

Preface

In the early days of mechanics, it was not uncommon for investigators to develop mathematical methods to fit their immediate needs. Modern mathematicians and mechanicians, on the other hand, are often too specialized to understand each other at all. This book is an attempt to relate continuum mechanics with some methods of contemporary mathematics, and to present the latter in a sound mechanical context.

Of course, each topic we treat (e.g., functional analysis, the calculus of variations) has its own vast literature, and we naturally restrict ourselves to those portions needed for our purposes. But even a mathematically prepared reader can benefit from seeing how abstract notions arose from applications. For example, generalized solutions to boundary value problems are closely related to solutions obtained via the extremal principles of mechanics. S.L. Sobolev, who pioneered the use of such solutions and introduced the functional spaces that now bear his name, understood this link because he discovered generalized solutions while solving a hydromechanics problem. But later mathematicians who developed the theory did not maintain this relationship with mechanics. The same thing happened throughout mathematics; ultimately the phenomenon can be attributed to overspecialization of researchers working in the area.

Narrow specialization has led us to the point where experts in different areas cannot communicate. Even those working in closely-related subareas can fail to perceive common points between their disciplines. These points exist precisely because they were inherited from older versions of scientific theory, of course, but many specialists lack the historical perspective necessary to see this. They regard the methods they employ as essentially independent of other areas. This partially explains why we decided to start with classical mechanics, although many readers will be familiar with much

of it already. Our plan is to examine continuum mechanics as a growing discipline in which one model arises on the basis of pre-existing models, and therefore inherits certain properties while disregarding others. Size constraints prevent us from covering classical mechanics in detail, so we merely stress those points that show how continuum mechanics developed. We take a somewhat unified view of mechanics as a science rooted in the simple models of a particle and rigid body, and then move to more complicated models. We hope this will assist readers coming from various areas in mechanics, engineering, and mathematics.

Plan of the Book. The book is meant to partially bridge the gap between mathematics and mechanics. It presents a set of topics that were developed in mechanics as mathematical tools.

The purpose of Chapter 1 is to review some elementary ideas of classical mechanics. Although the objects of classical mechanics — particles and rigid bodies — are elementary, the mathematical techniques needed to describe them are not. Hence we also introduce some older portions of mathematics that were developed for application in mechanics. These include the calculus of variations, the elements of Lagrangian mechanics (which we will not use in the sequel but which is important in its own right in many applications), and basic functional analysis. Thus the first chapter is preparation for the introduction and discussion of continuum mechanics.

Equipped with the necessary tools, we can begin working with the objects of continuum mechanics in Chapters 2 and 3. Some — like the spring, string, and bar — are quite elementary. Others — like the beam, membrane, and linearly elastic body — are more advanced. Each time we start by deriving the equations that describe the object. Then we introduce well-posed formulations for practical problems. Finally, we study these formulations using mathematical methods. From Chapter 2 onward, the models considered are not those of classical mechanics. We use results and equations from classical mechanics, but apply them only after introducing certain assumptions that may be regarded as additional axioms for the deformable objects. The two most important hypotheses are the continuum assumption and the solidification principle. These permit us to apply the tools of calculus and the equations of mechanics to deformable objects. Next, we introduce other assumptions such as the smallness of deformations and linearity in the form of Hooke's law. In this way we encounter a few of the models mentioned above. While it might seem that the model of a three-dimensional elastic body should encompass all the other models,

this is not the case. The string and membrane models are independent of the model of a three-dimensional body. And the bar and beam models are based on additional hypothesis, so these can be regarded as independent of the three-dimensional model as well.

Unlike classical mechanics where the focus is on initial value problems, in continuum mechanics we encounter boundary value problems for statics and initial-boundary value problems for dynamics. The statics problems for the bar, beam, and string are described by ordinary differential equations and associated boundary conditions. Although the resulting boundary value problems for these objects are relatively simple and can be solved using elementary tools, we take the opportunity to introduce a more powerful approach that also applies to problems involving membranes and three-dimensional bodies. This is the notion of generalized solution as the point of minimum total energy for the “body-force system.” Using this mechanical principle, we convert each boundary value problem for an equation or system of equations to a problem of minimizing the total energy. In the process, we come to adopt a very different point of view. Indeed, the mechanics of a problem can dictate certain conditions that are necessary for solution but that may not be evident from a purely mathematical point of view. Moreover, questions such as solvability or uniqueness of a generalized solution will naturally lead us into the realm of functional analysis with its generalized derivatives, Sobolev spaces, and many abstract results.

Despite the abstractness of these studies, they lead to very practical outcomes. The various finite element methods, along with many other numerical methods upon which engineers have come to rely, have these generalized solutions as limits of the resulting numerical approximations. So to understand how practical numerical methods work, one should study the questions presented in this book.

For each mechanical object we cover in this book, we present a derivation of the governing equations that permit engineers to solve practical problems. We also present mathematical tools necessary to study the related boundary value problems. Many of the objects under consideration require only the calculus of functions in one or two variables. The linearly elastic body, however, is presented in the context of tensor analysis. The tensor apparatus, while not strictly required in this relatively simple case, is needed to develop the theories for the corresponding nonlinear and inelastic models. Moreover, the tensorial notation is of value in its own right because of the compact and vivid representation it provides.

In Chapter 3 we touch on some questions concerning vibration. Here the

reader can see how the abstract theorems of functional analysis yield results basic in engineering such as the discrete nature of the eigenfrequencies of certain systems while also providing a firm background for topics such as Fourier analysis and the method of separation of variables.

In this book we attempt to present only a minimal set of mathematical tools needed for the qualitative investigation of mechanics problems. We demonstrate that a knowledge of mechanics can assist the pure mathematician, while at the same time a knowledge of mathematics can lead to a deeper understanding of purely mechanical questions. We accomplish these things in an unconventional but organized manner, progressing from simple models to three-dimensional linear elasticity.

Clearly, a great many questions in continuum mechanics are of mutual interest to mathematicians and engineers. A large number of these lie outside the scope of this small book, but are important nevertheless. We ask the reader to bear in mind that the present book is only a brief introduction to the interaction between mechanics and pure mathematics. As such, it cannot be complete.

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