

Chapter 1

Mathematical Problem Solving in Singapore Schools

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This opening chapter provides a view of the development of mathematical problem solving in Singapore schools. From a research and curriculum development perspective, this chapter shows how research and development elsewhere had impacted upon the emergence and subsequent development of mathematical problem solving in Singapore schools. From a pedagogical perspective, the chapter shows the range of problem-solving processes students engage in, the variety of pedagogy options available to teachers and the array of tasks that can bring the processes and pedagogy together. From an assessment perspective, the chapter suggests how tasks used in national examinations have a direct influence on the implementation of a problem-solving curriculum. From an economic perspective, this chapter argues that an effective implementation of a problem-solving curriculum equips students with the necessary competencies for a knowledge-based economy.

1 Introduction

In 1992 mathematical problem solving was made the primary goal of the school mathematics curriculum in Singapore. Since then, though the curriculum has been revised twice, in 2001 and 2007, mathematical problem solving has remained its primary goal. Figure 1 shows the mathematics curriculum framework for Singapore schools (Ministry of Education, 2006a, 2006b). The emphasis on mathematical problem

solving was influenced by recommendations in documents such as An Agenda for Action (National Council of Teachers of Mathematics, 1980) and the Cockcroft Report (Cockcroft, 1982) from the United States and the United Kingdom respectively. Today, it is rare to find a mathematics curriculum that does not place emphasis on mathematical problem solving.

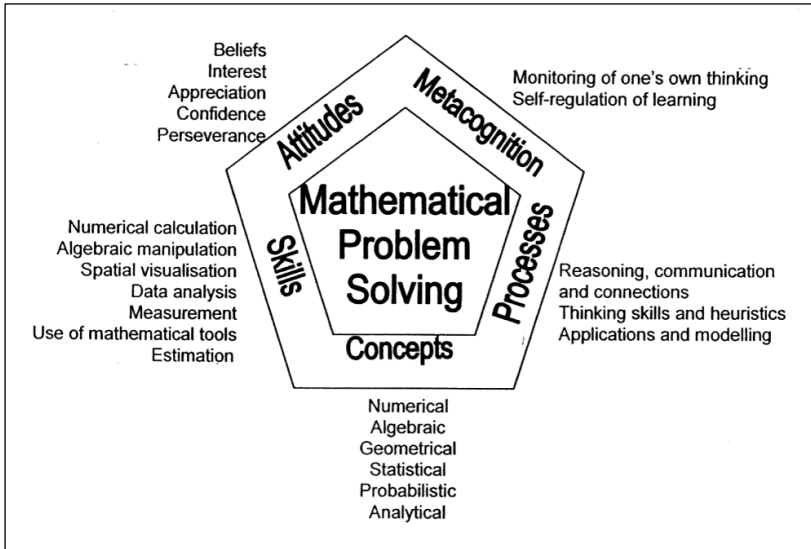


Figure 1. Framework of the Singapore school mathematics curriculum

The seminal doctoral work of Kilpatrick (1967) involving the analysis of solutions of word problems in mathematics at Stanford University and subsequent work by himself and other researchers have established mathematical problem solving as a research field. In particular, Kilpatrick's (1978) classic paper, *Variables and Methodologies in Research on Problem Solving*, outlined key research variables in the field. Since then, mathematical problem solving as a research field has grown and matured to some extent (Lester, 1994; Lesh & Zawojewski, 2007). This has certainly been the case in Singapore

(Foong, 2009). In a state-of-the art review in the early 1990s, Chong, Khoo, Foong, Kaur and Lim-Teo (1991) found that research in mathematics education in Singapore, in general, and problem solving, in particular, to be in its state of infancy. Since then, significant work had been done. Early studies in mathematical problem solving on students (Kaur, 1995) and teachers (Foong, 1990) have stimulated further research into the domain. Kaur (1995) investigated the strategies used by middle school students in solving non-routine problems and clarified the relationship between students' ability to perform particular mathematical procedures and their ability to solve problems. Foong (1990) investigated the problem-solving processes used by pre-service teachers in solving non-routine problems. A recent review of research, by Foong (2009), on mathematical problem solving in Singapore has indicated that our knowledge on problem-solving approaches and tasks used in the classroom, teachers' beliefs and practices, and students' problem-solving behaviours have grown. It is important that such rich research findings find their way into the classrooms. This book showcases several research findings and theories translated into classroom practice.

2 Mathematical Problem Solving

Mathematical problem solving occurs when a task provides some blockage (Kroll & Miller, 1993). Lester (1983) describes a mathematical problem as a task that a person or a group of persons want or need to find a solution for and for which they do not have a readily accessible procedure that guarantees or completely determines the solution.

How does the mathematics textbooks used in Singapore encourage problem solving? Ng (2002) found that the majority of the problems in the primary textbooks were word problems that are closed and routine. Open-ended problems were not common. Fan and Zhu (2000) found that while the lower secondary textbooks provided students with a strong foundation in problem solving, more open-ended problems as well as authentic real-life problems could be included. It is, thus, timely that several chapters in this book attempts to broaden the conception of what

it means to engage in mathematical problem solving. The chapter by **Yeo Kai Kow** describes the importance of open-ended problems in lower secondary levels. The chapter by **Yeo Boon Wooi** and **Yeap Ban Har** clarifies the relationship between mathematical problem solving and mathematical investigation. The chapter by **Ang Keng Cheng** helps readers understand the role of mathematical modeling in real-world mathematical problem solving. **Yeap Ban Har** described the processes in mathematical problem posing to show its relationship to mathematical problem solving.

3 Pedagogy and Practice in Mathematical Problem Solving

Textbook analysis studies and classroom studies have shown that the vast majority of textbook tasks are well-structured tasks (Ng, 2002; Fan & Zhu, 2000) and classroom instruction is mostly teacher-led (Ho, 2007). Foong (2002) has found that teachers in Singapore tend to adopt the teaching for problem solving approach where the emphasis is learning mathematics content for the purpose of applying them to a wide range of situations. Ho's (2007) case studies of four primary-level teachers confirmed, and provided more information for, this finding. With the call for a wider repertoire of teaching methods, in general, and of problem-solving instruction, in particular, it is necessary for teachers to explore alternative pedagogies for mathematical problem-solving instruction.

In the chapter by **Manu Kapur**, it is interesting to note that the use of ill-structured problems as well as students experiencing productive failure resulted in students performing significantly better in problem-solving tasks. The chapter by **Lillie Albert**, **Christopher Bowen** and **Jessica Tansey** describes note taking as a pedagogical tool to develop mathematical problem solving. The chapter by **Yoshinori Shimizu** provides an insider's perspective to the findings from an international study about the way mathematics lessons are conducted in typical Japanese classrooms and describes a typical mathematics lesson in Japan that is best described as structured problem solving. In the chapter by

Yeap Ban Har, how mathematical problem posing was used in several primary-level classes in Singapore is described.

With advances in information and communication technology, it is not possible to avoid the impact of technology on mathematical problem solving. Chua (2001) described the processes of social construction of mathematical ideas as students solved problems in pairs in a computer-mediated environment. In this book, the chapter by **Sarah Davis** shows the immense potential of a technology-supported classroom pedagogy that requires students to work together. The chapter by **Ang Keng Cheng** also emphasizes the central role of technology in mathematical modeling processes.

These chapters show how teachers in Singapore and elsewhere used pedagogy that departs from typical well-structured tasks and teacher-led classroom instruction. Such pedagogical practices provide readers with a repertoire of instructional models to teach mathematical problem solving in their own classrooms. The chapter by **Peter Sullivan, Judith Mousley** and **Robyn Jorgensen** provides research-based teacher actions that can facilitate mathematical problem solving.

4 Mathematical Problem-Solving Tasks

The Singapore mathematics curriculum defines problems to include a wide range of situations, including non-routine, open-ended and real-world problems (Ministry of Education, 2006a, 2006b). Figures 2, 3 and 4, show problems that students had to solve in the national examinations of recent years. The problem in Figure 2 was from the sixth grade national examination (Primary School Leaving Examination). The problem in Figure 3 was from the tenth grade national examination (General Certificate of Education Ordinary Level Examination). The problem in Figure 4 was from the twelfth grade national examination (General Certificate of Education Advanced Level Examination). Each of the problems was novel in that it was the only time a task of that type was posed in the respective examinations.

Table 1

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56

Table 1 consists of numbers from 1 to 56. Kay and Lin are given a plastic frame that covers exactly 9 squares of Table 1 with the centre square darkened.

- (a) Kay puts the frame on 9 squares as shown in the figure below.

3	4	5
11		13
19	20	21

What is the average of the 8 numbers that can be seen in the frame?

- (b) Lin puts the frame on some other 9 squares.
The sum of the 8 numbers that can be seen in the frame is 272.
What is the largest number that can be seen in the frame.

Figure 2. A problem from the grade six national examination
(Singapore Examination and Assessment Board, 2009)

A fly, F, starts at a point with position vector $(\mathbf{i} + 12\mathbf{j})$ cm and crawls across the surface with a velocity of $(3\mathbf{i} + 2\mathbf{j})$ cm s^{-1} . At the instant the fly starts crawling, a spider, S, at the point with position vector $(85\mathbf{i} + 5\mathbf{j})$ cm, sets off across the surface with a velocity of $(-5\mathbf{i} + k\mathbf{j})$ cm s^{-1} , where k is a constant. Given that the spider catches the fly, calculate the value of k .

Figure 3. A problem from the grade 10 national examination
(Ministry of Education, 2007)

Four friends buy three different kinds of fruit in the market. When they get home they cannot remember the individual prices per kilogram, but three of them can remember the total amount that they each paid. The weights of fruit and the total amounts paid are shown in the following table.

	Suresh	Fandi	Cindy	Lee Lian
Pineapple (kg)	1.15	1.20	2.15	1.30
Mangoes (kg)	0.60	0.45	0.90	0.25
Lychees (kg)	0.55	0.30	0.65	0.50
Total amount paid in \$	8.28	6.84	13.05	

Assuming that, for each variety of fruit, the price per kilogram paid by each of the three friends is the same, calculate the total amount that Lee Lian paid.

Figure 4. A problem from the grade 12 national examination
(Singapore Examination and Assessment Board, 2008)

Given that test items in Singapore's national examinations comprises of some problems, it is a challenge for teachers to generate such novel tasks for their students to attempt during instruction. The chapter by **Dindyal Jaguthsing** describes problems for secondary level-students and the processes students engage in when attempting them. The chapter by **Toh Tin Lam** shows tasks that have the ability to spark the curiosity in students. **Yeo Kai Kow** presents open-ended tasks that require students to delve into their conceptual understanding. **Catherine Vistro-Yu** shows how a familiar task can be systematically transform to

generate a set of related tasks, some of which are novel. This technique is useful to Singapore teachers who often need to design worksheets comprising of a set of problems for students to consolidate their mathematical problem-solving ability. In the chapter by **Yoshinori Shimizu** readers are able to see how good lessons can be constructed around carefully-selected problems. The use of a set of related problems as well as centering lessons around good problems give students opportunities to have prolonged and deep engagement with the tasks.

5 Mathematical Problem Solving and the Education System in Singapore

The vision of the Ministry of Education in Singapore is Moulding the Future of the Nation i.e. education is perceived as critical to the survival of the country. Mathematics and other school subjects are platforms for students to develop a set of competencies that hold them in good stead to function well in the type of economy that Singapore engages in. It is no wonder that the Ministry of Education has over the years introduced a slew of initiatives, two of which are Thinking School, Learning Nation (TSLN) and Teach Less, Learn More (TLLM). TSLN aims to develop good thinking through school subjects. TLLM encourages teachers to reduce the content taught via direct teaching but instead engage students in meaningful activities so that they use knowledge to solve problems and whilst solving problems extend their knowledge through inquiry. Thus, a shift in the emphasis of mathematics teaching and learning from acquisition of skills to “development and improvement of a person’s intellectual competence” (p.5, Ministry of Education, 2006a), makes it necessary for mathematics education to make mathematical problem solving and its instruction its focus. It is the aim of this book to provide readers with a range of ideas on how this can happen in the mathematics classroom.

6 Concluding Remarks

It has been 17 years since mathematical problem solving was introduced as the primary aim of learning mathematics in Singapore schools. While

many teachers are now familiar with the notion of mathematical problem solving as well as various problem-solving heuristics used during problem solving, the challenge of balancing between developing fluent basic skills and problem-solving ability remains. Some teachers may perceive these as mutually exclusive. There are several chapters in this book that provide the alternate perspectives that acquisition of basics is not mutually exclusive with the development of mathematical problem-solving ability. Given that teachers are already familiar with the notion of mathematical problem solving, it is timely to step back and examine what it means to learn mathematics, and in the process, derive implications for mathematics education research and practice as well as some of the critical issues that the AME yearbooks could focus on in the coming years. Chapter 14 by **Manu Kapur** aims to do precisely this. By drawing on the folk categories of “learning about” a discipline and “learning to be” a member of the discipline (Thomas & Brown, 2007), Kapur proposes a move beyond the *pedagogy* of mathematics to include the *epistemology* of mathematics. To this end, he puts forth three essential research thrusts: a) understanding children’s inventive and constructive resources, b) designing formal and informal learning environments to build upon these resources, and c) developing teacher capacity to drive and support such change.

Several chapters in this book arose out of the keynote lectures and workshops conducted during the annual Mathematics Teachers Conference of 2008 which was jointly organized by the Association of Mathematics Educators in Singapore and the Mathematics and Mathematics Academic Group at the National Institute of Education in Singapore. The annual conference is very well attended by mathematics teachers in Singapore with an increasing number of foreign teachers joining the event each year. The yearbook, of which this is the first in the series, provides multiple perspectives to a selected aspect of mathematics education – mathematical problem solving. Such a treatment of mathematical problem solving is done with a purpose of bring mathematical problem-solving instruction to the next level.

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