

Contents

<i>Preface</i>	vii
1. Introduction	1
1.1 Reality and Philosophy	1
1.2 Summary of the Main Claims	2
1.2.1 Critique of the frequency and subjective theories .	3
1.2.2 Scientific laws of probability	6
1.2.3 Statistics and philosophy	8
1.3 Historical and Social Context	10
1.4 Disclaimers	12
2. Main Philosophies of Probability	15
2.1 The Classical Theory	16
2.2 The Logical Theory	16
2.3 The Propensity Theory	18
2.4 The Subjective Theory	18
2.4.1 Interpreting subjectivity	19
2.4.2 Verification of probabilistic statements	20
2.4.3 Subjectivity as an escape from the shackles of ver- ification	22
2.4.4 The Dutch book argument	23
2.4.5 The axiomatic system	25
2.4.6 Identification of probabilities and decisions	26
2.4.7 The Bayes theorem	27
2.5 The Frequency Theory	27
2.6 Summary of Philosophical Theories of Probability	29

2.7	Incompleteness—The Universal Malady	30
2.8	Popular Philosophy	31
3.	The Science of Probability	35
3.1	Interpretation of (L1)-(L5)	36
3.2	A Philosophy of Probability and Scientific Verification of (L1)-(L5)	40
3.3	Predictions	43
3.4	Is Symmetry Objective?	53
3.5	Symmetry is Relative	54
3.6	Moderation is Golden	55
3.6.1	A sixth law?	57
3.7	Circularity in Science and Philosophy	58
3.8	Applications of (L1)-(L5): Some Examples	59
3.8.1	Poisson process	60
3.8.2	Laws (L1)-(L5) as a basis for statistics	60
3.8.3	Long run frequencies and (L1)-(L5)	61
3.8.4	Life on Mars	62
3.9	Symmetry and Data	64
3.10	Probability of a Single Event	65
3.11	On Events that Belong to Two Sequences	66
3.12	Deformed Coins	67
3.13	Symmetry and Theories of Probability	68
3.14	Are Coin Tosses i.i.d. or Exchangeable?	70
3.15	Physical and Epistemic Probabilities	71
3.16	Countable Additivity	72
3.17	Quantum Mechanics	74
4.	Decision Making	77
4.1	Decision Making in the Context of (L1)-(L5)	77
4.1.1	Maximization of expected gain	78
4.1.2	Maximization of expected gain as an axiom	80
4.1.3	Stochastic ordering of decisions	81
4.1.4	Generating predictions	83
4.1.5	A new prisoner paradox	84
4.2	Events with No Probabilities	86
4.3	Law Enforcement	88
4.4	Utility in Complex Decision Problems	90

4.4.1	Variability of utility in time	91
4.4.2	Nonlinearity of utility	91
4.4.3	Utility of non-monetary rewards	93
4.4.4	Unobservable utilities	94
4.5	Identification of Decisions and Probabilities	95
5.	The Frequency Philosophy of Probability	97
5.1	The Smoking Gun	98
5.2	Inconsistencies in von Mises' Theory	98
5.3	Collective as an Elementary Concept	100
5.4	Applications of Probability Do Not Rely on Collectives	101
5.5	Collectives in Real Life	103
5.6	Collectives and Symmetry	105
5.7	Frequency Theory and the Law of Large Numbers	106
5.8	Benefits of Imagination and Imaginary Benefits	107
5.9	Imaginary Collectives	108
5.10	Computer Simulations	109
5.11	Frequency Theory and Individual Events	110
5.12	Collectives and Populations	111
5.13	Are All i.i.d. Sequences Collectives?	112
5.14	Are Collectives i.i.d. Sequences?	113
6.	Classical Statistics	115
6.1	Confidence Intervals	115
6.2	Estimation	118
6.2.1	Estimation and (L1)-(L5)	121
6.3	Hypothesis Testing	121
6.3.1	Hypothesis tests and collectives	122
6.3.2	Hypothesis tests and the frequency interpretation of probability	123
6.3.3	Hypothesis testing and (L1)-(L5)	124
6.4	Experimental Statistics—A Missing Science	126
6.5	Hypothesis Testing and (L5)	129
6.6	Does Classical Statistics Need the Frequency Theory?	129
7.	The Subjective Philosophy of Probability	131
7.1	The Smoking Gun	131
7.2	How to Eat the Cake and Have It Too	133

7.3	The Subjective Theory of Probability is Objective	136
7.4	A Science without Empirical Content	137
7.5	The Weakest Scientific Theory Ever	139
7.5.1	Creating something out of nothing	139
7.5.2	The essence of probability	140
7.6	The Subjective Theory Does Not Imply the Bayes Theorem	143
7.6.1	Sequential decisions in statistics	144
7.6.2	Honest mistakes	145
7.6.3	The past and the future are decoupled	147
7.6.4	The Dutch book argument is static	150
7.6.5	Cohabitation with an evil demiurge	153
7.6.6	The Bayes theorem is unobservable	155
7.6.7	All statistical strategies are Bayesian	156
7.7	The Dutch Book Argument is Rejected by Bayesians	158
7.8	No Need to Collect Data	159
7.9	Empty Promises	160
7.10	The Meaning of Consistency	161
7.11	Interpreting Miracles	162
7.12	Science, Probability and Subjectivism	163
7.13	A Word with a Thousand Meanings	165
7.14	Apples and Oranges	169
7.15	Arbitrage	171
7.16	Subjective Theory and Atheism	172
7.17	Imagination and Probability	173
7.18	A Misleading Slogan	175
7.19	Axiomatic System as a Magical Trick	176
8.	Bayesian Statistics	177
8.1	Two Faces of Subjectivity	177
8.1.1	Non-existence vs. informal assessment	177
8.1.2	Are all probabilities subjective?	178
8.1.3	Conditioning vs. individuality	178
8.1.4	Nonexistent decisions	179
8.2	Elements of Bayesian Analysis	179
8.3	Models	180
8.4	Priors	181
8.4.1	Objective priors	182
8.4.2	Bayesian statistics as an iterative method	183

8.4.3	Truly subjective priors	184
8.5	Data	187
8.6	Posteriors	187
8.6.1	Non-convergence of posterior distributions	188
8.7	Bayesian Statistics and (L1)-(L5)	190
8.8	Spurious Predictions	190
8.9	Who Needs Subjectivism?	191
8.10	Preaching to the Converted	192
8.11	Constants and Random Variables	195
8.12	Criminal Trials	196
9.	Teaching Probability	199
9.1	Teaching Independence	202
9.2	Probability and Frequency	203
9.3	Undergraduate Textbooks	204
10.	Abuse of Language	207
11.	What is Science?	211
11.1	From Intuition to Science	214
11.2	Science as Service	216
11.3	Decision Making	217
11.4	Mathematical Foundations of Probability	218
11.5	Axioms versus Laws of Science	220
12.	What is Philosophy?	221
12.1	What is Philosophy of Probability?	223
12.2	Is Probability a Science?	226
12.3	Objective and Subjective Probabilities	227
12.4	Yin and Yang	228
12.5	What Exists?	229
12.6	Who Needs Philosophy?	230
13.	Concluding Remarks	231
13.1	Does Science Have to be Rational?	231
13.2	Common Elements in Frequency and Subjective Theories	232
13.3	On Peaceful Coexistence	233
13.4	Common Misconceptions	233

14. Mathematical Methods of Probability and Statistics	237
14.1 Probability	237
14.1.1 Law of Large Numbers, Central Limit Theorem and Large Deviations Principle	238
14.1.2 Exchangeability and de Finetti's theorem	239
14.2 Classical Statistics	240
14.3 Bayesian Statistics	241
14.4 Contradictory Predictions	242
15. Literature Review	245
15.1 Classics	245
15.2 Philosophy	246
15.3 Philosophy and Mathematics	246
<i>Bibliography</i>	249
<i>Index</i>	253