

CONCERNING A SYMMETRY PROPERTY OF THE NEW GELL-MANN THEORY

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RECENTLY Gell-Mann¹ has proposed a new theory of the interaction between elementary particles. In this theory all baryons have spin $\frac{1}{2}$, and the same mechanical mass and parity. They form a supermultiplet which is split up only when moderately-strong interactions occur by way of K mesons. The Hamiltonian of the interaction with π mesons is written in the following way:

$$H_{\pi} = ig \{ (\bar{p}\gamma_{\mu}\rho - \bar{n}\gamma_{\mu}n) \pi^0 + \sqrt{2} (\bar{p}\gamma_{\mu}n\pi^+ + \bar{n}\gamma_{\mu}\rho\pi^-) + (\bar{\Xi}^0\gamma_{\mu}\Xi^0 - \bar{\Xi}^-\gamma_{\mu}\Xi^-) \pi^0 + \sqrt{2} (\bar{\Xi}^0\gamma_{\mu}\Xi^-\pi^+ + \bar{\Xi}^-\gamma_{\mu}\Xi^0\pi^-) \\ + (\bar{\Sigma}^+\gamma_{\mu}\Sigma^+ - \bar{Y}^0\gamma_{\mu}Y^0) \pi^0 + \sqrt{2} (\bar{\Sigma}^+\gamma_{\mu}Y^0\pi^+ + \bar{Y}^0\gamma_{\mu}\Sigma^+\pi^-) + (\bar{Z}^0\gamma_{\mu}Z^0 - \bar{\Sigma}^-\gamma_{\mu}\Sigma^-) \pi^0 + \sqrt{2} (\bar{Z}^0\gamma_{\mu}\Sigma^-\pi^+ + \bar{\Sigma}^-\gamma_{\mu}Z^0\pi^-) \}. \quad (1)$$

(see Ref. 1 for the notation). In our note we shall show that in addition to the so-called global symmetry discussed by Gell-Mann, there is another symmetry property of his theory even in the absence of K coupling.

In the Hamiltonian (1) we make the substitutions

$$p \rightarrow \Xi_c^-, \quad n \rightarrow \Xi_c^0, \quad \Xi^- \rightarrow \rho_c, \quad \Xi^0 \rightarrow n_c, \quad \Sigma^+ \rightarrow \Sigma_c^-, \quad Y^0 \rightarrow Z_c^0, \quad \Sigma^- \rightarrow \Sigma_c^+, \quad Z^0 \rightarrow Y_c^0, \quad \pi^+ \rightarrow \pi^+, \quad \pi^- \rightarrow \pi^-, \quad \pi^0 \rightarrow -\pi^0, \quad (2)$$

where A_c denotes the charge-conjugation operator of the field of particle A. After this substitution the Hamiltonian takes the form

$$H_{\pi} = ig \{ (\bar{\Xi}_c^0\gamma_{\mu}\Xi_c^0 - \bar{\Xi}_c^-\gamma_{\mu}\Xi_c^-) \pi^0 + \sqrt{2} (\bar{\Xi}_c^-\gamma_{\mu}\Xi_c^0\pi^+ + \bar{\Xi}_c^0\gamma_{\mu}\Xi_c^-\pi^-) + (\bar{\rho}_c\gamma_{\mu}\rho_c - \bar{n}_c\gamma_{\mu}n_c) \pi^0 + \sqrt{2} (\bar{n}_c\gamma_{\mu}\rho_c\pi^+ + \bar{\rho}_c\gamma_{\mu}n_c\pi^-) \\ + (\bar{Z}_c^0\gamma_{\mu}Z_c^0 - \bar{\Sigma}_c^-\gamma_{\mu}\Sigma_c^-) \pi^0 + \sqrt{2} (\bar{\Sigma}_c^-\gamma_{\mu}Z_c^0\pi^+ + \bar{Z}_c^0\gamma_{\mu}\Sigma_c^-\pi^-) + (\bar{\Sigma}_c^+\gamma_{\mu}\Sigma_c^+ - \bar{Y}_c^0\gamma_{\mu}Y_c^0) \pi^0 + \sqrt{2} (\bar{Y}_c^0\gamma_{\mu}\Sigma_c^+\pi^+ + \bar{\Sigma}_c^+\gamma_{\mu}Y_c^0\pi^-) \}. \quad (3)$$

The following relations exist between the field operators A and B:²

$$\bar{A}B = \bar{B}A_c, \quad \bar{A}\gamma_{\mu}B = -\bar{B}_c\gamma_{\mu}A_c, \quad \bar{A}\gamma_{\mu}\gamma_5B = -\bar{B}_c\gamma_{\mu}\gamma_5A_c, \quad \bar{A}\gamma_{\mu}\gamma_5B = \bar{B}_c\gamma_{\mu}\gamma_5A_c, \quad \bar{A}\gamma_5B = \bar{B}_c\gamma_5A_c. \quad (4)$$

With the help of (4) we can satisfy ourselves that the Hamiltonian (3) agrees with (1). In the same way it can be shown that the Hamiltonian of the interaction of baryons with photons

$$H_{\gamma} = -ie \{ \bar{p}\gamma_{\mu}\rho - \bar{\Xi}^-\gamma_{\mu}\Xi^- + \bar{\Sigma}^+\gamma_{\mu}\Sigma^+ - \bar{\Sigma}^-\gamma_{\mu}\Sigma^- \} A_{\mu} \quad (5)$$

is invariant under the substitution (2), with the addition of the substitution $A_{\mu} \rightarrow A_{\mu}$. This symmetry property of the Gell-Mann theory provides us with selection rules for certain processes. Let us consider processes in which there are only π mesons and photons in the initial and final states. For every Feynman diagram there will be another diagram in which the lines of the inner baryons are replaced by the propagation lines of the baryons appearing in the right-hand sides of the substitution (2). For example, proton lines are replaced by the lines of the baryon described by the field operator Ξ_c^- (i.e., by the lines of an anti Ξ^- -particle), antiproton lines by the lines of a Ξ_c^0 -particle, etc. If the number of outer π^0 mesons is odd, then the matrix elements of these two graphs will cancel each other when they are added. Consequently in the new Gell-Mann theory the process of the decay of a π^0 meson into two γ -rays takes place only through a K interaction.

If it is assumed that β interactions of baryons also have Gell-Mann's global symmetry, then we can obtain a few other selection rules. Consider, for example, a tensor β interaction:

$$H_{\beta} = g_{\beta} \{ \bar{p}\gamma_{\mu}\gamma_5n + \bar{\Xi}^0\gamma_{\mu}\gamma_5\Xi^- + \bar{\Sigma}^+\gamma_{\mu}\gamma_5Y^0 + \bar{Z}^0\gamma_{\mu}\gamma_5\Sigma^- \} e_{\gamma_{\mu}\gamma_5\nu}.$$

It is clear that this Hamiltonian changes sign under the substitution (2). Thus the matrix elements for the β -decay of a π^+ meson, that is

$$\pi^+ \rightarrow e^+ + \nu + n_{\gamma}, \quad n = 0, 1, \dots$$

through a virtual baryon-pair, cancel each other in pairs. These processes also are allowed only through a K interaction.

If only the interactions of π mesons with nucleons are taken into account, then the theoretical probabilities of the processes

$$\pi^0 \rightarrow 2\gamma \text{ (Ref. 3) and } \pi^+ \rightarrow e^+ + \nu + \gamma \text{ (Ref. 4),}$$

obtained from perturbation theory, are greater than those observed. The considerations above indicate that it is possible to reduce the discrepancy between theory and experiment by allowing also for the interactions of π mesons with all baryons.* Of course, without a study of the interaction with a K meson we are still unable to say with certainty that the new Gell-Mann theory gives better agreement with the experimental data for these processes.

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*This same result for the decay process $\pi^0 \rightarrow 2\gamma$ was also obtained by Gell-Mann.¹ An analogous result was obtained earlier by Kinoshita.³

¹M. Gell-Mann, Phys. Rev. 106, 1296 (1957).

²R. H. Good, Jr., Rev. Mod. Phys. 27, 187 (1955).

³T. Kinoshita, Phys. Rev. 94, 1384 (1954).

⁴S. B. Treiman and H. W. Wyld, Jr., Phys. Rev. 101, 1552 (1956).

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