1

Mathematics Education in Russia before the 1917 Revolution

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1 The History of the Inception of Russian Mathematics Education

The period from the 10th to the 17th centuries can be called the Age of the Inception of Mathematics Teaching in what is now Russia. It was during this period that conditions gradually changed, permitting mathematics education to emerge in the 18th century as a broad national concern. The historical records of this period lack specific evidence of the content or methods of mathematics education and most of the individuals who contributed to the emergence of mathematics education remain unknown. Still, historical evidence of commerce, government, and military activities indicates that mathematical activity was ongoing and was transferred by some means from person to person and generation to generation. Development proceeded in a haphazard manner, however; so the following account describes only the vital stages in the development of mathematical education in Russia.

The first stage took place in Kievan Rus', which in the 10th–12th centuries reached its zenith in terms of both culture and sheer power. Byzantium was a principal influence for both "intellectual and literary activity" in Kievan Rus' (Kostomarov, 1995, p. 9); it brought the

Cyrillic alphabet to Russia, which stimulated the development of a unified system of letters and numbers.

Prince Vladimir, who brought Christianity to Rus', and his son, Yaroslav the Wise, were the first to realize the importance of education. The Orthodox church expected educated people to support the newly accepted religion. On the strength of this belief public schools were founded in Kiev, in Novgorod and in other prominent cities, primarily for the children of priests and the secular upper classes, but not for the rest of the population. Education was mandated aggressively by the state. The result was the first few generations of educated people in Russia.

There are a few literary sources documenting the quality of education at that time. The best are the juridical collection "Russian Truth" by Yaroslav the Wise (Grekov, 1947) and the first mathematical essay in Rus' by the monk Kirik (Istoriko-matematicheskie issledovaniya, 1952).

"Pravda Russkaya" (Russian Truth) contains many articles with mathematical calculations. Some of these are quite complicated for that time (computation of percentages, evaluation of areas, increases in livestock, and other chattel). The collection's wide circulation implies that the educated segment of society could both understand and use its mathematical laws.

The very highest level of mathematical scholarship in Rus' is demonstrated in the mathematical–chronological essay "The manual of how a person comes to know numeration of years" (Nastavlenie, kak cheloveku poznat' schilslenie let) written by Kirik Novgorodets in the beginning of the 12th century. It contains professional-level mathematical–chronological calculations and even an example of a geometric progression with a common ratio of five.

At this time, however, mathematics education was most likely subordinate to other forms of education and had only a utilitarian intention. Its contents were limited to elementary information from practical geometry and also the rudiments of arithmetic. In the 10th–12th centuries in Rus', among the elite, in any case, there was an acceptance of the value of education and the quality of mathematical education fully comparable with Byzantine and European models.

The second stage (13th–14th centuries) coincides with the Tatar-Mongol invasion. The general cultural decline of this period included a decline in all levels of education at all levels of society. For all intents and purposes, schools virtually ceased their existence. Even the most educated societal group, the clergy, experienced this decline. The chronic insufficiency of literate people was so severe that even positions as priests were unfilled. Additionally in the 15th century, the clergy became the savage enemy of the dissemination of mathematics, all but banning mathematical books.

The only city that retained high culture in Old Rus' was Novgorod, which was virtually untouched by the Tatar-Mongol invasion. Many groups of society there had strikingly high levels of education, as evidenced by the sensational archeological discovery of writings on birch bark in Novgorod in the middle of the 20th century. The writings had sufficiently many numerical figures, including some written by children, to testify to the high level of mathematical education. This discovery, which evidenced the first appearance of the so-called "numerical alphabet," marks the first educational material in mathematics; the birch bark writings were presumably used for the study of numeration and exercises in the writing of numbers (Simonov, 1974, p. 80).

The third stage is linked with the consolidation of power in Moscovian Rus' (15th–17th centuries). The church's prohibition notwithstanding, literature appeared in which there was mathematical material. Geometry and arithmetic were apparently used in practical activities and art (Polyakova, 1977, p. 33). The clergy was the first to embrace the value of education, and monasteries played an important role in the development of enlightenment. As a result, libraries and schools were created in conjunction with monasteries.

The ecclesiastical system of education grew. In 1639, the first establishment of higher education was opened: the Kiev-Mogilyansky Academy. "In Russia, as in other countries, the need for higher education was satisfied earlier than the need for middle or lower education" (Brockhaus and Efron, 1898, p. 382). In the final year of the Academy, the curriculum included elements of geometry. Moscow's Slavic–Greek–Latin Academy, which was opened later, did not include mathematics in its curriculum.

In the 17th century, mathematical education functioned on a high level, even outside the clerical educational system, as evidenced by a considerable quantity of mathematical manuscripts from the period. The overwhelming majority of these manuscripts appear to be educational textbooks in mathematics. In turn, arithmetic manuscripts constituted the majority of these textbooks. The quality of the arithmetic textbooks matches that of contemporary European prototypes (Yushkevich, 1968, p. 24). Their contents included numeration, rules of operations with whole numbers and fractions, calculation, rules of commercial arithmetic, and elements of entertaining arithmetic (puzzles). These arithmetic manuscripts were largely responsible for the dissemination in Rus' of the Indo–Arabic system of numeration, which was of paramount importance for the general cultural development of the country.

Methodologically speaking, arithmetic was approached dogmatically. As in other countries, artificial rules were applied (the rule of three, the rules of regula falsi, etc.) to problem solving. The Russian abacus appeared as the basic calculating instrument. Arithmetic manuscripts used the traditional Russian system of measurement. Problems often were rooted in specific Russian reality (Polyakova, 1977, pp. 60–61).

The matter of geometry was different. Geometry was usually included in arithmetic manuscripts and a few practical manuals providing information for solving practical problems; the rules in these manuscripts were often inexact and occasionally incorrect and their foundations were absent. Only two manuscripts of that period were dedicated entirely to geometry. One was a textbook of practical geometry containing reasonably reliable rules of measuring distance and area. It also included problems on construction and isometric transformations of figures. The level of geometric mastery in this book is quite low, markedly lower than that of its European contemporaries (Polyakova, 1977, p. 67).

However, this is not the case with the second geometric manuscript. "Sinodal'naya No. 42" (Belyj and Shvetsov, 1959) occupies a prominent place in the history of mathematical education. For the first time in Russia this geometry textbook, created by order of the sovereign

Mikhail Fyodorovich, contains a systematic account of geometry presented in a manner similar to its European counterparts. It contains definitions of geometric figures, theorems with diagrams and elements of proofs, and solutions of problems on construction and calculation. The textbook was not printed nor widely distributed, and although it did not influence the development of mathematical education in Rus', it was an indicator of an existence of a layer of educated people in 17th century Rus', who not only were interested in mathematics but were also interested in its dissemination (Polyakova, 1977, p. 74).

In the 17th century, for the first time since the time of the Kievan Rus', the idea of the value of education caught the attention of the country's leadership. Political will, however, proved insufficient to cause real educational change. Tsar Boris Godunov intended to open schools and even universities, but he was unsuccessful; Mikhail Romanov commissioned a modern geometric textbook but did not facilitate its publication. Finally Peter I, often referred to as Peter the Great, had the kind of political will necessary to bring about real change.

2 The 18th Century: The Period in which Mathematical Education in Russia Came to a Halt

2.1 *Mathematical Education in the Epoch of Peter I*

Once the governmental reform began, Peter I was thwarted by the absence of literate people who were adequately prepared to bring his plans to fruition. For this reason, he began the preparation of specialists to form a regular army, to build a fleet and to open factories, and to reconstruct the apparatus of government. Peter's direct participation brought about the first secular public schools; moreover, he established a clear dominant position for mathematics in every secular school. In doing so, Peter I set the precedent for governmental patronage of mathematical education (Polyakova, 2000, p. 175).

From the beginning remembering his own European journeys, Peter I tried to utilize the scientific-educational potential of Europe.

Before Peter's rule, only diplomats and merchants were authorized to cross the border. Under Peter, however, travel abroad were encouraged and even mandated. As it turned out, Peter's initiative was largely unsuccessful. Only a few young Russians aspired to study abroad; even fewer proved capable.

The lack of books, necessary for the dissemination of knowledge, was another obstacle. To remedy the situation, in 1700 Peter I gave Yan Tessing, a businessman from Amsterdam, the right to publish and sell books in Russian secular stores. Losses were suffered: there were not very many booklovers in Russia and the quality of books was not high. All the same, the first academic books appeared in Russian, and they were mathematical. Despite this limited success of book production, however, by and large, attempts to utilize the scientific-educational potential of Europe did not produce the desired effect.

In the beginning of the 18th century, Peter I embarked on the first organization in Russia of a national, secular, public, and professional educational system. Peter I was sufficiently competent in mathematics to appreciate its role in military-technical education. For this reason, mathematics was one of the fundamental subjects in organized schools, the teaching of which he and his comrades-in-arms followed personally.

In 1701, mathematical–navigational and artillery schools were opened in Moscow. In 1707, a surgical school was opened, affiliated with a military hospital. In 1711–1712, an engineering school was opened. Subsequently a few training colleges appeared in conjunction with factories in Karelia and the Urals, where metallurgical craftsmen were trained.

2.1.1 *The mathematical–navigational school*

On 14 January 1701, Peter I issued a decree for the foundation of a school in Moscow "for mathematical and navigational, that is, seafaring scientific skills." It produced young people "for all sorts of service, military and civil, that demanded scientific knowledge or even Russian language-based knowledge; from the navigational school there emerged, besides sailors: engineers, artillerists, teachers for other new schools, geodesists, architects, civil servants, clerks, craftsmen and others" (Veselago, 1852, p. 7). The school was intended for children of gentry and civil servants. However, the gentry and civil servants were not overly anxious to educate their children; besides, they did not want their children to study with others of "ignoble birth." As a result a significant number of the school's students were from a lower class of society. The school enrolled a total of 500 students from 12 to 17 years of age and was housed in Sukharev Tower. Sukharev Tower acquired a wide reputation and was considered a major center in both mathematics and general scholarship.

During the time the Tsar was in England, he became acquainted with a professor from Aberdeen University named Fargwarson and invited him and his two colleagues to Russia. There, Fargwarson became one of the founders of the mathematical–navigational school where he established a thriving program. He took part in the development of the educational system and brought in programs in arithmetic, algebra, geometry, plane, and spherical trigonometry; he himself taught and also wrote the textbooks. But the English people did not speak Russian and the students not only did not speak English, they frequently could not read or write Russian also. Subsequently Leontij Filippovich Magnitsky, whom Peter I both knew and respected, was invited to accept a teaching post. Magnitsky was one of the most educated people of his time. He knew Latin, Greek, German, and Dutch and was acquainted with the achievements of European mathematicians. He taught arithmetic, geometry, and trigonometry, fulfilling his duties with exceptional conscientiousness. Magnitsky became the senior teacher and head of the academic part of the school. He was the leader of the mathematical–navigational school for the remainder of his life.

2.1.1.1 Magnitsky's "Arithmetika"

Several handbooks in mathematics were written by L. F. Magnitsky, of which the most important, "Arithmetika," was printed in Moscow in January 1703. Twenty four hundred copies were printed, which was a large number for that time. In the next half-century it was popular in schools, but enjoyed even wider popularity among other readers, particularly autodidacts. This was largely due to its linking of traditional, Moscow-based educational literature with new European influences. Since Magnitsky knew foreign languages so well, he was able to master a large quantity of European textbooks, books by Greek and Latin authors, and manuscripts by Russian mathematicians. He incorporated all of these materials in his textbooks.

The textbook's name conveys some but not all of the book's contents. It also introduced significant algebraic and geometric material and elements of plane and spherical trigonometry in addition to arithmetical knowledge. Because of its breadth, "Arithmetika" functions as an encyclopedia of mathematical knowledge of the time.

In accordance with the tradition of Russian educational literature, Magnitsky included the "Deed of Peter" in "Arithmetika." It functions on some level as a textbook of the latest Russian history (although an apologetic variant thereof). Also, it had many general philosophical debates and advice to the reader (frequently in verse). "Arithmetika" contained information on meteorology, astronomy, and navigation, as well as numerous data on natural science and engineering. "Arithmetika" was a precursor of popular scientific literature in addition to its other merits.

2.1.1.2 Organization of instruction

Normative measures by today's standards, such as lesson plans and programs, were not found in the mathematical–navigational school. The content of education was defined on the basis of L. F. Magnitsky's "Arithmetika" and on the geometry textbook "Priyomy tsirkul'a i linejki" ("Using compass and straightedge") and "Geometriya praktika" ("Geometry for the practitioner") which were translated into Russian by Ya. V. Brius. Instruction was dogmatic: it demanded that students memorize rules and be able to apply them to problem solving. At the same time, Magnitsky's "Arithmetika," which defined the method of instruction, was not devoid of methodical merit: its examples were selected with increasing difficulty and presented an interesting array of problems.

Study at that time was not easy. Classes were taught in a poorly-understood language, instructional equipment was scarce, and instructors interacted with students sternly, often using corporal punishment. For these reasons students frequently left the school.

In 1715, navigational classes were moved to Petersburg where the Morskaya (Navy) Academy was founded. At that time, Russia was a formidable naval power. With the Academy's opening came the reconstruction of the curriculum: war-related science was studied in the Academy. In the Moscow school, they studied mathematics only, which prepared the students for the Academy's course.

Difficulties notwithstanding, the mathematical–navigational school secured a prominent role not only in Russian mathematical educational history, but also in Russian education as a whole. It provided rapid production of high-quality mathematicians and navigators, as well as specialists of wider description. It became the first center for promoting secular schooling at that time, first and foremost for mathematics. It also became the first institution of teacher training, preparing mathematics teachers for a large number of learning institutions.

2.1.2 *Arithmetic (Tsifirnye) schools*

Arithmetic (Tsifirnye) schools (schools devoted to teaching counting and arithmetic in general) were established in 1714 in provincial cities in conjunction with bishoprics and large monasteries. Their student composition was relatively heterogeneous, including children of nobles and civil servants, those in the priesthood, and merchants. "Education was free, but at the conclusion of study, before issuing a certificate, a teacher had the right to collect a ruble for each student. Without this certificate it was forbidden to marry" (Kostomarov, 1995, p. 351). As mentioned above, the mathematical–navigational school prepared the first teachers for the arithmetic schools: Peter I ordered that two students from the navigational school well-versed in geometry and geography were to be sent to each of the provinces to teach there.

In 1716, 12 arithmetic schools were opened; in 1720–1722, another 30. A few more than 2000 people were conscripted into these schools, both voluntarily and by force. Educational reforms had formerly met with opposition from society. "Arithmetic study" was posted as mandatory for upper and middle social classes, but gradually children of nobility, townspeople, and clergy were freed from mandatory attendance from these schools, and subsequently 14 of the 42 existing schools were closed. The remaining students were almost exclusively children of civil servants. Toward 1727 only 500 students remained. In 1744, arithmetic schools were merged with garrison schools, opened in 1716 by Peter's decree for the education of soldiers' children.

2.1.2.1 Mathematical education in arithmetic schools

Students in arithmetic schools studied arithmetic and geometry. There were no established textbooks. It appears that the only source of mathematical knowledge was the teacher. Frequently one teacher conducted lessons for 20–30 students who all were studying totally different subjects at the same time — the equivalent of the American one-room schools. There were neither demonstrations nor intelligible explanations. The teacher would formulate basic definitions and rules and would provide solutions for model problems. The students were required to memorize a series of rules and solve problems.

2.1.2.2 Difficulties in teaching

About 15% of students left the arithmetic school, although it was not uncommon for them to be put in prison, in chains, for doing so (Gnedenko, 1946, p. 51). The causes for this were diverse. The first group of causes was linked with the opposition of society. As previously discussed, the population reluctantly allowed their children to go to school because it sharply changed old family structures, customs, and habits. The school regime was very cruel; poor results and carelessness were treated with corporal punishment. The second group of causes was related to the fact that the system of instruction was still "a work in progress." Textbooks, tablets, and instruments were in short supply. Teachers had no special preparation. There was no teaching methodology. Russian educational terminology, including a terminology of mathematics, was poorly cultivated. Despite relatively low efficacy, a few thousand Russian students were educated in State schools in Peter's time.

2.2 *Leonhard Euler and Mathematical Education in Russia*

2.2.1 *The academic educational system*

Earlier mathematical education in Peter's time was characterized as practical. Leonhard Euler took part in the foundation of the educational system that may be called "academic" (Polyakova, 1977, p. 127). It arose linked with the 1724 establishment of the Imperial Petersburg Academy of Sciences; this system functioned for the entire 18th century and ceased at the beginning of the 19th century.

Peter I decided to strengthen the Academy not only scientifically, but also in its teaching capacity, thereby providing the economic basis for the university and *academic gymnasium* (Gnedenko, 1946, p. 71). This was an original idea, as was the affiliation of the university with the Academy. In contrast to European universities, Peter's university consisted only of juridical, medical, and philosophical departments, with the exclusion of a theological department. The Academy, university, and gymnasium were therefore wholly secular institutions.

Still another distinction of the Petersburg Academy from European academies was that it was not public, but rather an organ of the government, supported firmly by a governmental budget. In fact, one of Peter I's final decrees set a tradition of governmental patronage of science. Since the caliber of invited scientist-mathematicians was excellent, so the development of mathematics as a science was strongly effected and bolstered.

After Peter I's death, the idea of the value of education to some extent lost importance in the eyes of the country's highest leadership. Educational politics caused external support to languish, but for the most part the material stimuli for the development of education continued due to inertia (Polyakova, 2000, pp. 177–178). Moreover, up until that time different levels of society considered the Academy (and its educational foundations) a very important institution and so it enjoyed some self-sufficiency and was able to function. It was important for the extension of the governmental patronage that the Academy focused not only on research but also on instruction.

The appearance of Leonhard Euler was extremely fortunate for Russian mathematics and mathematics education. As Peter I is considered responsible for a vigorous organizational influence on Russian education, so Euler imparted a great strength of content and methodology, creating and incorporating the mechanism of research mathematics' patronage of mathematical education. This patronage is fully demonstrated in the activity of the Mathematical–Methodological School of Leonhard Euler, which is a unique phenomenon in Russian intellectual history.

2.2.2 *The mathematical–methodological school of Leonhard Euler*

In the second quarter of the 18th century, the informal mathematical–methodological school, founded by Euler, played an ever-increasing role in the development of mathematical education. Though it is easy to identify the time when this school began its work — it coincided with the beginning of Euler's activities — it is a bit difficult to identify its end, largely because the ideas of Euler's methodological school developed over the course of the entire 18th century, and in many respects continued to be significant in the 19th century. Mathematics education as a scientific field was born later; therefore, the usage of the term "mathematical–methodological school" itself might have been put into question. Nevertheless, understanding the whole evolution of this term, we accept this term here.

The first Russian academicians-mathematicians S. K. Kotel'nikov, S. Y. Rumovskij, and M. E. Golovin, and the academician-secretary of the academy, N. I. Fuss, assembled the skeleton of the Euler school (Lankov, 1951, pp. 15–18). It is considered to be the first methodological school in Russia, since Magnitsky was a one-man methodologist. Moreover, although one of his students, N. G. Kurganov, developed the methodological ideas of Magnitsky, he also developed those of Euler, and factually belongs more to the methodological school of the latter. The school's activity affected the development of mathematical teaching in Russia in the following substantial ways:

- First, it provided operational access to the pedagogical and methodological ideas of Europe, in which the idea of proof and the systematization of the exposition of mathematics dominated.
- Second, having acquired these ideas, the methodologists of the school enriched and made sense of them. Adapting the ideas to Russian reality facilitated their quick introduction.
- Third, they fairly swiftly put non-translated, original Russian educational mathematical literature into use instead of foreign sources.

Euler arrived at the Petersburg Academy of Sciences in 1727. In the course of his service in addition to intensive scientific activity, he occupied himself with the teaching of mathematics in the academic educational system. He attended to the selection of the content of mathematical education, having had experience in writing several mathematics textbooks specifically for the academic gymnasium. Among them, "Manual in arithmetic for use in the gymnasium of the Imperial Academy of Sciences" bears mention. It was published in German (1738–1740) and in Russian translation (1740, 1760). This book had significant influence on academic mathematical literature, standing as a precedent for the foundation of prominent academically-accessible school textbooks written at a high scientific level.

Later Euler wrote his algebra textbook, which, again, stood as a prototype for all subsequent textbooks. Euler published "Full introduction to algebra" in Petersburg in German in 1770. In 1768–1769, a Russian translation was published named "Universal arithmetic." This book contained material that would have been sufficient even for a university course, but some sections of this book were used successfully in the academic gymnasium.

There is reason to suppose that Euler also wrote a geometry textbook. There is reference to it in a number of bibliographic sources and a few manuscript fragments have been found (Belyj, 1961, p. 186). Euler also developed textbook materials in modern trigonometry, presented in almost the same form as is studied today in schools. Thus, Euler had textbooks in practically all mathematical academic subjects which boasted the most modern methodical ideas of the time.

The first such idea was the reconciliation of mathematical educational content with modern mathematics. It found its embodiment in an algebra textbook by Euler, subsequently reworked by N. I. Fuss; in trigonometry textbooks by M. E. Golovin and S. Y. Rumovskij; and in a textbook of mathematical analysis by S. K. Kotel'nikov. For the first time in academic courses, the newest achievements of mathematicians were included such as Euler's modern trigonometry, and his work in differential and integral calculus. (Everything referenced here and further textbooks of Euler are characterized in detail in Polyakova, 1977, pp. 143–157, 184–198.)

The second important methodological idea realized by Euler was the introduction of fundamental mathematical disciplines in school mathematics education (i.e., arithmetic, geometry, trigonometry, and algebra) as separate and specific subjects. This approach helped to discourage unnecessary diversity in previously-used mathematics textbooks (for example, a popular in Europe textbook by Christian Wolff included the study of 19 disciplines, all of which were considered mathematical). Euler's approach helped to clear textbooks of materials foreign to the course of each school's particular discipline. In this way, the arithmetic textbooks of Euler and Kurganov were cleansed of elements of algebra and geometry.

The third methodological idea realized by Euler in his textbooks is the building of mathematical courses on the basis of progressive (for that time, but even for today) didactic principles including systematicity, scientific foundation, and accessibility to the students of the exposition of mathematical discipline. Importantly, Euler speaks not only about these principles but (in modern language) about finding the optimal combination of a high-level scientific foundation and accessibility by the student.

Representatives of Euler's methodological school, with few exceptions (N. G. Kurganov, M. E. Golovin), were academicians — the intellectual elite of society. Having organized mathematics instruction in all types of schools and the creation of textbooks, they actively participated in scientific-organizational (Kotel'nikov, Rumovskij, Fuss), educational-cultural and popularizing (Kurganov, Kotel'nikov, Rumovskij, Fuss), and instructional-organizational (Kurganov, Kotel'nikov, Rumovskij, Golovin) activities. Euler and Kotel'nikov, in addition, authored original studies on the reorganization of high school education.

2.3 *Mathematical Education in Russia in the Second Half of the 18th Century*

In the second half of the 18th century the most advanced levels of Russian society began to realize the value of education. There were several educational systems in Russia at that time: professional, academic (described above), university, public schools, and schools affiliated with the church (which will not be covered here).

2.3.1 *The professional educational system*

The professional educational system was reconstructed during this period to serve increasingly the children of the upper class and the nobility. The system included military schools (both the navy and the army "noble-born" cadet corps), military-technical schools (engineer–artillery noble-born cadet corps), technical (mining) schools, and others. Towards the middle of the 18th century the largest viable educational field was mathematics. Mathematics was considered to be a field of high priority and was distinguished by its high quality.

2.3.1.1 The naval cadet corps

The teaching of mathematics in the naval cadet corps was particularly good. Teachers in these schools were particularly well-prepared. The most famous pedagogue-mathematicians of Euler's methodological school worked in naval corps. Professor N. G. Kurganov devoted his entire career to the education of future marine officers. The academician S. K. Kotel'nikov combined teaching in the academic gymnasium with delivering lectures to those in the naval corps and to writing textbooks for their usage. The academician N. I. Fuss wrote several mathematics textbooks and taught a course not only in the navy corps, but also in the army cadet corps.

In the 1760s the navy corps began to offer to the upper class a course on higher mathematics in addition to the elementary mathematics course. The founder of this tradition was S. K. Kotel'nikov, who started teaching sections of higher mathematics especially for naval cadets. In 1766, his textbook "First Foundations of Mathematical Science" ("Pervye osnovaniya matematicheskikh nauk") was published. At the end of the 18th century, the course in mathematics was supplemented with analytic geometry and mathematical analysis and was taught by N. I. Fuss.

2.3.1.2 The engineering–artillery corps

The director of the *engineering–artillery* cadet corps M. I. Mordvinov, believed that mathematics should take precedence in officer preparation. He included mechanics, arithmetic, the beginnings of algebra, and higher geometry in the syllabus. Mordvinov was concerned with teacher quality, primarily in mathematics.[1] One of the very best teachers there was Y. P. Kozel'skij, well-known for his philosophical essays. Kozel'skij had attended the academic gymnasium and the university where he was a student of the famous Russian scholar Lomonosov (after whom Moscow State University was named later). It is interesting to note Kozel'skij's opinions on instruction as expressed in the prefaces to his textbooks and philosophical works where he emphasized the link between theory and practice. In 1764, Kozel'skij published an arithmetic textbook, "Arithmetic propositions" ("Arifmeticheskie predlozheniya"), which was distinguished by its clarity and concreteness with particular attention to material that would be needed in everyday life.

[1]The wages of the faculty were determined by the importance of their subject and the availability of the teachers. It is interesting to note the following list of wages. Mathematics teacher: 800 rubles per year; political science: 600 rubles; Russian language: 500; foreign language: 400; and dance teacher: 300 rubles per year.

N. V. Vereshchagin, a disciple of Kozel'skij, was one of the best educated teachers in the corps. He had extensive knowledge of mathematics and military disciplines, and was also extraordinarily knowledgeable about natural science, philosophy, and history. He knew French, German, Italian and Latin, which gave him the ability to follow developments in mathematics and to reflect in his lectures upon the latest mathematical achievements. Vereshchagin familiarized his cadets with Euler's *Introduction to Infinitesimal Analysis* and also taught them, among other things, methods of solving systems of linear equations with determinants. He was one of the first people in Russia to teach analytic geometry. Vereshchagin set an example in Russia for disinterested service. He gave lectures to students and amateurs who had an inclination toward mathematics without any compensation. In addition, he exerted substantial influence not only on mathematical education in the engineering–artillery corps but also on the development of mathematical education in general in the second half of the 18th century.

2.3.2 *The university educational system*

The most important element of the system was *Moscow University.* With the opening of the Naval Academy in Petersburg (1715), the center of mathematical education moved from Moscow to Petersburg. After the foundation of the Academy of Sciences, Petersburg became the definitive focal point of the development of mathematics and of mathematical education. New military and technical schools were opened, followed by a teaching seminary. Petersburg was the center of Euler's scientific-methodological school; it was the site where he published his works about mathematics and its teaching.

The founding of Moscow University in 1755 renewed Moscow's perspective concerning the development of mathematics and mathematical education. The university, however, did not have a mathematics department; one could study mathematics only as an ancillary subject. In the course of almost half a century, only minimal amounts of mathematics necessary for medicine and natural science were included in the Moscow University curriculum. These courses also were taken by students who wished to be considered generally well-educated.

A specialized chair of mathematics was established in 1758 at Moscow University at the request of the medical faculty. Professors of medicine noticed that students had deficient knowledge of physics because they were unable to *apply* their mathematical knowledge (since the only course offered in mathematics at the time was pure mathematics). The first specialist-mathematician to occupy the chair of mathematics was D. S. Anichkov, who was the head of the mathematics faculty and who had published textbooks in many mathematical subjects. Anichkov taught a course in pure mathematics in a two-year cycle, with two two-hour lectures per week. In the first year, he taught arithmetic and geometry; in the second, a continuation of geometry and trigonometry. Surprisingly, algebra was not always offered. After Anichkov the mathematician V. K. Arshenevskij taught with the same time arrangement per week, but his course had a three-year cycle which included algebra in the third year. I. A. Rost and his successor, M. I. Pankevich, taught applied mathematics in a three-year cycle of four two-hour lectures per week.

The level of mathematics teaching was lower than in academic educational establishments and professional schools of the time. The principal merit of university mathematics education was the active preparation of prospective teachers. At the end of the 1760s there were so called "gymnasia informers" (*Biographical dictionary*, p. 46), students prepared for future pedagogical work, to whom special courses were given. After a decade a pedagogical seminary was built, which, beginning in the 1790s, was called the "Teachers' Seminary." The mathematical education curriculum there included arithmetic, geometry, planar trigonometry, and elements of algebra. The Teachers' Seminary prepared teachers to teach in Moscow University's gymnasia, and eventually for gymnasia teaching in general.

"With a university there should be a gymnasia," wrote M. V. Lomonosov, the founder of Moscow University, "without which the university is like a plowed field without seeds" (Lomonosov, 1957, p. 514). In 1755, two gymnasia were founded for children of nobility and for students of other societal levels. Shortly after 1755, the university opened a gymnasium in Kazan'. These gymnasia were successful because less-affluent nobles were happy to educate their children there instead of hiring expensive (and frequently poorly-prepared) foreign teachers. In 1779, a boarding school was founded for the nobility. Based on this model, schools for “children of noble birth” were created in Nizhnij-Novgorod, Tver’, Ryazan’, Kursk, and other provincial cities. In this fashion, Moscow University became a leading supporter of school education since the university supplied teachers, textbooks, and occasionally money.

Two gymnasia in Moscow shared a single rector. Instruction was carried out separately in each school until the last class (the so-called “Rector class”), for which the students from both schools, often including children of lower rank or children of impoverished nobles, were brought together for preparation to enter the university. The children of the affluent nobility on completion of their first classes typically prepared for military or civil service rather than attending the Rector class. Once enrolled in the University, student “received a sword and with it a noble rank. Upon finishing university, the student left with the rank of an officer” (Brockhaus and Efron, 1898, p. 383). The provision of an officer’s rank inspired the lower-ranking students to obtain a university education, thereby gaining not only intellectual advancement, but also material resources and prestige.

Although generally education at that time emphasized the humanities, there was not just one educational trajectory for all gymnasia students. Gymnasia students could select trajectories of training themselves. Technically, the gymnasium was divided into three schools: Russian, Latin, and the school of new European languages. The Russian school was an introductory one. After graduating the student could choose between the Latin school (leading to the University) and the school of new languages. Mathematics was not taught in the Russian school. Later, students had a course in arithmetic (twice a week for 4 hours). In the next class they studied fractions, elements of geometry, and algebra (4 hours per week). In the “highest” class they continued the study of arithmetic, algebra, and geometry for 4 hours per week. In the beginning of the 1790s, algebra and trigonometry were added to the gymnasia’s curricula.

The general level of mathematical education in the university educational system, including gymnasia in large cities and in Moscow, was extremely low. There was one sole exception, and that was Moscow University's boarding school for children of noble birth. The focus in this school was "practical" studies rather than humanities.

2.3.3 *The general education system: Public schools at the end of the 18th century*

In the first half of her reign, Katherine II was very interested in questions of upbringing, following the ideas of the French encyclopedists. She attempted to realize various educational projects. The organization of networks of homogeneous schools of general education which covered the entire country proved of particular importance for Russian education, including mathematical education. In 1782, the "Commission to establish public schools" began its work on a "plan to establish public schools in the Russian empire." The Commission followed an Austrian educational model and invited an Austrian Serb, Yankovich de Mirievo, a follower of the ideas of Jan Amos Komensky, to carry out the reforms. De Mirievo knew Russian and had experience with school organization gained during the foundation of a system of training colleges for Austrian Serbs living in Hungary. In 1786, in accordance with "Rules of education for the people," schools were opened in the cities in Russia, including so-called "chief schools" in the capital of provinces (guberniya) and so-called "small schools" in the major towns of districts (uezd).

For this project to be successful, it was essential to prepare teachers, and so, in 1782, a special "teachers' seminary" was opened. Students were recruited from Russia's religious schools — the best students from the Alexander Nevsky, Smolensk, and Kazan' orthodox seminaries as well as students from the Slavic–Greek–Latin Academy were invited to study there. Yankovich tried to acquaint students with new methods of teaching, outlined in his "Handbook for Teachers of First and Second Classes of the Public Schools" ("Rukovodstvo uchitel'am pervovo i vtorovo klassov narodnykh uchilishch"), 1783. This book was the first methodological handbook in the history of Russian education in which requirements for a method of teaching were found. In this way, a foundation for the organization of the

Russian educational system and its characteristic methodology was prepared.

Mathematical subjects were among the most important in the curriculum, according to the rules of the schools. In the small schools, students studied arithmetic: in the first class, they would do oral and written calculations; in the second, there was a continuation of arithmetic including the rule of three. In the first and second classes in the chief schools, students would review the program of the small schools (to avoid any omissions); in the third class, the faculty taught fractions, decimals, and also exercises in the rule of three and others. In the fourth class, students studied basic geometry in an abbreviated form.

In his "Handbook for Teachers…" Yankovich proposed a method of teaching oral and written calculation. Teachers would have to clarify each new rule with examples and detailed commentary. After that, they would give an assignment with that rule to the best student who would complete it on the classroom board and then comment on its solution. Then the teacher would give an assignment that the class would solve on their individual boards, or one student would record the solution on the classroom board, prompted by the class. Later, when basic skills were acquired, the students were given tasks based on different rules without instructions and were asked to determine which of them should be used for the solution. These activities were expected to instill appropriate applications of different rules. The questions selected by the teacher were not to be abstract, but rather related to life. At home, students were supposed to solve new problems that were always checked during the next lesson.

Textbooks available for the public schools were not attributed to specific authors. Later, this practice proved to be a source of discussion among educational historians. The majority of historians attribute the authorship (in full or in part) to M. E. Golovin, who belonged to the methodological school of Euler. The organization of these textbooks was quite traditional. In the first part of the "Handbook for Arithmetic," for example, numeration and operations were presented with whole and nominative numbers; in the second part, fractions, decimals, proportions, and "exercises with rules" were discussed. The geometry textbook contained only 16 theorems with proofs. The bulk of the material consisted of fairly interesting problems. Substantial attention was given to measurement, for which the usage of physical models was recommended. In the summer, Golovin offered what we would now call "lesson-excursions."

3 The Formation of Russia's Classical System of School-Based Mathematical Education: 19th–early 20th Centuries

3.1 *Mathematical Education in Russia in the First Quarter of the 19th Century*

Following the foundation of the Ministry of People's Education in 1802, Russia was divided into six academic regions, with the leading role in each entrusted to that region's university. In 1803, "four types of schools were identified: 1) parish-based schools, 2) district-based schools, 3) provincial schools (called gymnasia), and 4) universities" (Yushkevich, 1948, p. 15). In the parish-based schools, instruction lasted one year, in the district schools two, and in the gymnasia four. Continuity was provided between them, although that continuity deteriorated almost completely toward the beginning of the 20th century. In 1804, the rules of the universities and other academic establishments were accepted, which consolidated the universities' scientific-methodical role, providing in this way the most qualified possible leadership of early and secondary education.

The value of education at that time was accepted by both the higher leadership of the country and a wide portion of society. Educational politics of the leadership oscillated depending on exterior and internal circumstances: from clearly-declared democratic tendencies at the beginning of the century to the idea of classicism, toward the end of the first quarter of the century, which was considered as a counterbalance of democratic tendencies.

Changes occurred also in the school's class-based restrictions. At the beginning of the 19th century, almost all class-based restrictions were removed. The government took over the financial support of the gymnasium. Toward the beginning of the 1820s, regulations became more rigid; in particular, people now were required to pay for their education.

During this period in Russian education, the opposition between the classicist approach (based on the study of ancient languages) and the "real" approach (natural-mathematical) became pronounced. This opposition was particularly acute at critical moments in Russian history. Its influence upon the quality of mathematical education was indicated by:

(1) the number of students enrolled,
(2) the position of mathematics in a general system of education.

In the beginning of the century, gymnasia education, on the whole, included aspects of practical ("real") education. Mathematics occupied the primary position in the schedule of lessons. Six hours per week were allotted to the study of mathematics during the first three (out of four) years. During the last two years of education, students studied statistics as well (4 and 2 hours per week, respectively). Mathematics teachers (as opposed to teachers of language or drawing) were called senior teachers and occupied the highest rank of governmental civil service a teacher could have, that is the ninth rank (according to Peter I's ranks system). A teacher of mathematics held a rank equal to the military rank of captain. A mathematics teacher's elevated rank allowed mathematics to rank the highest place among the academic subjects. Moreover, the government officially proclaimed education free of class and cost. In the beginning of the century, the government provided, practically speaking, the ideal conditions for the development of education in general and mathematical education in particular.

Its content at this time was determined neither by the preparation nor interests of the teacher, as it had been at the beginning of the 18th century, nor by mathematical textbooks, as it was the end of the century, but by rules of the academic institution. In the parish schools the first operations of arithmetic were studied; in the district schools, arithmetic and beginning geometry were presented; in the gymnasia "pure and applied mathematics and experimental physics" were part of the curriculum. The names of the subjects themselves speak to the fact that the model of mathematical education characterized by the 18th century had not yet been overcome.

Pure mathematics included geometry, plane trigonometry, and algebra up until third degree equations with applications to geometry and conic sections. Courses in pure and applied mathematics included some sections of physics, and also elements of analytic and descriptive geometry, with the beginnings of differential and integral calculus. In the first year of gymnasia, students studied pure mathematics consisting of algebra, geometry, and plane trigonometry; in the second year, the study of pure mathematics was completed and applied mathematics and experimental physics were introduced, to be concluded in the third year.

The leading influence on the content of mathematics education, as before, was the content of mathematics textbooks. For the improvement of old textbooks and to prepare new ones, the Academic committee was appointed in 1803. One of the appointed members was academic-mathematician N. I. Fuss. Beginning in 1805, the first two volumes of "Course in Mathematics" by T. F. Osipovsky was recommended as the textbook for gymnasia. The first volume contained arithmetic and algebra; the second, geometry, rectilinear, and spherical trigonometry and an introduction to curvilinear geometry. The textbook "Course in Mathematics" presented rich content with simplicity and accessibility. It was very popular in secondary and higher educational institutions and was reprinted many times. This success notwithstanding, this textbook was not actually intended for gymnasia, but as a university textbook in mathematics. The Academic Committee decided to put together textbooks specifically for gymnasia, the preparation of which was commissioned to N. I. Fuss.

N. I. Fuss reworked earlier publications and published "Beginning Foundations of Pure Mathematics," which from 1814 until 1828 functioned as the first stable textbook of mathematics for gymnasia in Russia. Fuss included the foundations of algebra and geometry, applications of algebra to geometry, plane trigonometry, conic sections, and the foundations of differential and integral calculus.

The tradition of support of mathematical education by research mathematics continued in this period. This tradition of patronage was evidenced by the work of practically all of the eminent mathematicians. In particular, S. Y. Rumovsky and N. I. Fuss were the first members of the Academic Committee. S. E. Gur'yev published the first mathematical–methodological essays in Russia (Polyakova, 2002, pp. 219–237). T. F. Osipovsky, the future rector of Khar'kov University and one of the directors of the academic district, provided important assistance to mathematical education on all levels.

3.2 *The Development of the Gymnasia System of Mathematical Education in the Second Quarter of the 19th Century*

Although there was an attempt to bring uniformity to gymnasia education, the quest for uniformity was not successful. In fact, this very question had been part of the discussion of the reform of the educational system in Russia over many years. In 1826, the "Committee to organize academic institutions" ("Komitet ustrojstva uchebnykh zavedenij") was established in order to bring about this reform: by establishing new regulations for academic institutions. Gymnasia regulations, established in 1828, announced a seven-year period of study. Three years of elementary school were included in the gymnasia course. Strict class-based boundary restrictions were established: only children of nobility and merchants of the first guild were allowed to enroll in gymnasia. The cost of studies was increased; the classical underpinnings were emphasized. These "innovations" had chiefly a negative effect on mathematical education. The only positive effect of the 1828 rules was the elimination of vagueness of topics included in mathematical courses. Applied mathematics was eliminated. Pure mathematics was limited to cover only topics through conic sections (additionally, the course of descriptive geometry was introduced).

There was also a bifurcation in gymnasia education. It was divided into two types: the first, or Variant A, included the study of Latin. The second, or Variant B, included both Latin and Greek. It could be surmised that the first variant embodied a practical approach, whereas the second a classical one.

The content of mathematical education in these types of gymnasia was diverse. In the regulations of 1828, besides the schedule of lessons, a plan of study was included for the first time in the history of Russian mathematical education. This showed not only academic discipline, but also contained subjects of study that would provide consistency. In gymnasia of the first type, students studied arithmetic in the first and second classes; in the third, the beginnings of algebra were studied, including second degree equations; in the fourth, students finished algebra and started geometry; in the fifth, they finished geometry; in the sixth, they studied elements of descriptive geometry and the beginning of applications of algebra to geometry; in the seventh, they completed applications of algebra to geometry up to conic sections. At the completion of the course, the teacher conducted a review of the whole course of mathematics. In the second type of gymnasia, the course in mathematics was shortened to 15 hours per week (combined for all grades)[2] and included only arithmetic in the first and second classes, algebra in the third, and geometry in the fourth, fifth, and sixth classes, and a review of these offerings in the final year of studies.

Despite deficiencies, the regulation of 1828 played a positive role in the development of Russian mathematical education by providing firm limits on its content. The implementation of these changes shows that society as a whole was beginning to appreciate the general cultural meaning of mathematical education.

The first concise unified programs in mathematics introduced in 1832 were intended to guarantee uniform mathematics instruction in gymnasia across the entire country. It succeeded in providing relative uniformity only on the level of the academic regions. Each region's trustees delegated the task of developing precise instructions on mathematics teaching in the gymnasia to the regions' universities.

[2] By comparison, 46 hours per week (combined for all grades) were allotted to the study of Greek and Latin. (Here and below, we frequently indicate the combined number of hours per week for all grades, i.e., if during the first four years of study, 3 hours per week were allotted to some subject, and during the subsequent three years, 2 hours per week were allotted to the same subject, the combined total for all seven grades would be 18 hours per week.)

From the end of the 1820s until the beginning of the 1830s, there was a delicate equilibrium between the classical and "real" (practical) models of gymnasia education. This bifurcation was bolstered by the regional interpretations of the national regulations for the gymnasia.

The position changed significantly in 1834 with the appointment of S. S. Uvarov to the post of Minister of Education. He was active in educational politics, central to which was the famous triad "Orthodoxy, Autocracy, Nationality." The political remedy chosen to comply with this triad was to tighten class boundaries and raise the price of education. The academic plan was revised to exclude topics such as statistics and logic. The dominant model for gymnasia education was the classical system with its basis in ancient languages.

As a result of the Uvarov "reforms," mathematics, in large part, lost its former privileged status. In 1845, a circular governmental letter appeared called "On the shortening of the education of mathematics in gymnasia" ("Ob ogranicheniyakh v gimnaziyakh prepodavaniya matematiki"), in which the teaching of analytic and descriptive geometry in the gymnasia was abolished. The usual number of hours per week in mathematics combined for all four classes was 20 hours. A new allocation of mathematical classes was proposed in 1846. Based on this allocation, F. I. Busse developed his project of an exemplary program in mathematics. In it he broadened the course of arithmetic and algebra and extended the time allotted for this course. Trigonometry was introduced in Busse's project in an effort to reinforce the applied side of the mathematical disciplines.

The 1848 anti-monarchist revolutions in Europe brought new changes to educational politics in Russia. The Ministry of Education published a memorandum intended to eradicate "free thinking" in high schools and universities. This time, the conflict between classical and practical ("real") gymnasia education was settled in favor of the latter. Instead of Greek, students were required to study natural-mathematical disciplines. At the same time, the number of hours per week devoted to mathematics combined for all four classes was increased from 20 to 30, however, in 1852 that number was reduced to 22.5 hours.

Once again a new program of mathematics was introduced but it was not significantly different from the program of 1846. Special attention was given to practical applications of mathematics and the strengthening of interdisciplinary links between the sub-fields of mathematics.

The textbook, "Beginning foundations of pure mathematics" ("Nachal'nye osnovaniya chistoj matematiki") by N. I. Fuss, was replaced by arithmetic textbooks and exercise books written by F. I. Busse and "Arithmetic leaflets" ("Arifmeticheskie listki") by P. S. Gur'yev. A translation of "Course in pure mathematics" ("Kurs chistoj matematiki") by the French mathematician Bellavein also was used in Russian gymnasia.

"Handbook for arithmetic" ("Rukovodstvo k arifmetike") and "A collection of arithmetic problems for gymnasia and district schools" ("Sobranie arifmeticheskikh zadach dlya gimnazij i uezdnykh uchilishch") by F. I. Busse were the textbooks of a new type (Polyakova, 2002, p. 277). For the first time, the complete set of books for students and teachers united by one methodological approach was created. This set included a textbook, an exercise book, and a teacher guide entitled "Handbook for instruction in arithmetic" ("Rukovodstvo k prepodavaniyu arifmetiki"). This handbook was the first methodological handbook for Russian mathematics teachers. Complete sets of textbooks, problem books, and teacher guides did not become customary until the end of the 20th century, suggesting that the methodological provision of F. I. Busse's arithmetic course can be considered a "breakthrough" in the history of mathematical education. F. I. Busse's textbook appeared in a record number of 18 editions from 1830 to 1875. It is worth noting also that Busse's collection of problems was one of the first such collections published in Russia, establishing a tradition continued to the present.

P. S. Gur'yev published "Arithmetic leaflets" with the primary goal of developing students' self-sufficiency and assisting teachers in providing differentiated instruction. Their original form utilized individualized sheets with portions of learning materials to be distributed to students in accordance with their ability.

"A course in pure mathematics" by Bellavein was a reworked and supplemented variant of a course that was quite full, but compact. The most successful part of this text was on algebra, which was published separately and accepted officially for usage in gymnasia until the 1850s.

Despite efforts to provide uniformity, various academic districts used different textbooks, or occasionally local manuscript versions, that reflected the preferences of the region. Thus, in Moscow gymnasia textbooks by D. M. Perevoshchikov were used; in Kazan', manuscripts and textbooks by N. I. Lobachevsky were preferred.

The textbook "A gymnasia course in pure mathematics" by D. M. Perevoshchikov included practically the whole university course in mathematics — arithmetic, elementary algebra, beginning geometry, rectilinear trigonometry, and conic sections. In the gymnasia, courses in algebra and trigonometry were offered widely. The textbook "The foundations of algebra" was published in 1854, and was recommended by the Ministry as a textbook for gymnasia. Excerpts in trigonometry from Perevoshchikov's prior text were recommended for gymnasia as late as the 1860s.

Textbooks in elementary geometry and elementary algebra by N. I. Lobachevsky (1823, 1825), which appeared as manuscript, were used in gymnasia in the Kazan' academic district. These manuscripts generalized the gymnasia course and were used in gymnasia in an abbreviated form as an introduction to university study. Lobachevsky's geometry textbook was developed along the lines of the author's original methodological system, the basis of which is laid out in the methodological essay "A survey of teaching pure mathematics." Beginning with the measurement of circumference, Lobachevsky used the theory of limits. He utilized the idea of the parallel study of plane and three-dimensional geometry. This essay defines the methodical innovation of N. I. Lobachevsky.

N. I. Lobachevsky's textbook on elementary algebra was not published at that time. In 1835, only his "Algebra, or calculation of finites" was printed, in which topics such as the foundations of operations with whole numbers, a general approach to functions, an original treatment of transcendental functions, and other topics were presented with great self-sufficiency and methodical innovation.

In 1844, textbooks developed especially for gymnasia were published. In geometry, "Mastery of beginning geometry" by F. I. Busse and the exercise book "Practical exercises in geometry" by P. Gur'yev and A. Dmitriev appeared. In arithmetic, the text "Arithmetic" by V. Y. Bunyakovsky appeared.

Busse's textbook was the first geometry textbook in the history of Russian mathematical education with modern structure and contents. It did not contain a separate treatment of straight line geometry, including only two- and three-dimensional geometry. The Busse textbook corresponded to the program of 1846 and was well designed methodologically. The exercise book included problems devoted to constructions and calculation, arranged in increasing order of difficulty.

V. Y. Bunyakovsky's "Arithmetic" appeared in three editions (1844, 1849, 1852) and was used in the majority of academic regions and enjoyed deserved popularity due to its recognized methodological merit. To its credit, it had systematic and comprehensive examples as well as complete and understandable explanation of rules. It was well organized with a limited amount of basic materials and with all supplementary materials moved to the appendix. All irrelevant traditional materials were excluded. There were methodical notes for readers, concentrated primarily in the preface and then generalized subsequently in "Program and abstract of arithmetic."

The tradition of the research mathematicians' patronage of mathematics was strengthened by the universities' dominant role in academic–methodological leadership in the schools. V. Y. Bunyakovsky was a prominent mathematician of this period who wrote textbooks for the schools, taking part in methodological meetings and commissions. Bunyakovsky taught in military-academic institutions, and even published one of the first books on the methods of teaching mathematics. The great mathematician, N. I. Lobachevsky, who was mostly unrecognized by his contemporaries, fulfilled his obligation as one of the leaders of the Kazan' academic region and wrote textbooks in elementary geometry and elementary algebra. He also wrote serious methodical works and even published a pedagogical essay (the published speech "On the most important subject in upbringing").

We should take special notice of the role in the development of mathematical education of mathematician–university rectors D. M. Perevoshchikov and N. I. Lobachevsky. They stood at the head of the Moscow and Kazan' academic regions, providing competent academic–methodological leadership in schools that put school-based mathematical education in their regions on a high level. In addition, they wrote first-class mathematical textbooks.

3.3 *Mathematical Education in Russian Schools in the Second Half of the 19th Century and the Beginning of the 20th Century*

In 1858, an Academic Committee appointed by the Ministry of Education issued new school regulations, foreseeing the bifurcation of gymnasia education into philological and physical–mathematical specializations. The prominent mathematician and founder of the Petersburg mathematical school, P. L. Chebyshev, guided the creation of the academic plan in mathematics. His plan was formulated with the following goals:

(1) the development of the intellectual capability of students;
(2) the mastery of knowledge vital to any cultured person;
(3) preparation for specialized occupations in physical–mathematical sciences with attention to their practical application.

For this plan, the gymnasia course in mathematics was partitioned into two parts: "general," for all students in the 1st–5th classes; and "specialized," for those wishing to continue their education in the "practical" vein in the 6th–8th classes. The specialized course included continuation of algebra, trigonometry, analytical and descriptive geometry, mathematical geography, optics, and mechanics. The right to develop a specific syllabus was granted to the faculty committee of each gymnasium. However, this project did not come to fruition. The Academic Committee rejected the idea of bifurcation in 1859.

A new project of gymnasia regulations was proposed in 1860 in which "real" (more practical) education was favored. Latin was begun only in the third class, Greek in the fifth. The number of hours per week allotted to mathematics was 27.5 hours (combined for all years of the gymnasia studies). Twenty hours per week were allocated for natural science and physics. This project was altered twice, in 1862 and 1864, finally resulting in a decrease of hours allocated to mathematics.

The problems of education, including mathematical education, were discussed widely and publicly in the 1860s in pedagogical and public-political periodicals. A professional discussion of the problems of gymnasia education in general took place in 1864 at a pedagogical conference for school directors and teachers in Odessa. In particular, at the Odessa meeting, the issue of gymnasia mathematical education was discussed. Concerns about content, in terms of the overload of the program of 1852, were resolved by a decision to abbreviate the course in mathematics. The conference formulated methodological principles in the teaching of mathematics, such as striving for consciousness in learning and visualization; in addition, teachers' intervisitations with a full exchange of ideas were recommended.

New regulations for gymnasia and programs in accordance with the foundation of three types of gymnasia with seven-year terms of study were established in 1864. These regulations recognized three types of curricula: "classical" with two ancient languages, classical with one ancient language, and "real" or applied curriculum. The regulations provided the opportunity to choose among the three, granting equal rights to graduates.[3] Corporal punishment also was abolished while the authority of teachers' and faculty committees was elevated.

P. L. Chebyshev revised the pedagogical guidelines in 1865, sharply defining the framework of the teaching of mathematics, physics, and cosmography. In the program for the classical gymnasia 22 hours per week were allotted to mathematics; in the real gymnasia, the allocation for mathematics was 25 hours per week. The content of the course in mathematics was defined by general characteristics only. The teachers had the right to design their syllabi with subsequent approval by faculty committees. This allowed the teaching of mathematics to distinguish itself not only in gymnasia in different regions, but also in a single region.

[3]All of the real gymnasia, however, guaranteed admission only to higher specialized academic institutions and, for the first time, to physical–mathematical university departments. The classical schools did not have an analogous limitation.

In 1871, the liberal school regulations of 1864 were replaced by regulations which remained in place without substantial change until 1917. All of the gymnasia were transformed into classical gymnasia with an eight-year term of study including two ancient languages. Mathematical study in gymnasia was abstract-deductive, and formal scholastic instruction methods were encouraged. Mathematics, physics, mathematical geography, and natural science together were allotted just 37 hours out of the 206 hours per week. By comparison, Latin was allotted 49 hours and Greek was allotted 36 hours.

"Real schools" were established to replace "real gymnasia." After the sixth class, students in real schools selected one of three directions: commercial, mechanical–technical or chemical. Real school did not provide the study of ancient languages. However, the course in mathematics was substantial. Further, many hours were allotted to technical drawing. Upon completion of the real school, students were denied the right of admission to a university. The battle between classical and real education, which went on for the entire 19th century, concluded with a victory for the classical model of school education. The quantity of hours devoted to mathematics in school did not decrease, however, but even increased. In addition, Russian schools had, for the first time, stable national curricula in all subjects, including mathematics.

Published in 1872, this syllabus in mathematics, which owed its existence to P. L. Chebyshev, defined the following order of mathematics study in gymnasia. In the first through third classes, students studied arithmetic. In the third class they began the study of algebra, which was completed in the seventh year of education. In the eighth year students repeated the entire courses of algebra and arithmetic. The geometry course began in the fourth class and was completed in the seventh year with a review of everything that had been studied and with the study of the application of algebra to geometry. Trigonometry was studied during the eighth year.

The mathematical content remained roughly the same as that described in the 1852 syllabus. Changes were introduced with the goal of eliminating parts considered either too difficult or unimportant. Real schools' academic plans and curricula in all subjects including mathematics, mechanics, and practical mechanics were approved in 1873. New mathematics curricula for real schools were based on the academic plan prepared by P. L. Chebyshev that was briefly characterized earlier. Considerable attention was paid in the curriculum to descriptive geometry, series, and theory of limits.

New regulations for educational institutions were issued for real schools in 1888 and for gymnasia in 1891. However, they had practically no influence on the teaching of mathematics. Several not particularly substantive amendments were added to the 1872 curriculum in 1890. In 1890, for the first time in the history of Russian schools, an explanatory note on teaching methodology was included in all curricula. Also, it was emphasized that, as an exact and abstract science, mathematics should be a means of intellectual development. For this reason, basic attention should be paid to the study of theory, while practical examples should serve as an illustration of theory and for the acquisition of calculating skills. This approach substantially weakened the representation of the role of applications of mathematics.

The 1890 arithmetic curricula included the following sections: the decimal system of numeration, arithmetic operations with rational numbers, their application to the study of quantities, the metric system of measurement, and the Russian abacus. The course was completed with the section "Problems based on rules." In the next class, students studied elements of theoretical arithmetic: systems of numeration, theorems about divisibility and factoring of numbers, indicators of divisibility, repeating decimals, and approximations.

The syllabus of algebra included the following sections: polynomials, algebraic fractions, first-degree equations, the theory of proportion, equations and second-degree trinomials, radicals, quadratic and cubic roots of numbers, quadratic roots of polynomials, progressions, logarithms, equations and systems of equations, first-degree inequalities, indeterminate first-degree equations, continuous fractions, the theory of combination, and the binomial theorem. The next class included divisibility of polynomials by binomials, equivalence of equations, applications of algebra to geometry, the theory of limits, and the existence of logarithms.

The syllabus in the real schools was distinguished by the inclusion in the last class of complex numbers, solutions of binomial equations with applications to the construction of inscribed regular polygons, analysis of the extrema of some rational functions, and the method of indeterminate coefficients.

The syllabus in geometry was partitioned in two parts:

Plane geometry — the study of angles, parallel lines, triangles, polygons, circumference, incommensurate segments, similarity, inscribed and circumscribed polygons, metric properties of triangles, calculation of area, the concept of limit and the calculation of circumference and area of circles, and geometric constructions.

Solid geometry — straight lines and planes in space; polyhedra (prisms, parallelepipeds, pyramids); regular polyhedra; congruence and similarity of prisms and pyramids; cylinder, cone, sphere, and sections of these bodies.

Although the 1890 mathematical syllabus remained in Russian schools until 1917, several attempts at reform took place early in the 20th century. At some point "classicism" was somewhat weakened in the gymnasia — Greek became no longer mandatory, the number of hours in Latin was decreased, and the written examination in Latin was eliminated. The usual number of hours per week allotted to mathematics hovered around 30.

At the turn of the century, the Ministry of education decided to bring the problem of secondary education to the forum of public opinion. As a result, academic plans and syllabi were composed for four types of schools: gymnasia with two ancient languages, gymnasia with one ancient language, real school, and a new type of secondary school. The following goal of the teaching of mathematics was formulated: "mathematics would be adopted as a science in its own right and as a scientific method of the exploration." A mathematical subcommission carried out the bulk of the work (Metel'skij, 1968, p. 12) with the belief that mathematical content in the gymnasia should be identical to that in real schools.

Change in the syllabi of the real schools took place in 1906, when the academic plan of the senior classes was altered to include the beginning of analytic geometry (except for analysis of second-order curves) and mathematical analysis.

Toward the 1890s, a system took shape that can be called an international classical system of mathematical education following I. K. Andronov (1967, p. 6). Here are the particulars of its mathematical content:

(1) The study of so-called elementary mathematics in secondary school and higher mathematics in higher educational institutions.
(2) The division of elementary mathematics into four academic subjects: arithmetic, algebra, geometry, and trigonometry.
(3) The study in elementary school of arithmetic only and with only an informal empirical approach.
(4) The study in higher institutions of the bases of mathematics from the 17th and 18th centuries — analytical geometry and mathematical analysis, known as "higher mathematics."

The methodical particulars of this system were:

(1) The establishment of two goals of education: "purely educational," directed at mastery of mathematical facts; and "developmental," aimed at the development of the student's formal-logical thought.
(2) A sharp delimitation of the functions of the teacher and student: the teacher would deliver prepared lessons, the student would memorize and recall passively.
(3) The availability of textbooks and workbooks for every academic sub-division in mathematics.

This classical system of mathematical education, which endured for a very long time, had some outstanding results, although, at the same time, it had extreme inherent deficiencies:

(1) The disparity between the developing "science of mathematics" and the very stable "academic subject of mathematics."
(2) The "divide" between elementary and higher mathematics.
(3) The absence of an informal preparatory course in mathematics (in geometry, in particular).

(4) Weak connections between the four sub-divisions of elementary mathematics.
(5) The prevalence of formal-logical goals for the study of mathematics and negligence of other goals of its study.
(6) The prevalence of problems and exercises of artificial character, poorly related to practice or even theory.

As a result of this development in history, a pyramidal system emerged in Russia where only 40–50% of those who had enrolled in the first class graduated from school. This extensive lack of success in mathematics engendered an extreme point of view — to exclude mathematics completely from the general education school course.

3.4 *The Reform Movement in Russian Mathematics Education at the Turn of the Century (19th–20th)*

By the beginning of the 20th century, difficulties and clashes in school mathematical education had accumulated. As a result, a movement toward reform emerged. This movement gained an international character linked with the name of Felix Klein. In 1897, at the International Mathematical Congress in Zurich, Klein established the necessity of reform in mathematical education and formulated its principles which formed the foundation of the "Meran Programme," developed in 1905 under Klein's direction.

Even earlier, similar ideas were widely discussed in Russia. During 1891, a work called "The goal and means of the teaching of elementary mathematics from the point of view of general education" by S. I. Shokhor-Trotskij was published in the journal "*Russkaya Shkola*" (Russian School) (Nos. 2, 3, 9, 10). In this work, the existing system of mathematics education was criticized, and the introduction of a beginning course in geometry, the enrichment of arithmetic with approximating calculations, and the introduction of the study of functions and the theory of limits in the algebra course were proposed. A lot of attention was attracted by the publication of V. P. Sheremetevsky's article "Mathematics as a science and its school-based surrogates" in

1895, in which the author emphasized that modern mathematical ideas were ignored by the gymnasia curriculum. Sheremetevsky believed that the elementary course in mathematics should be based on the concept of functional relation and fought for the introduction of elements of analytical geometry and mathematical analysis into the school course. A great amount of attention was devoted to the problems of reform in mathematical education in pedagogical and methodological publications and in academic-pedagogical discussions in Petersburg and Moscow. For example, special summer courses were organized for teachers; in 1911, the journal "Matematicheskoe obrazovanie" (Mathematical education) appeared, which became a center of educational discussions. Toward the end of the 19th century, Russian mathematical education not only became a part of the international classical system of mathematical education, but was focused on development, reform, and principles which coincided with global tendencies.

The international reform movement was organized officially at the Fourth International Congress of Mathematicians held in Rome (1908). At that meeting Klein was elected president of the executive committee of the International Mathematical Commission. Well-known mathematicians including the academician N. Y. Sonin and professor K. A. Posse headed the Russian subcommittee of this Commission.

Two All-Russia congresses of mathematics teachers contributed to the advancement of the reform movement. During the winter holidays of the 1911–1912 academic year the first All-Russia congress of mathematics teachers met in Petersburg with 1217 educators in attendance. In all, 71 presentations were given, followed by discussions. The proceedings of the congress were published (Publications, 1913). The well-known mathematician, philosopher, mathematical historian, and pedagogue D. D. Morduchay-Boltovsky (1912) provided a detailed report on the conference. Among the important items discussed at the congress were the following:

(1) It was suggested that elements of analytical geometry and especially mathematical analysis were to be introduced into school "only

with the assistance of graphical, visual methods to clarify this idea and only to the extent that it can be done in that way" (Morduchay-Boltovsky, 1912, p. 9). M. G. Popruzhenko, F. B. Filippovich, and D. D. Morduchay-Boltovsky presented serious objections to this position.

(2) The reconstruction of the school course in geometry with more rigor, and the list of axioms, in particular, did not receive support; as opposed to the idea of an informal preparatory course in geometry built on a visual and intuitive approach rather than on logical reasoning.

(3) General problems and issues attracted considerable interest, including philosophical aspects of mathematics, psychological questions on the teaching of mathematics, the introduction of elements of history of mathematics, and the practical and applied directions of school mathematics.

The second All-Russia meeting of mathematics teachers took place in Moscow during the winter holidays of academic year 1913–1914 with 1200 participants. In all, 22 presentations were delivered (Reports, 1915). Seven resolutions were accepted, the first of which concerned the preparation of mathematics teachers. The following proposal was made:

(a) That people entering the teaching mathematics should possess preparation both in subject matter and in pedagogy.

(b) That in physical–mathematical departments of higher academic institutions courses should be taught that would illuminate, from an academic point of view, the foundations of aspects of elementary mathematics.

(c) That regional conferences of mathematics teachers should be established.

(d) That short-term and long-term pedagogical courses for mathematics teachers should be established.

(e) That higher academic institutions and mathematical societies should assist in organizing these courses (Reports, 1915, pp. 163–164).

The following proposals were made with the goal of improving the curriculum in mathematics:

(a) That the right be granted to gymnasium faculty committees to permit teachers to deviate from existing official syllabi with the condition that changes would be approved by schools' academic committees.
(b) That change of the syllabi and of the plan of the teaching of mathematics in school be carried out in their entirety and not in parts. It would be imperative that such a project not only would introduce new topics, but also would eliminate some topics that had become outdated and irrelevant.
(c) That the teaching of mathematics in girls' gymnasia be organized in exactly the same manner as in boys' gymnasia.
(d) That reworking of the plans and syllabi of teaching should be done in conjunction with representatives from the universities and scientific and teachers' societies (Reports, p. 164).

The Congress identified analytical geometry and analysis as essential to schools of all types, recommending:

(a) A review of syllabi of analytical geometry and analysis.
(b) The allocation of sufficient time to the study of these topics.
(c) The establishment of connections between analysis and previous parts of the course.
(d) Improvements in the teaching of analytical geometry and analysis (Reports, p. 164).

Many of the problems discussed in the Congress were not resolved. A few fruitful ideas were not put into effect due to World War I. Also a few of the ideas proved unsupportable in practice. The Congress's work showed that the Russian reform movement in mathematics education had a wide and rich content and various forms. To a great extent, the influence of the Congress was magnified by the extensive increase during the 1912–1915 period in the publication of academic and mathematical–methodological literature with new content and ideas from the Congress.

4 Conclusion

The kernel of Russian mathematical education — the Russian model of the international classical system of school-based mathematics education — was established before the 1917 Revolution. It immensely influenced everything that happened in Russian mathematical education in the 20th century. From 1918–1931, there were attempts to eradicate the classical system of mathematical education, accompanied by a generally unsuccessful search for new models. From 1931–1964, there was a restoration of Russian traditions with the formation of the Soviet version of the classical system of school-based mathematics education. This Soviet version achieved its greatest functionality in the 1940s–1950s, but practically exhausted its resources by the 1960s. From 1964–1981 there was a reformation of the Soviet model of the classical system of mathematical education (often referred to as the "Kolmogorov Reform"). In 1982, a counter-reformation movement began, which in large part returned mathematical education to the classical model.

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