Chapter 1

Introduction

1.1 High-energy nuclear collisions

Physics of the ultra-relativistic heavy-ion collisions is an interdisciplinary field which connects the high-energy physics of elementary particles with the nuclear physics. The name "heavy-ions" is used for heavy atomic nuclei, whereas the term "ultra-relativistic energy" denotes the energy regime where the kinetic energy exceeds significantly the rest energy. Typically, the high-energy particle physics deals with single particles (leptons, quarks, hadrons), and the interactions are derived from first principles. On the other hand, the nuclear physics deals with extended, complicated objects (nuclei), and the interactions are described by effective models. In the new field of the ultra-relativistic heavy-ion collisions one tries to analyze the properties of hot and dense nuclear/hadronic matter in terms of elementary interactions. Of the special importance are experimental searches for theoretically predicted new phases of hadronic matter, identification of the phase transitions between those phases, and a possible reconstruction of the phase diagram of strongly interacting matter in the broad range of the thermodynamic parameters such as temperature or baryon chemical potential.

In the last thirty years the nuclear physics has changed its character in a very significant way. In the 1970s and at the beginning of 1980s several accelerators used by the particle physics community were modified to accelerate heavy ions. For example, the Bevatron in Berkeley was coupled with the SuperHilac to form the Bevalac [1]. The SuperHilac was a linear accelerator where the ions of heavy elements were created and sent for further acceleration to the Bevatron. In this way the relativistic energies of 1–2 GeV per nucleon were achieved. Similarly, the Dubna Syncrophasotron was converted to accelerate heavy ions. On the other hand, in the same time a number of accelerators used in the nuclear research were developed, yielding relativistic beams of heavy ions in many places, for example, at the Gesellschaft für Schwerionenforschung (GSI) in Darmstadt.

The first experiments with the ultra-relativistic heavy ions (with energies exceeding 10 GeV per nucleon in the projectile beam) took place at the Brookhaven National Laboratory (BNL) and at the European Organization for Nuclear Re-

Table 1.1 Summary of the RHIC runs in the years 2000–2006: number of the run, date, colliding systems, and energy.

Run	Year	Species	$\sqrt{s_{NN}}$ [GeV]
01	2000	Au+Au	130
02	2001–2	Au+Au	200
		p+p	200
03	2002–3	d+Au	200
		p+p	200
04	2003–4	Au+Au	200
		Au+Au	62
05	2004–5	Cu+Cu	200
		Cu+Cu	62
		Cu+Cu	22.5
		p+p	200
06	2006	p+p	200
		p+p	62

search (CERN) in 1986. The Alternating Gradient Synchrotron (AGS) at BNL accelerated beams up to ^{28}Si at 14 GeV per nucleon. At CERN, the Super Proton Synchrotron (SPS) accelerated ^{16}O at 60 and 200 GeV per nucleon in 1986, and ^{32}S at 200 GeV per nucleon in 1987. In 1990 a long-term project on heavy-ion physics was realized at CERN with several weeks of ^{32}S beams. In the spring of 1992 the experiments with ^{197}Au beams at 11 GeV per nucleon were initiated at BNL. In 1995 the completely new experiments took place at CERN with ^{208}Pb beams at 158 GeV per nucleon. These were for the first time really ultra-relativistic "heavy" ions providing large volumes and lifetimes of the reaction zone.

In 2000 the first data from the Relativistic Heavy Ion Collider (RHIC) at BNL were collected. RHIC was designed to accelerate fully stripped Au ions to a collision center-of-mass energy of 200 GeV per nucleon pair, i.e., for $\sqrt{s_{NN}} = 200$ GeV (for the history of the construction of RHIC see [2]). The design luminosity corresponds to approximately 1400 Au+Au collisions per second. During the first run in 2000, the maximum energy of 130 GeV per nucleon pair was achieved, with 10% of the designed luminosity. In the years 2001–2004 the next three runs took place with the maximum energy of 200 GeV per nucleon pair. One of those runs was devoted to the study of the deuteron-gold collisions which were analyzed in order to get the proper reference point for the more complicated gold on gold collisions. Altogether, in the years 2000–2006 six runs took place with different colliding systems and at different beam energies, see Table 1.1.

There are four experiments at RHIC. The two smaller experiments are BRAHMS and PHOBOS, and the two larger experiments are PHENIX and STAR. The experimental aim of BRAHMS is particle identification over a broad rapidity range.

Table 1.2 *Quark Matter* Conferences

Number	Date	Place	Proceedings
1th	Aug. 24–31, 1980	Bielefeld, Germany	[3]
2nd	May 10–14, 1982	Bielefeld, Germany	[4]
3rd	Sept. 26–30, 1983	Upton, USA	[5]
4th	June 17–21, 1984	Helsinki, Finland	[6]
5th	April 13–17, 1986	Pacific Grove, USA	[7]
6th	Aug. 24–28, 1987	Nordkirchen, Germany	[8]
7th	Sept. 26–30, 1988	Lenox, USA	[9]
8th	May 7–11, 1990	Menton, France	[10]
9th	Nov. 11–15, 1991	Gatlinburg, USA	[11]
10th	June 20–24, 1993	Borlänge, Sweden	[12]
11th	Jan. 9–13, 1995	Monterey, USA	[13]
12th	May 20–24, 1996	Heidelberg, Germany	[14]
13th	Dec. 1–5, 1997	Tsukuba, Japan	[15]
14th	May 10–15, 1999	Torino, Italy	[16]
15th	Jan. 15–20, 2001	Stony Brook, USA	[17]
16th	July 18–24, 2002	Nantes, France	[18]
17th	Jan. 11–17, 2004	Oakland, USA	[19]
18th	Aug. 4–9, 2005	Budapest, Hungary	[20]
19th	Nov. 14–20, 2006	Shanghai, China	[21]
20th	Feb. 4–10, 2008	Jaipur, India	[22]
21th	Mar. 30–Apr. 4, 2009	Knoxville, USA	
22th	2011	Annecy, France	

The PHOBOS experiment measures total charged particle multiplicity and particle correlations. The PHENIX experiment is designed to measure electrons, muons, hadrons and photons. The STAR experiment concentrates on measurements of hadron production over a large solid angle.

The future of the field is connected with the construction of the Large Hadron Collider (LHC) at CERN (Pb on Pb reactions at $\sqrt{s_{\rm NN}} = 5.5$ TeV). Nevertheless, the performance of new experiments at lower energies is also very important, since this allows us to study the energy dependence of many characteristics of the particle production. Within the more recent project at the SPS, in the years 1999–2003, the NA49 Collaboration recorded Pb+Pb collisions at 20, 30, 40 and 80 GeV per nucleon. More information on the development of the experimental situation in the last thirty years may be found in the series of the *Quark Matter Proceedings* whose list is given in Table 1.2 [1].

[1] It is not quite clear what the first conference in this series was. An alternative for the first position listed in Table 1.2 is the *First Workshop on Ultra-Relativistic Nuclear Collisions*, organized at the Lawrence Berkeley Laboratory in 1979, LBL report 8957.

1.2 Theoretical methods

In the ultra-relativistic heavy-ion collisions very large numbers of particles are produced (we deal with so called large particle *multiplicities*). For example, in the central Au+Au collisions at RHIC, at the highest beam energy $\sqrt{s_{\rm NN}} = 200$ GeV, the total charged particle multiplicity is about 5300 [23]. Hence, the number of produced particles exceeds the number of initial nucleons by a factor of 10. In this situation, different theoretical methods are used, which are suitable for description of large macroscopic systems, e.g., thermodynamics, hydrodynamics, kinetic (transport) theory, field theory at finite temperature and density, non-equilibrium field theory, Monte-Carlo simulations.

Many estimates of the effects in high-energy nuclear collisions are done on the basis of purely *thermodynamic* or *statistical* considerations. However, the hadronic systems produced in the collisions are not static. The need for the dynamical description involves rich applications of *relativistic hydrodynamics.* Furthermore, since the matter produced in the collisions lives only for a short while, it is natural to expect that its spacetime evolution proceeds far away from equilibrium. Consequently, there exists a growing interest in applying and developing *transport theories* which are suitable for the description of non-equilibrium processes. In the similar spirit, one tries to describe the heavy-ion reactions with the help of *microscopic Monte-Carlo simulations* which usually represent an extrapolation of low energy models of hadron-hadron collisions. Last but not least, the physics of the ultra-relativistic heavy-ion collisions triggered fast development of the *quantum theory of fields in and out of equilibrium.* Using the methods of field theory one can study the in-medium properties of particles. Moreover, this approach allows also for the formulation of the kinetic equations satisfied by the particle distribution functions.

1.3 Quantum chromodynamics

Generally speaking, during high-energy nuclear collisions a many-body system of *strongly* interacting particles is produced. The fundamental theory of the strong interactions is *Quantum Chromodynamics* (QCD), the theory of quarks and gluons which are confined in hadrons, i.e., baryons and mesons.

From the historical perspective we may say that the development of QCD started with the 1963 proposal of Gell-Mann [24] and Zweig suggesting that the structure of hadrons could be explained by the existence of smaller particles inside hadrons (at that time u, d and s quarks). In 1964 Greenberg [25] and in 1965 Han with Nambu [26] proposed that quarks possessed an additional degree of freedom, that was later called the *color charge.* Han and Nambu noted that quarks might interact via exchanges of an octet of vector gauge bosons (later *gluons*). Feynman and Bjorken argued that high-energy experiments should reveal the existence of partons, i.e., particles that are parts of hadrons. Those suggestions were spectacularly veri-

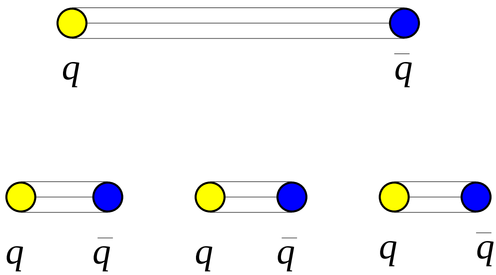

Fig. 1.1 Schematic view of the confinement mechanism. The separating $q\bar{q}$ pair stretches the color string until the increasing potential energy is sufficient to create another $q\bar{q}$ pair. More pairs may be produced in this way, which leads to the formation of final state hadrons.

fied in the *deep inelastic scattering* of electrons on protons, the experiments carried out at the Stanford Linear Accelerator Center (SLAC) in 1969. The partons were identified with quarks. The discovery of asymptotic freedom in the strong interactions by Gross, Politzer and Wilczek [27–29] allowed for making precise predictions of the results of many high-energy experiments in the framework of the perturbative quantum field theory [30] — the asymptotic freedom is the property that the interaction between particles becomes weaker at shorter distances.

Probably the most striking feature of QCD is the color confinement [31–33], which is the other side of the asymptotic freedom. This is the phenomenon that color charged particles (such as quarks and gluons) cannot be isolated as separate objects. In other words, quarks and gluons cannot be directly observed. The physical concept of confinement may be illustrated by a string which is spanned between the quarks when we try to separate them, see Fig. 1.1. If the quarks are pulled apart too far, large energy is deposited in the string which breaks into smaller pieces. As a result the quarks form new hadrons produced from the pieces of the initial string.

Such a qualitative picture of confinement is supported strongly by the numerical calculations. On the other hand, at the moment there is no analytic approach or approximation that describes the behavior of QCD at large distances. Lacking this, we often feel that we do not understand fully the mechanism of confinement and its proof is missing. This situation is reflected by the fact that the Clay Mathematics Institute of Cambridge includes the confinement problem as one of the seven Millennium Problems and offers a prize of one million dollars for the proof [34].

In some sense, the *nuclear* force between baryons and mesons can be viewed as a residual force acting between quarks and gluons, in the analogous way as the *chemical* (van der Waals) force is the residual electromagnetic interaction. Since QCD is a complicated non-linear theory, the *complete* description of the relativistic heavy-ion collision based exclusively on first principles is impossible in practice. Almost in all cases we have to use models, although QCD may be successfully applied to describe subprocesses of complicated collisions or to deliver an input for modeling. In particular, the lattice simulations of QCD give us information about the equation of state of strongly interacting hot matter, which may be then used as an input for the hydrodynamical codes. In addition, theoretical calculations based on QCD inspire new measurements.

More information about QCD (although still very much restricted) will be given in Chap. 5. We refer also to the general textbooks discussing QCD, for example, Refs. [35, 36].

1.4 Quark-gluon plasma

The main challenge of the ultra-relativistic heavy-ion collisions is the observation of the two phase transitions predicted by QCD, i.e., the *deconfinement* and *chiral phase transitions*. As we have mentioned above, at Earth conditions (i.e., at low energy densities) quarks and gluons are confined in hadrons. However, with increasing

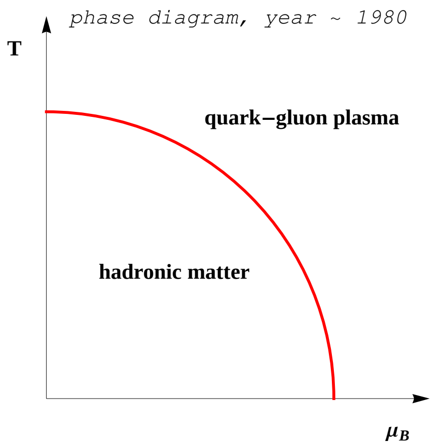

Fig. 1.2 The first phase diagram of strongly interacting matter was introduced in the paper by Cabibbo and Parisi in 1975 [37]. It looked similar to the plot shown here. The line distinguishes two regions in the two-dimensional space of temperature, T, and baryon chemical potential, μ_B. For smaller values of T and/or μ_B (the points below the curve) the matter is made out of hadrons, whereas at sufficiently high values of T or μ_B (the points above the curve) the matter is made up of deconfined quarks and gluons.

temperature (heating) and/or increasing baryon density (compression), a phase transition may occur to the state where the ordinary hadrons do not exist anymore, and where quarks and gluons become the correct degrees of freedom. In 1975, soon after the discovery of asymptotic freedom in the strong interactions, Collins and Perry argued that "superdense matter (found in neutron-star cores, exploding black holes, and the early big-bang universe) consists of quarks rather than of hadrons" [38]. In the same year, Cabibbo and Parisi identified the limiting Hagedorn temperature with the temperature of the phase transition from hadronic to quark matter [37]. They also sketched the first phase diagram of strongly interacting matter, see Figs. 1.2 and 1.3. The collective phenomena in gauge theories were studied then by Kislinger and Morley [39, 40]. Freedman and McLerran computed the three-loop contributions to the thermodynamic potential [41–43]. Calculations at finite temperature were performed by Shuryak [44, 45], who in 1978 introduced the name *quark-gluon plasma* (QGP), and by Kapusta [46]. The first quantitative considerations concerning the possibility of the formation of hot quark matter in relativistic heavy-ion collision were given by Chin [47] [2].

The present experimental evidence indicates that in the ultra-relativistic heavy-ion collisions an extended and very dense system of strongly interacting matter is indeed formed. It differs in many aspects from the systems formed in elementary hadron-hadron reactions. On Feb. 10, 2000, CERN announced officially [48] that "a compelling evidence now exists for the formation of a new state of matter at energy densities about 20 times larger than in the center of atomic nuclei and temperatures about 100000 times higher than in the center of the sun". This announcement followed the analysis of many experimental data collected during 15 years of heavy-ion experiments at the SPS.

The first RHIC data confirmed the overall picture that emerged from the studies at lower energies. However, several new features of the collision process were observed, e.g., higher particle multiplicities, increased production of antiparticles, strong collective phenomena, and lower baryon number density in the central rapidity region. The data collected in the next runs, especially in the deuteron-gold collisions which were used as a reference measurement, brought the evidence for strong quenching of very energetic particles traversing the medium created in the central Au+Au collisions. The streams of such particles, called jets, appear in elementary collisions where they form two, flying back-to-back, groups of hadrons. The RHIC data indicate that such back-to-back correlations between very energetic hadrons are lost in central Au+Au collisions. This effect may be understood easily by the assumption that one of the jets is absorbed by the medium because it has a longer distance to traverse in the medium.

The notorious question is asked if the quark-gluon plasma has been indeed dis-

[2] Probably, for the first time the idea of using the ultra-relativistic heavy-ion collisions to produce and study new forms of matter was introduced during the *Workshop on BeV Collisions of Heavy Ions: How and Why*, which was held in Bear Mountain, New York, Nov. 29 - Dec. 1, 1974, BNL-AUI report, 1975.

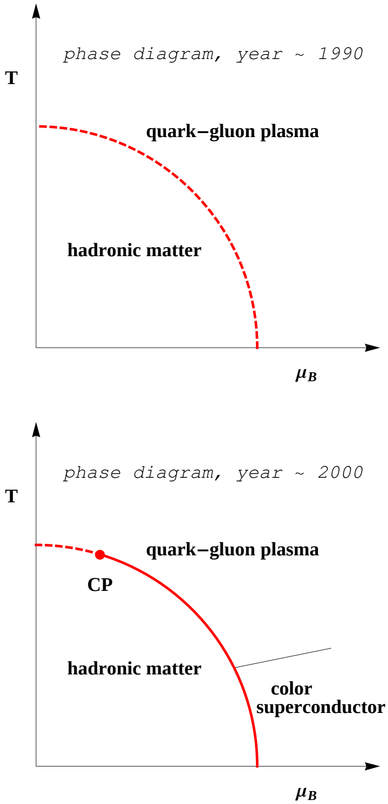

Fig. 1.3 Time evolution of our views on the phase diagram of strongly interacting matter. Around 1990 most physicists believed that there was a crossover between the hadronic matter and the quark-gluon plasma, i.e., a sudden change in the energy density, but not real phase transition (dashed line in the upper part). By the year 2000 this opinion had changed. At present, we expect that there is a line of the first order phase transition ending in a critical point (solid thick line in the lower part). In addition, at very high baryon density (large baryon chemical potential) there is a color superconductivity region, for a review see [49]. The crossover transition takes place at low baryon density and high temperature (dashed line in the lower part).

covered in the relativistic heavy-ion collisions. If we have in mind the asymptotic state, where due to the asymptotic freedom the plasma is treated as an ideal gas of quarks and gluons, then the question seems to be quite difficult or even impossible to answer. Although different "plasma signatures" have been proposed, we have no definite proof that such an asymptotic state has been reached. On the other hand, the phenomena such as the absorption of jets or simple estimates of the energy density accessible at the center of relativistic heavy-ion collisions indicate that the produced matter cannot consist of hadrons — the hadronic sizes are simply too big to allow for the treatment of hadrons in the initial stages of the collisions as well isolated degrees of freedom. Consequently, the matter produced in the ultra-relativistic heavy-ion collision is definitely a system of interacting quarks and gluons. It is much denser than that produced in more elementary hadronic or proton-nucleus collisions. Moreover, the system created in heavy-ion collisions exhibits high level of thermalization and shows strong collective behavior. Accepting these facts, it is natural to admit that the produced matter is an interacting quark-gluon plasma [50–53]. This point of view has been adopted in this book. The remaining problem is, however, to establish more precisely the physical properties of the plasma. In particular, it is important to conclude how strongly interacting system it is [54–56].

Clearly, further systematic studies are necessary now to extract more detailed information about the dense medium found in the heavy-ion experiments. Certainly, only now this field of physics has come into its mature age. Lead on lead collisions at the LHC, offering the initial energy density 50 to 100 times larger than that of normal nuclear matter, will be the next source of very intriguing data.

1.5 Chiral symmetry

There exist six different types (flavors) of quarks: *up, down, strange, charmed, bottom* and *top.* The bottom and the top quark are sometimes called the beauty and the true one. Let us now restrict our considerations to the subsector of QCD describing only up and down quarks. At normal Earth conditions, the most common hadrons, i.e., protons, neutrons and pions are made of these two flavors. The masses of up and down quarks are very small so they are usually ignored in most of the practical calculations, see Table 1.3.

In the limit of vanishing masses the left- and right-handed quarks become decoupled from each other and QCD becomes invariant under their interchange. One of the consequences of this fact is that there are left- and right-handed quark currents which are separately conserved, instead of only the vector current which is conserved in the massive case. Symmetry between the left- and right-handed quarks implies also that each state of the theory should have a degenerate partner of the opposite parity. On the other hand, we know that hadrons have well defined parity, and no such parity partners are observed! The paradox is resolved by the phenomenon of the *spontaneous breakdown of chiral symmetry*: the chiral symmetry of the interaction is broken by the true ground state of the theory, see Fig. 1.4.

This mechanism was recognized first by Nambu, already in the pre-QCD times [57–59]. There are many examples of such situations in other fields of physics. For instance, the exact translational symmetry is broken by the ground states of solids which are periodic crystals. According to the famous Goldstone theorem [60, 61], spontaneous breaking of any continuous symmetry is connected with the existence of *soft modes.*

Table 1.3 Mass, electric charge, the third component of the isospin, strangeness (S), charm (C), bottom (B), and top (T) of quarks. All quarks have spin $\frac{1}{2}\hbar$, baryon number $\frac{1}{3}$, and lepton number 0. The values of the masses are taken from Ref. [62]. We note that these are *current masses* that appear in the perturbative QCD calculations. They differ from the *constituent masses* that are effective masses of the strongly interacting quarks.

flavour	up (u)	down (d)	strange (s)	charmed (c)	bottom (b)	top (t)
m	1–5 MeV	3–9 MeV	75–170 MeV	1.15–1.35 GeV	4.0–4.4 GeV	174.3 ± 5.1 GeV
Q	$\frac{2}{3}e$	$-\frac{1}{3}e$	$-\frac{1}{3}e$	$\frac{2}{3}e$	$-\frac{1}{3}e$	$\frac{2}{3}e$
I_3	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0
S	0	0	-1	0	0	0
C	0	0	0	+1	0	0
B	0	0	0	0	-1	0
T	0	0	0	0	0	+1

In QCD such soft modes correspond to pseudoscalar pions. Their existence solves the paradox — infinitely soft pseudoscalar modes can be added to any state, changing the parity without any change in energy. In reality pions have a small mass and are sometimes called pseudo-Goldstone bosons. This is due to the fact that the masses of up and down quarks are not exactly zero and the chiral symmetry is approximate.

In a very hot and dense hadronic medium, ordinary hadrons lose their identities and the quark-gluon plasma is produced. In this case the ground state of the strong interactions is significantly modified and the chiral symmetry is expected to be restored; in the natural way the left- and right-handed (practically massless) quark excitations in the plasma are the chiral partners to each other.

Observation of the signals of the chiral phase transition is an exciting perspective of the experiments with heavy ions. It is possible that the two phase transitions, i.e., deconfinement and chiral restoration, do not happen simultaneously. One can imagine that with the increasing temperature we first have deconfinement (but still the quarks have non-zero effective masses) and later the chiral phase transition to massless quarks. In 1983 the lattice simulations of both SU(2) and SU(3) gauge theories indicated, however, that the two phase transitions took place at almost the same temperature [63]. The coincidence of the two temperatures has been confirmed by more recent calculations, for example, see [64]. The issue whether this

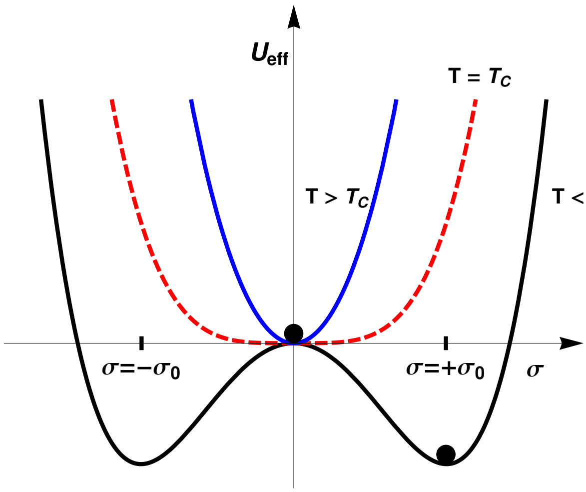

Fig. 1.4 The mechanism of the spontaneous symmetry breaking may be illustrated by a symmetric, temperature dependent potential $U_{\rm eff}(\sigma)$. For $T > T_{\rm c}$ the potential has one (global) minimum and the ground state of the field σ corresponds to the case $\sigma = 0$. If the temperature becomes smaller than $T_{\rm c}$, the potential develops two different local minima. Although the potential is still a symmetric function of σ, the ground state of the field is randomly changed to either $+\sigma_0$ or $-\sigma_0$ (to $+\sigma_0$ in this plot).

coincidence is exact or approximate is under discussion now, but in any case the two temperatures seem to be very close to each other. This is why most physicists identify the two phase transitions. A theoretical possibility still exists, however, that the two phase transitions differ significantly at a finite baryon chemical potential.

1.6 Hot and dense nuclear matter

The study of high-energy nuclear reactions gives us important information about properties of hot and dense hadronic matter — by this we mean here the matter present at the late stages of the collisions where hadrons may be regarded as the correct degrees of freedom. Heavy-ion collisions are the only way to compress and heat up nuclear matter in laboratory conditions. Information extracted from data can be useful for the construction of adequate models of neutron stars and supernova explosions. Already at energies of the order of a few GeV per nucleon one encounters many interesting and well established phenomena like, e.g., collective flows or subthreshold production of particles. Central collisions of symmetric heavy ions at 1 GeV per nucleon (so called *relativistic regime*) are likely to yield about 3 times normal nuclear matter density. The particles inside such a system do not propagate completely freely: their Compton wavelength may be comparable with their mean free path. In this situation, we expect that some of the particle properties (e.g., hadron masses, widths or coupling constants) can be changed. These *in-medium modifications* can lead to the experimentally observed phenomena. For example, the change of the ρ meson mass and/or width in dense matter can influence the measured dilepton spectrum. Nowadays, one attempts to connect in-medium modifications of hadron properties with the *partial* restoration of chiral symmetry.

1.7 Units and notation

In this book we use the natural system of units where the velocity of light in vacuum, c, the Planck constant h divided by 2π, $\hbar$, and the Boltzmann constant, $k_{\rm B}$, are all equal to unity, $c = \hbar = k_{\rm B} = 1$. The exception are the sections devoted to the kinetic theory where the convention $h = 1$ is sometimes more useful. With the choice $c = \hbar = k_{\rm B} = 1$ it becomes unnecessary to write $c, \hbar$, and $k_{\rm B}$ in the equations, thus, we save space and trouble. The dimensional analysis may be always used to unambiguously reinsert those constants into various expressions. It is suggested to the readers who are not familiar with the natural system of units that they first work out Ex. 7.1. For electromagnetic quantities we have adopted the Heaviside-Lorentz system of units where ϵ_0 is set equal to unity and the factors 4π are absent in the Maxwell equations.

The field of the relativistic heavy-ion collisions includes many subfields with commonly accepted notation. In the book discussing phenomena belonging to such different subfields it becomes difficult to use simple and unequivocal notation. We have tried to avoid the situations where the same symbols are used for different physical quantities but sometimes such situations are inevitable. We hope that it does not lead to much confusion, since the proper meaning follows usually from the physical context. The most common symbols used in the book are presented in Tables 1.4–1.7. We note that spatial three-vectors and color two-vectors (in the space of color isotopic charge and color hypercharge) are indicated by letters in boldface.

In the text we often refer to Lorentz transformations. By this we mean the Lorentz boosts, mostly in the direction of the beam axis, or the proper orthochronous Lorentz transformations. Again the correct meaning follows from the context.

Table 1.4 Symbols of the physical quantities used in the book (part 1).

spacetime variables	
$g_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$	metric tensor
$x^\mu = (x^0, x^1, x^2, x^3) = (t, x, y, z)$	spacetime coordinates
$r = \sqrt{x^2+y^2}$	distance from the collision axis
$\phi = \arctan(y/x)$	azimuthal angle
$\theta = \arctan(r/z)$	polar angle
$\tau = \sqrt{t^2-z^2}$	(longitudinal) proper time
$\eta_\parallel = \frac{1}{2}\ln\frac{t+z}{t-z}$	spacetime rapidity
kinematical variables describing single particles	
$m \quad (m_\mathrm{N}, m_\pi)$	mass (nucleon mass, pion mass)
$p^\mu = (p^0, p^1, p^2, p^3) = (E, p_x, p_y, p_z)$	four-momentum
$p^0 = p_0 = E$	energy
$E_p = \sqrt{m^2+p^2}$	mass-shell energy
$p^3 = -p_3 = p_z = p_\parallel$	longitudinal momentum of a particle
$p_\perp = \sqrt{p_x^2+p_y^2}$	transverse momentum of a particle
$\phi_p = \arctan(p_y/p_x)$	momentum azimuthal angle
$m_\perp = \sqrt{m^2+p_\perp^2}$	transverse mass
$\mathrm{y} = \frac{1}{2}\ln\frac{E+p_\parallel}{E-p_\parallel}$	rapidity
$\eta = \frac{1}{2}\ln\frac{\lvert\mathbf{p}\rvert+p_\parallel}{\lvert\mathbf{p}\rvert-p_\parallel}$	pseudorapidity
variables characterizing nucleus-nucleus, nucleon-nucleus, and nucleon-nucleon collisions	
A, B	atomic mass numbers (also labels characterizing nuclei)
ρ_A	Woods-Saxon function
N_part, N_spec	number of participants, number of spectators
n, w	number of binary collisions, number of wounded nucleons
$\mathbf{b}$, b	impact vector, impact parameter
c	centrality
$s = (p_1+p_2)^2$, $t = (p_1-p_1')^2$, $u = (p_1-p_2')^2$	Mandelstam variables
$f(s,t)$	scattering amplitude
θ	scattering angle
σ_tot, σ_in, σ_el	nucleon-nucleon total, inelastic, elastic cross section
σ_incl	inclusive cross section
$\delta(s,\mathbf{b})$	phase shift
$t(\mathbf{b}), T_A(\mathbf{b}), T_{AB}(\mathbf{b})$	thickness functions
R_{AB}	nuclear modification factor
$\mathcal{P}_1, \mathcal{P}_2$	invariant one- and two-particle inclusive distributions

Table 1.5 Symbols of the physical quantities used in the book (part 2).

kinetic-theory variables	
$f(x,p)$	phase-space distribution function
$\tilde{f} = (2\pi\hbar)^3 f = h^3 f$	
ϵ	parameter of the equilibrium distribution functions $\epsilon = +1$ for bosons and $\epsilon = -1$ for fermions
$\bar{f}(x,p) = 1 + h^3 \epsilon f(x,p)$	statistical correction factors in collision terms
$\Phi[f(x,p)]$	functional used to define entropy current
$W_{\alpha,\beta}(x,p)$	Wigner function
$S(x,p)$	emission (source) function
$d\Gamma_{\rm cl} = \frac{dV d^3p}{(2\pi\hbar)^3}$	classical phase-space element
$d\Gamma_{\rm inv} = dV\, dt\, \frac{d^3p}{E_p}$	Lorentz invariant phase-space element
$N^\mu = (n^0, \mathbf{n})$	particle number current
$j^\mu = (\rho, \mathbf{j})$	electric current
$T^{\mu\nu}$	energy-momentum tensor
S^μ, $\Delta\overline{\Gamma}$	entropy current, statistical weight
$\Delta\Gamma$	differential transition rate
$\Delta\sigma$	differential cross section
$\Phi_{\rm i}$, $F_{\rm i}$, $v_{\rm M}$	flux, invariant flux, Møller velocity
C, C_{kl}	collision terms
$W(p,p_1\|p',p'_1)$, W_{kl}, $W_{kl\|ij}$	transition rates
QED and QCD variables	
A^μ, A^μ_a	four-potential, color four-potential
$F^{\mu\nu}, F^{\mu\nu}_a$	field tensor, color field tensor
$\mathcal{E}$	longitudinal electric field F^{30}
$\boldsymbol{\mathcal{E}} = \left(\mathcal{E}^3, \mathcal{E}^8\right)$	"neutral" components of longitudinal chromoelectric field $\left(F^{30}_3, F^{30}_8\right)$
$\boldsymbol{\epsilon}_i$, $\boldsymbol{\eta}_{ij}$	color charges of quarks $(i = 1,2,3)$ and "charged" gluons $(i,j = 1,2,3; i \neq j)$
λ^a	Gell-Mann matrices
g, $\alpha_s = g^2/(4\pi)$	strong coupling constants
B	bag constant

Table 1.6 Symbols of the physical quantities used in the book (part 3).

thermodynamic variables	
N	multiplicity, baryon number
n	multiplicity, particle density, baryon number density
$T \quad (T_\mathrm{i}, T_\mathrm{f})$	temperature (initial temperature, final temperature)
$\mu \ (\mu_B)$	chemical potential (baryon chemical potential)
$P, \quad V$	pressure, volume
$E, \quad \varepsilon = E/V$	energy, energy density
$W = E + PV$	enthalpy
$w = W/N$	enthalpy per baryon
$\tilde{w} = W/V$	enthalpy density
S	entropy
$s = S/N$	entropy per baryon
$\sigma = S/V$	volume entropy density
c_s	sound velocity
$\lambda = c_\mathrm{s}^2$	sound velocity squared
T_H	Hagedorn limiting temperature
$\rho(m)$	hadron mass spectrum
g_π	degrees of freedom of the pion gas
g_qgp	degrees of freedom of the weakly-interacting quark-gluon plasma
	thermodynamic variables characterizing relativistic gas of
$n_\mathrm{cl}, P_\mathrm{cl}, ...$	classical massive particles (cl),
$n_\mathrm{b}, P_\mathrm{b}, ...$	massless bosons (b), and
$n_\mathrm{f}, P_\mathrm{f}, ...$	massless fermions (f)

hydrodynamic variables	
$u^\mu = (u^0, \mathbf{u}) = \gamma(1, \mathbf{v})$	fluid four-velocity
$\mathbf{v} = (v_x, v_y, v_z),$	fluid three-velocity
$\gamma = (1 - v^2)^{-1/2}$	gamma Lorentz factor
$v_\perp = \sqrt{v_x^2 + v_y^2}$	transverse flow
$\vartheta_\perp = \frac{1}{2} \ln \frac{1+v_\perp}{1-v_\perp}$	transverse fluid rapidity
$\vartheta = \frac{1}{2} \ln \frac{1+v}{1-v}$	fluid rapidity (note that this definition does not require that $\mathbf{v}$ has only the longitudinal component)
$\frac{d}{d\tau} = u^\mu \partial_\mu$	total time derivative
c_s	sound velocity
$\Phi(T)$	potential used in the Baym formalism
$\Phi_\mathrm{L}(t, z)$	potential used in the Landau formalism
$\chi(T, \vartheta)$	Khalatnikov potential
$Y = \ln(T/T_\mathrm{i})$	logarithm of temperature

Table 1.7 Symbols of the physical quantities used in the book (part 4).

terminology for interferometry	
$C(\mathbf{p}_1, \mathbf{p}_2)$	two-particle correlation function
$\rho(\mathbf{p}_1, \mathbf{p}_2)$	density matrix
$\mathbf{k} = \frac{1}{2}(\mathbf{p}_1 + \mathbf{p}_2)$	average three-momentum
$\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_2, \quad q = q_1 - q_2$	relative three- and four-momentum
$\boldsymbol{\beta}$	velocity of a pair
$\mathbf{q}^*, \quad \mathbf{r}^*$	relative three-momentum and relative distance in the pair rest frame
$i =$ out, side, long	
q_i	ith component of $\mathbf{q}$ in the out-side-long frame
$\tilde{q}_i$	relative four-momentum in the out-side-long frame for the case where only q_i is different from zero
R_i	HBT radii
special functions	
$\theta(x)$	Heaviside function (unit step function)
$\delta(x)$	Dirac delta function
$I_n(x)$	modified Bessel functions of the first kind
$K_n(x)$	modified Bessel functions of the second kind
$\Gamma(x)$	Euler gamma function
$\zeta(x)$	Riemann zeta function
$\mathrm{Li}_n(x)$	polylogarithm function
B_n^*	Bernoulli numbers

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