing procedure is proposed. The procedure is based on the presence of such characteristics of the ray arrival distribution in the time–depth plane that remain stable even under conditions of ray chaos.

In Chap. 5 we give a brief description of those basic concepts and methods in the Hamilton chaos theory that have been used in this book. It is a kind of Glossary with a large number of illustrations, facilitating the understanding of abstract theoretical concepts, and short articles ranged in the alphabetic order. We hope that simple explanations of the important mathematical issues make reading easier. Throughout the text of the book, we give references to the Glossary in parenthesis (see the corresponding article in the Glossary) just after the term or the notion which are needed to be explained in more detail. Technical details of derivation of some equations and their solutions can be found in the appendicies.

In Appendix A, we describe a few analytic models of the sound-speed profile in an USC including the Munk profile, commonly used for modeling variations in the velocity of sound in real environment, and more artificial profiles — linear, bilinear, and biexponential ones used to provide analytic derivation of the action and angle variables. Details of derivation of ray equations and solution to the transport equation are given in Appendices B and C. In Appendix D, a stationary phase technique is described.

The book is intended for postgraduate students and researches, physicists, oceanographers, and applied mathematicians, who are interested in ocean acoustics and in the applications of ideas and methods of ray and wave chaos to other disciplines.

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